# Vasicek Model

Interest rate process:

$$dr = (b - ar)dt + \sigma dz$$

Affine Term Structure

$$P(t,T) = e^{A(t,T) - B(t,T) r_0}$$

In some literature the Affine bond prices are written as

$$P(t,T) = A(t,T) e^{-B(t,T) r_0}$$

b/a: long-term equilibrium of mean reverting spot rate process

In the vasicek model we have A(t,T); B(t,T); like this.

$$A(t,T) = \exp\left[ (B(t,T) - T + t)\left( \frac{ba - \sigma^2/2}{a^2} \right) - \frac{\sigma^2 B(t,T)^2}{4a} \right]$$

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$

Value of zero coupon bond price can also be written as:

$$p(0,T) = \exp\left\{ \frac{1}{a}\left[1 - e^{-a(T-t)}\right]\left( \frac{b}{a} - \frac{1}{2}\left(\frac{\sigma}{a}\right)^2 - r(0) \right) - (T-t)\left[ \frac{b}{a} - \frac{1}{2}\left(\frac{\sigma}{a}\right)^2 \right] - \frac{\sigma^2}{4a^3}\left[1 - e^{-a(T-t)}\right]^2 \right\}$$

# Graph Vasicek model:

## steady state probability density function for spot-rate r.

**P is normally distributed with**

$$P \sim N\left(\frac{b}{a}, \frac{\sigma}{\sqrt{2a}}\right)$$

We can also write as follows:

$$p = \sqrt{\frac{a}{\pi}}\,\frac{1}{\sigma}\,e^{-\frac{a(r_0-b)^2}{\sigma^2}}$$

**Vasicek volatility of zero rate:**

$$\sigma_{Y(t,T)} = \sigma\frac{1-e^{-a(T-t)}}{a(T-t)} = \sigma\frac{B(t,T)}{(T-t)}$$

## Vasicek Discount function

$$P(t,T) = A(t,T)e^{-B(t,T)r_0}$$

## Vasicek Term Structure Interest

**Infinitely-long Rate:**

$$Y = b - \frac{\sigma^2}{2a^2}$$

**Vasicek zero rate:**

$$r = \frac{\ln \dfrac{1}{A(t,T)e^{-B(t,T)r_0}}}{(T-t)}$$