EXOTIC OPTIONS

Selected types

Analytical Finance I
Department of Mathematics and Physics
Mälardalen University
2006

This paper is part of a seminar project and contains introduction to selected types of exotic options that will be valuated using VBA/Excel in a class seminar Oct 12th.

Supervisor
Jan M. Röman

Students
Michael Wennermo
Piotr Liszewski
Tomasz Piotrowski

Introduction to Exotic options

Exotic options are an important feature of the over-counter derivatives market. Exotic refers to the non-standard character of this financial instrument. The majority of options is of American- or European style and follows similar patterns for valuation and hedging.

It happens that brokers bring together both sides of a contract and construct a product which does not exist as an exchange-traded option. One party may also market and sell the options to clients. From above it is easy to conclude that exotic options in many cases could have more suitable features which investors search for. For the writer of the exotic option it is possible to request higher margin for that kind of a tailor made protection or investment.

On the other hand, the relatively low liquidity of the exotic option market may sometimes cause a problem in buying or selling adequate numbers of options in order to hedge an investor's portfolio.

In order to create an exotic option, financial engineers modify some of the properties of common American and European puts and calls. The changes may include strike, expiration terms, type of settlement, and type of an underlying instrument or payout.

Generally speaking, an exotic option is an option that is not a vanilla put or call.

Many of the exotic options are path dependent which means that their payout at exercise or expiry depends, in some non trivial way, on the past history of the underlying asset price as well as its spot price at exercise or expiry. However, while a vanilla American call is path dependent, it is not considered exotic, whereas a European binary option is not path dependent but is considered exotic.

Here is an overview of some common types of exotic options. Some of them are discussed in detail later in the paper.

- ✓ Binary options
- ✓ Knock-out options
- ✓ Barrier options
- ✓ Lookback options
- ✓ Asian options
- ✓ Chooser options

- ✓ Options on two underlying variables
- ✓ Options on options
- ✓ Currency options
- ✓ Forward options

The number of different options has been growing rapidly and includes range forwards, ladders, exchange, two-colour rainbow, and cliquet, among others. Some of these (rainbow, exchange) depend on the values of many underlying assets, rather than a single underlying asset.

The next section will cover three different types of exotic options in greater detail, namely binary-, barrier- and Asian options.

Binary/Digital Options

This is a quite common type of exotic option. Sometimes, they are also referred to as all-or-nothing options or bet options.

There are two kinds of digital options which differ in terms of the type of settlement. Cash-or nothing digital option pays a fixed amount of cash if they expire in the money. Otherwise the payout is equal to zero. Similarly, asset-or nothing options give its holder the right to buy (if it is a call) or sell (in case of put) the underlying asset at a specific discount to the strike. They may have either European or American style of the exercise.
It is possible to imagine binary option as a bet on whether the underlying asset would be above (cash-or-nothing call) or below (cash-or-nothing put) an exercise price.
On whole, the most characteristic feature of binary options is that the value of the payout is determined at the beginning of the contract and does not depend on the magnitude by which the price of the underlying moves.

Black-Scholes formula can be adjusted for pricing digital cash-or nothing options:

$P_{call} = e^{-rT} KN(d)$ Where, $d = \frac{\ln\left(\frac{S}{K}\right) + (r - q + \frac{\sigma^2}{2}) \cdot T}{\sigma \cdot \sqrt{T}}$

$P_{put} = e^{-rT} KN(-d)$

r –risk free interest rate

K –strike

S – stock price

T - maturity time

σ - volatility

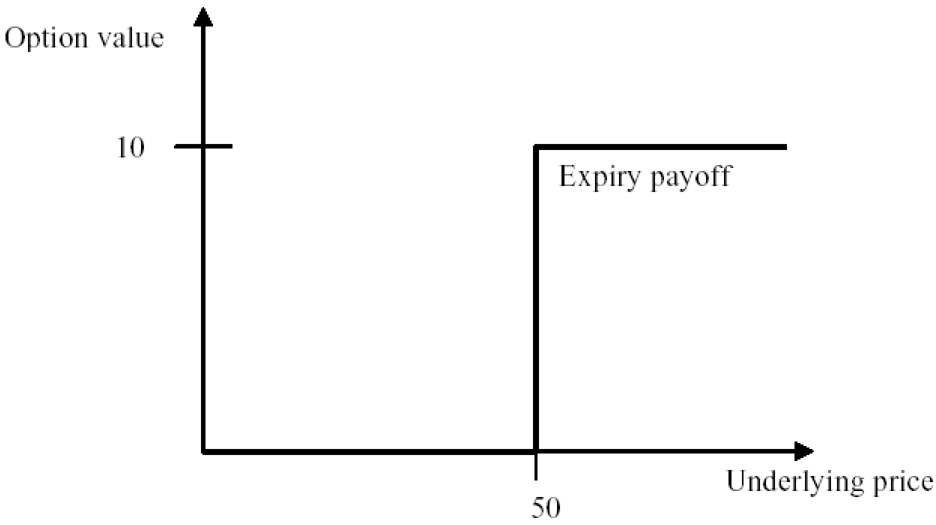
q- dividend rate

The price of the asset-or-nothing options is given by:

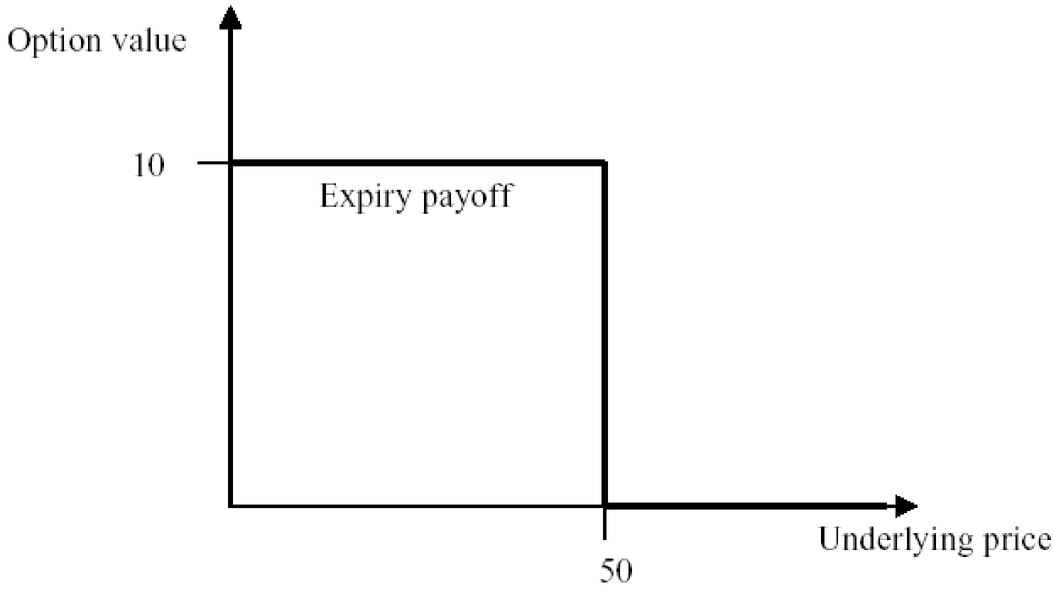
$$P_{call} = S_0 e^{-qT} N(d_1)$$
$$P_{put} = S_0 e^{-qT} N(-d_1)$$

Formulas for American type digital options were derived by Reiner and Rubenstein in 1991.

Here is the diagram of cash-or nothing call payout with strike 50 and cash payout 10



Payout for cash-or nothing put with strike 50 and cash payout 10



Considering digital options investors tend to be careful and limit their amount in portfolios. They also imply significant safety margins into their pricing in order to include additional risk. This is partially because that delta hedge in case of digitals is very difficult.

Barrier options

Barrier options are options depending not only on the price of underlying instrument with respect to strike but also on hitting or crossing certain threshold level. That's why we call them path-dependent options because the payoff is conditional – whether the barrier was touched/breached or not.

There are two general types of barrier contracts:

- Knock-in - when the options is void and activates when a barrier is reached/crossed
- Knock-out – when the option is active and becomes void when a barrier is reached/crossed

We can divide them into another subgroup with respect to direction of breaching the barrier:

- Up's – when the price starts below threshold and has to cross it when increasing
- Down's – when the price starts above threshold and has to cross it when decreasing

You have of course combinations of these kinds which give us following types:

- up-and-in – option becomes active when barrier is breached by increasing price
- up-and-out – options becomes void when barrier is breached by increasing price
- down-and-in – option becomes active when barrier is breached by decreasing price
- down-and-out - option becomes void when barrier is breached by decreasing price

Value of call down-and-in when H <= K is given by:

$$c_{\text{di}} = S_0 e^{-qT}(H/S_0)^{2\lambda} N(y) - Ke^{-rT}(H/S_0)^{2\lambda-2} N(y - \sigma\sqrt{T})$$

where

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

$$y = \frac{\ln[H^2/(S_0 K)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

Value of call down-and-out when H >= K is given by:

$$c_{\text{do}} = S_0 N(x_1)e^{-qT} - Ke^{-rT} N(x_1 - \sigma\sqrt{T}) - S_0 e^{-qT}(H/S_0)^{2\lambda} N(y_1) + Ke^{-rT}(H/S_0)^{2\lambda-2} N(y_1 - \sigma\sqrt{T})$$

and

$$c_{\text{di}} = c - c_{\text{do}}$$

where

$$x_1 = \frac{\ln(S_0/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \qquad y_1 = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

Value of call up-and-in when H <= K is given by:

$$c_{\text{ui}} = S_0 N(x_1)e^{-qT} - Ke^{-rT} N(x_1 - \sigma\sqrt{T}) - S_0 e^{-qT}(H/S_0)^{2\lambda}[N(-y) - N(-y_1)] + Ke^{-rT}(H/S_0)^{2\lambda-2}[N(-y + \sigma\sqrt{T}) - N(-y_1 + \sigma\sqrt{T})]$$

Value of put up-and-in when H >= K is given by:

$$p_{\text{ui}} = -S_0 e^{-qT}(H/S_0)^{2\lambda} N(-y) + Ke^{-rT}(H/S_0)^{2\lambda-2} N(-y + \sigma\sqrt{T})$$

Value of put down-and-in when H < K is given by:

$$p_{di} = -S_0 N(-x_1)e^{-qT} + Ke^{-rT}N(-x_1 + \sigma\sqrt{T}) + S_0 e^{-qT}(H/S_0)^{2\lambda}[N(y) - N(y_1)] - Ke^{-rT}(H/S_0)^{2\lambda-2}[N(y - \sigma\sqrt{T}) - N(y_1 - \sigma\sqrt{T})]$$

When we talk about barrier options we should mention rebates which are predetermined sums of money paid to recompense the possible loss, for example in the case of knock-out's rebates can be paid when the option becomes void (so when underlying asset breaches the threshold) and it can compensate (of course, not fully) the lost opportunity and price already paid. On the other hand when we deal with knock-in's a rebate can be provided when the option has never activated (it has never breached the threshold).

Asian Options

Asian options are one of the most commonly traded non-vanilla options on the over-the-counter market. The main feature of this option is the use of average stock prices and it is commonly referred to as the average option.

The main reason exotics developed was to suit certain purposes where plain options seemed insufficient. The average option was first introduced by a Banker's Trust company in Japan, which used a pricing average for oil contracts and hence the name "Asian" options.

The benefits with this kind of option are that the price of the option itself is much cheaper than European options in general and American options in particular. An Asian option is also less volatile than the underlying stock price because it is based on average prices. In other words it is less risky and thus popular on the currency- and commodity market. The negative side is when there is a major increase in stock price, resulting in a large money making situation for a call option, the payout is averaged out. It may initially come off as easier to calculate although, just like exotics in general, this is far from the truth.

Asian options can divided into different categories. It may be an average price call- or an average price put option, or an average strike call or put and the averages may be calculated differently. Thus, before an option contract is written a few terms must be agreed upon:

- *What kind* of option? Average strike or price and call or put.
- *What is the period of averaging*? The entire time until maturity or parts of it.
- *What are the samples in the average*? Price at the end of the months, weeks or days.
- *How do we measure the average*? Arithmetic, geometric or a weighted average.

An average price call has a payoff of *max(0, S(ave) – K)* while the average price put is payoff is calculated *max(0, K - S(ave)).* As an example we can think of an American company which will receive a large amount of cash from the company's British office. The average price options are first of all much cheaper and with a long position in a put option the company can guarantee themselves that the average exchange rate between the pound and the dollar is above some level.

An average strike option call has payoff *max(0, S(T) - S(ave))* and the same for put has a payoff *max(0, S(ave) – S(T)).* Although not as common as average price options, a long position in an average strike call will for example, guarantee that the average price for an asset is at least not larger than the final price at maturity. If it is larger, then we can use the right to execute the option.

There are several ways to calculate the average of the underlying variable. In most cases Asian options are based on an arithmetic average of the underlying price. The problem with this averaging method is that no analytical formulas are available. The reason is that although the stock price is lognormally distributed, the arithmetic average of these variables are not. Arithmetic averages are calculated as follows:

$$S_{(average)} = (S_1 + S_{2,}\ldots + S_n) / n$$

On the other hand the distribution is almost lognormal and therefore there exist fairly good estimates that are used to calculate Asian price options. On of the these approximations are made by S.M. Turnbull and L.M. Wakeman to calculate European style Asian/average price options. Also Levy has calculated an arithmetic approximation with option prices very similar to those of Turnbull and Wakeman.

Kemna and Vorst created an analytical solution based on geometrical averaging in 1990. The geometrical average of the underlying variables is calculated as follows:

$S_{(average)} = n^{th}\sqrt{}(S_1 S_{2,} \ldots S_n)$

The formula is a geometrical closed form solution to pricing of Asian options with a European style maturity date. Kemna and Vorst's solution works because the geometrical average of the underlying prices have a lognormal distribution. The formula is:

Call = $Se^{(b-r)(T-t)}N(d_1)$ – Ke-r(T-t)$N(d_2)$

Put = $Se^{(b-r)(T-t)}N(d_1)$ – Ke-r(T-t)$N(d_2)$

In a risk neutral world, the distribution of the geometrical average of an underlying price is the same as the asset price itself at the end of maturity if the growth factor is changed from (r) to $(r - \sigma^2/6)/2$. Also the volatility must be changed from sigma to an adjusted volatility $\sigma/\sqrt{(3)}$. Thus the variables in the analytical formula, with a dividend yield (d) included, are as follows:

$$\boldsymbol{\sigma_{(a)}} = \sigma / \sqrt{(3)}$$

$$\mathbf{b} = ½ (r - d - \sigma^2/6)$$

$$\mathbf{d_1} = \ln(K/S) + (b - ½\, \sigma_{(a)}{}^2)T$$

$$\mathbf{d_2} = d_1 - \sigma_{(a)}\sqrt{T}$$

References:

1. http://www.global-derivatives.com
2. http://janromandhis.org
3. http://www.investopedia.com
4. http://www.riskglossary.com
5. Math - Cambridge The Mathematics Of Financial Derivatives
6. John Hull – Option, Futures and Other Derivatives