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Kim Model

An Analytic Valuation of American Options

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Abstract

Due to the possibility of the early exercise, the option price and the optimal exercise boundary of American options are difficult to be determined. One of the American option valuations, developed by In Joon Kim in 1990, is analyzed economically and mathematically in this paper.

Based on mathematical and financial theories, the calculation method of Kim model is evaluated and compared with the other American option pricing models, especially the Binomial Model. Moreover, Kim model is programmed in Excel VBA to obtain the option prices. And the comparison of the option prices given by Kim model and the other American options pricing models has been conducted to exam the accuracy of the Kim model. The results of the contrast proved that Kim model provides a relatively accurate and fast way to compute American option prices.

Acknowledgement

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Table of contents

1. Introduction	6
2. European Options VS American Options	6
2.1 Assumptions and Notation	7
2.2 Factors affecting option price	8
2.3 Boundary for option prices	10
2.4 Optimal exercise	12
2.4.1 Options on a non-dividend-paying stock.....	12
2.4.2 Options on a dividend-paying stock.....	12
2.5 Pricing models for options	13
2.5.1 European option pricing models	14
2.5.2 American option pricing models	15
3. Kim model.....	16
3.1 Assumptions and Notations	16
3.2 The valuation of American Calls	17
3.2.1 Formulas of American calls	18
3.2.2 Two special cases for calls	19
3.3 The optimal exercise boundary.....	19
3.4 A valuation for American Puts.....	20
3.4.1 Formulas of American Puts	21
3.4.2 Two special cases for puts	21
3.5 Summary	22
4. Application in Excel VBA	22
4.1 Limitations:	23
4.3 Method in VBA	23
4.3 Results	25
5. Comparison	25
5.1 Kim model V.S. Binomial model.....	25
5.1.1 Algorithm	26
5.1.2 Computing results	26
5.1.3 Computing time.....	28
5.2 Kim Model V.S. Other models	28
6. Conclusion	29
7. References	30
8. Appendixes.....	31
Appendix 1	31
Appendix 2	32
Appendix 3	34

List of Figures

<i>Figure 1</i> A simple binomial tree.....	15
<i>Figure 2</i> The optimal exercise boundary region for American calls.....	20
<i>Figure 3</i> Convergence of Cox-Ross-Rubenstein BS RE.....	27

List of Tables

<i>Table 1</i> Notations.....	8
<i>Table 2</i> Summary of the effect on the price of a stock option of increasing one variable while keeping all other fixed.....	9
<i>Table 3</i> Boundary for option prices.....	11
<i>Table 4</i> Calculated values of American options when n increases doubly.....	25
<i>Table 5</i> Outputs comparison of Kim model and Binomial model.....	27
<i>Table 6</i> Outputs of American options.....	29

1. Introduction

There are several ways to calculate the American option price; however, there is no exact analytic solution exists for the valuation of American options written on the underlying assets pays continuous dividends for which early exercise may be optimal. Kim model¹ is the valuation for American option, which was based on the problem that the underlying assets pays continuous dividend. The key point is that Kim model has developed an implicit form of the integral equation for the optimal exercise boundary.

In order to understand Kim model, making a comparison between it and some common used American option pricing models is necessary. Binomial model is chosen to be the main comparator to compare with Kim model from the three aspects, numerical algorithm, computing results and computing time. And some other models for American options are also brought to contrast with Kim model. All the calculations of the option pricing models are applied in Excel VBA.

The remaining paper of proceeds as follows: section 2, the general concept of the American options will be given; introduction of Kim model specifically in the section 3; section 4, Kim model is applied by Excel VBA; section 5, the comparison of Kim model with some other models is made; the conclusion is presented in section 6 of the thesis.

2. European Options VS American Options

An option is the right either to buy or to sell a specified amount or value of particular underlying assets at a fixed exercise by exercising the option before or at its specified

¹ Kim model: the analytic valuation of American options developed by In Joon Kim, 1990.

expiration date.² There are two types of options, call option, which give the holder the right to buy at a specified date for a certain price; put option, which gives the holder the right to sell at a specified date for a certain price.

The European options and American options are the most common options in the financial market, which is not related with their locations. The most obvious difference between American options and European options is the choice of exercising date. A European option only can be exercised at the expiration date, but American option can be exercised at any time before (or at) its expiration date. Due to the flexibility, pricing an American option has always been placed in a more complicated case than pricing European option has. However, the property that the American option can be early exercised often gives it a higher price than European options.

Let us look into the European and American options, first, knowing the assumption and notation, and getting an overview of the factors affecting the option price; then understanding the boundary of option price and optimal exercise; last, getting to know some models for option pricing.

2.1 Assumptions and Notation

For both European and American types of options, they assume that there are no transactions costs for all the market participants; all trading profits are subject to have the same tax rate; borrowing and lending are possible at the risk-free interest rate; and there are no arbitrage opportunities.

In the Table 1, it shows the notations will be used in the American and European options. And these notations will also be applied in the following parts of the paper.

² ORC quant guide 71 p 7

Table 1 Notations

S_0	Current stock price
K	Stick price of option
T	Time to expiration of option, $T \geq 0$
S_T	Stock price on the expiration date
r^3	Continuously compounded risk-free rate of interest for an investment maturing in time, $r > 0$
C	Value of American call option
P	Value of American put option
c	Value of European call option
p	Value of European put option
σ	Volatility
D	Amount of future dividends
δ	Continuous dividend rate

2.2 Factors affecting option price

Among those factors is mentioned in previous section, there are six factors affecting the price of a stock option⁴. Table 2⁵ gives a general overview of the factors negative or positive influence for the option price.

Stock Price and Strike Price

For a call option, the payoff will be $S - K$, therefore call option become more valuable as S increases and less valuable as the K increase for both European and American call option. However, the payoff of a put option is equal to $K - S$; it is obvious that the put option become less valuable as S increases and more valuable as K increase.

³ It is assumed that $r > 0$ in this paper. The other situation of r will not be considered.

⁴ The options we discussed in this paper are not only the stock option, but almost all kinds of the options. Here stock option is mentioned because it is easy to explain the factors.

⁵ John C .Hull, 2006, *Option ,future and other derivatives*, 6th ed., p206

Perhaps in the real world, other variables should be considered in the situation, but the basically concept of the call option and put option will be always the same.

Table 2 Summary of the effect on the price of a stock option of increasing one variable while keeping all other fixed.

Variable	European call	European put	American call	American put
S	+	-	+	-
K	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
r	+	-	+	-
D	-	+	-	+

“+” indicates that increase the variable causes the option price go up; “-” indicates that the increase variable cause the option price go down. “?” indicate that the affect on the option price is not clear when the variable increase.

Time to Expiration

The time to maturity T has different effect on European and American option price. An American option can worth more as the time to maturity increases. Because it can exercises at any time during lifetime, the longer its lifetime is, the more opportunities will exit to better off the option holder’s profits.

Volatility

The volatility is a measure of how uncertain the future price movement of the underlying asset. As volatility increases, the uncertainty of price increases, and then the value of both calls and puts increase. The accurate value of the volatility is not easy to find out, usually, the volatility value is estimated from historical data of underlying price. For stocks, the volatility is between 15% and 60% typically.

Risk-free Interest Rate

Price of an option is obviously affected by the risk-free interest rate, because the present value of any future cash flow and the expected value will change due to the fluctuation of risk-free interest rate. The net effect of an interest rate increase and the accompanying stock price decrease can be to decrease the value of a call option and increase the value of a put option, and vice versa. Usually, the risk-free interest rate is 5% in calculation. It will be changed a little bit and to above or below 5% for fitting the various real world situation.

Dividends

For the Dividend “D”, it has the effect of reducing the stock price on the ex-dividend date. Therefore, the value of a call option is negatively related to the size of an anticipated future dividend; and the value of a put is in the opposite way, the larger size of the future dividend, the more value of a put.

However, when the amount of future dividends is unknown, instead of the continuous dividend “ δ ”, the situation is more complicated, especially for the American options. The amount of continuous dividend will grow as the time goes by, the value of American calls and puts may have big difference at different time node. “When is the optimal time to exercise the American options with or without dividend?” is a pretty interesting and tough question which it will be discuss in the following part of the paper.

2.3 Boundary for option prices

Boundary is a set of restriction for option prices; there are upper and lower bounds for both European and American options. If the option prices excess these bounds, the holder of the option can make risk-less profits by arbitraging. However, the boundaries for the European and American option are slightly different.

Table 3 Boundary for option prices⁶

	Call options	Put options
Upper bounds	$0 \leq c \leq C \leq S_0$	$0 \leq p \leq P \leq K$
Lower bounds without “D”	$c \geq S_0 - Ke^{-rt}$	$p \geq Ke^{-rt} - S_0$
Lower bounds with “D”	$c \geq S_0 - Ke^{-rt} - D$	$p \geq D + Ke^{-rt} - S_0$

The upper bounds for options: No matter what happens, the call option can never be worth more than the underlying assets; the put option can never be worth more than the strike price. The American option is always worth more than European option.

The lower bounds for options: European options with or without the amount of future dividend will not be the same, dividend should be subtracted from the boundary of a call and plus to the boundary of a put.

Put- call parity: a famous relationship between European call and put , see the equation below:

$$c + Ke^{-rt} = p + S_0$$

For American option, the put-call parity is not true, due to its possibility for early exercise. However, the following inequalities shows the boundary of American options,

$$S_0 - D - K \leq C - P \leq S_0 - Ke^{-rt}$$

Mathematically, changing the value of the variables in the formula of boundary will definitely change the value of calls or puts. Boundary is a clue to find out if there is arbitrage or not.

⁶ The formulas is from the book John C .Hull, 2008,*Option ,future and other derivatives*,7th ed., p205-215.

2.4 Optimal exercise

Compare to European options, the situation for American options are more complicated. There are many factors need to be considered before an early exercise decision. The key issue of the American options is to find out the optimal time to exercise based on the limit information. That has been discussed in two cases, without and with dividends.

2.4.1 Options on a non-dividend-paying stock

Firstly, for calls⁷, if the underlying asset does not pay dividend, it is never optimal to early exercise the American call option before its maturity on the stock. To demonstrate that, here present two points: 1. the call option provides insurance against stock price decline below strike price; 2. according to the theory of time value of money, it is better to pay out of the strike price later than earlier; otherwise the holder will lose the time value on the money the holder could achieve.

However, when come to American puts, the argument above vanishes. The holder of an American put can be better off when early exercised. Similarly, there are two reasons: 1. the puts insure the holder against any falls, but in some cases break down the insurance can lead a better situation; 2. from a perspective of a put holder, the money obtained now will be more worthy than obtained later.

2.4.2 Options on a dividend-paying stock

When dealing with the American options on a stock that pay dividend, the owner of stock on the ex-dividend date⁸ will be awarded the payment. After the ex-date⁹ has

⁷ Assume that $r, t > 0$

⁸ It is defined as the first date following the declaration of a dividend on which a stock buyer is not entitled to receive the next dividend.

⁹ Ex-dividend date

been declared, the stock value will decline by the amount of the dividend paid out.

For an American call holder, it may be optimal to early exercise the option on stocks with dividends. If the future dividend amount is higher than the time value remaining in the call's price, it might make sense to exercise the option; however, this strategy should be done prior to the ex-dividend date. Otherwise, after the ex-date, the stock value will fall below some certain level which makes the early exercise no longer profitable.

Opposed to calls, an approaching ex-dividend date is a deterrent against early exercise for puts. Mostly, a put holder won't choose early exercise before ex-date. By early exercising the put, the profits the owner receives now may be not enough to cover the dividend payments later, which lead to losses.

Analytic solutions can be derived relatively simple for the American options with discrete dividends; however, facing the continuous dividends, the simplification does not exist, because there is always the possibility of early exercise. Kim model provide analytic valuation of American option with continuous dividend, which is presented in theoretically and mathematically in section 4.

2.5 Pricing models for options

The value of an option can be estimated using a variety of quantitative techniques and implemented using a variety of numerical techniques. The European option pricing model is clear and understandable because European options have the exact date to exercise. On the other hand, American option pricing models are complicated, so that there are several ways for pricing the options according to the different limitations and the approximation method.

2.5.1 European option pricing models

Black-Scholes pricing model and Binomial model are the most important methods and foundations for the existing financial market in which the result is within the reasonable range. Both of them are based on the similar theoretical foundations and assumptions, such as the geometric Brownian motion theory of stock price and risk-neutral valuation.¹⁰ However, there are still some differences between these two models.

Black-Scholes

When B-S¹¹ is applied to calculate an option price, even though it ignoring dividends paid during the life of the option, it has a closed form solution. In the actual option trading, this model is clumsy because of its assumptions of dividend, interest rate, volatility. However, B-S model is still the foundations for the valuation of option price.

Binomial Model

The binomial option pricing model is often a superior alternative to B-S model. In this model, the value of the underlying asset is discrete, it moves either up or down to a given value in a time interval. It traces the evolution of the option's key underlying variables in discrete-time¹² by constructing a binomial tree. However, binomial option pricing models is non closed-form solutions.

In general, there are three steps for model¹³:

Step 1: Create the binomial price tree

Step 2: find the Option value at each final node

$$\text{Max}[(S_n - K), 0], \text{ for a call option}$$

¹⁰ <http://www.hoadley.net/options/bs.htm>

¹¹ The formulas of B-S in Appendix 1

¹² If Binomial model provides a discrete time approximation to the continuous, it process underlying the B-S model. In other words, the binomial model value converges on the B-S formula value as the number of time steps increases for European option without dividends.

¹³ There is an example for pricing European option by binomial model in Appendix 2

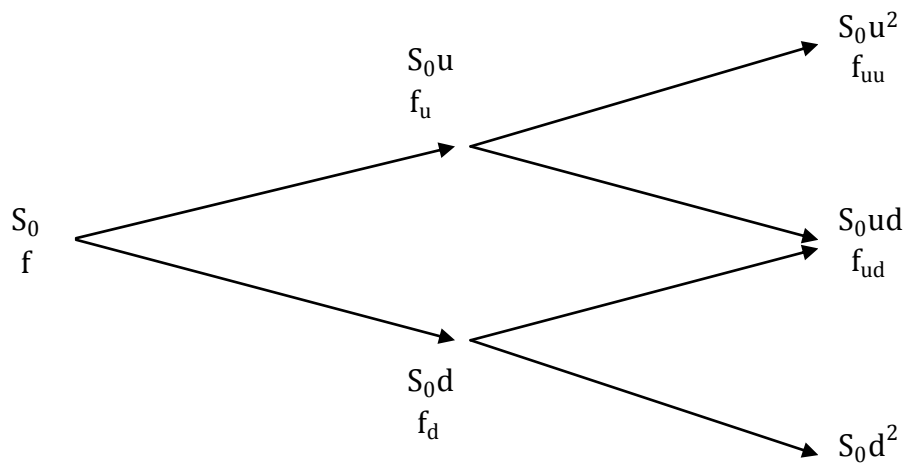
$$\text{Max}[(K - S_n), 0], \text{ for a put option}$$

S_n : is the spot price of the underlying asset at the nth period.

Step 3: Find option value at earlier nodes

The binomial tree is has two time intervals in the figure 1, during each time step, it either moves up to u times its initial value S_0 or moves down to d times its initial value. Each node has its own value of option f that can be calculate by the binominal formula.¹⁴

Figure 1A simple binomial tree



Binomial model is suit for both American and European options, but some difference exists certainly. The binomial value applies in all the nodes in European option; however, it is not in the case when binomial model is applied for American options on account of premature exercise.

2.5.2 American option pricing models

The valuation of American options has drawn a lot of attention since options are traded on organized exchanges are the American type. The pricing of American options using

¹⁴ $f = e^{-r\Delta t}[pf_u + (1 - p)f_d]$, here is a general formula to show how the binomial model works, the details of the Binomial model is in Appendix 2.

analytic models is more difficult than of European options. As is mentioned in section 2.5.1, the Binomial model is also can use to price the American option. However, since the option may either be held or exercised prior to expiry, the value at each node is: $\text{Max}(\text{Binomial Value}, \text{Exercise Value})^{15}$.

There three models for American options are the most widely used models: Roll-Geske- Whaley analytic solution, Black's approximation method and Barone – Adesi – Whaley quadratic approximation. Besides, Bjerklund – Stensland and Kim model are also the models for pricing American options.

In fact, most of the pricing models for American option are essentially a European option with adjusted for an early exercise premium. The difference is the various numerical ways to evaluate the early exercise premium under different assumptions. Due to the complexity, not all the models will be presented in the paper, but Kim model, the unique way to pricing the American option, is going to be introduced in next section. Some American option models will be applied in VBA in part 4 to observe the difference between those results.

3. Kim model

Kim Model is an analytic valuation of American options, it focus on the American options written on assets that pay continuous dividend. The model was articulated by In Joon Kim in his 1990 paper, "The Analytic Valuation of American Options".

3.1 Assumptions and Notations

Assumptions

It assumes that: markets are perfect; trading occurs continuously; interest rate r is

¹⁵ There is a example for pricing American put option by binomial model in Appendix 2

constant; American call written on assets that pay continuous proportional dividend at a rate of δ ; the assets price follows a lognormal diffusion process with volatility parameter σ ; the assets price S_t is satisfying $dS_t = \mu S_t dt + \sigma S_t dz_t$, and μ : the expected rate of return of the asset; z_t is one-dimensional standard Brownian motion defined on a suitable probability space; formulate the American put valuation problem as a free-boundary problem.

Notations

Some notation is mentioned and discussed in section 2.2 that will stay the same meaning in this section; here are the additional notations and their representing shown:

τ : the time to expiration

$G(\cdot)$: the optimal boundary for American calls

$B(\cdot)$: the optimal boundary for American puts

$C(S, \tau)$: the value of American call at $t = T - \tau$,

Domain: $\{(S, \tau); 0 < S \leq \infty, 0 < \tau \leq T\}$

$P(S, \tau)$: the value of American put at $t = T - \tau$

Domain: $\{(S, \tau); B(\cdot) < S \leq \infty, 0 < \tau \leq T\}$

$V(S, \tau; G(\cdot))$: the value of the call option

$W(S, \tau; B(\cdot))$: the value of the put option

$N(\cdot)$ is the unit normal distribution function

3.2 The valuation of American Calls

The value of an American call is shown to consist of two components: the value of a comparable European call and an early exercise premium.¹⁶

¹⁶ In Joon Kim, 1990, "The Analytic Valuation of American Options," The Review of Financial Studies, Vol.3, No.4, p549

Here denote the value of a live American call¹⁷ for the optimal exercise boundary $G(\cdot)$ by $V(S, \tau; G(\cdot))$ that is differentiable with respect to τ and twice – differentiable with respect to S with the domain $\{(S, \tau); 0 < S \leq G(\cdot), 0 < \tau \leq T\}$.

3.2.1 Formulas of American calls

$$EEP_{\text{call}} = \int_0^{\tau} [\delta S e^{-\delta(\tau-s)} N(d_1(S, \tau - s; G(s))) - r K e^{-r(\tau-s)} N(d_2(S, \tau - s; G(s)))] ds$$

$$V(S, \tau; G(\cdot)) = c(S, \tau) + EEP_{\text{call}} \quad (1)$$

EEP_{call} is the early exercise premium of American call option.

The optimal exercise boundary $G(\cdot)$ is implicitly defined by the following intergral equation:

$$G(s) - K = c(G(s), s) + \int_0^{\tau} [\delta S e^{-\delta(s-\varepsilon)} N(d_1(G(s), s - \varepsilon; G(\varepsilon))) - r K e^{-r(s-\varepsilon)} N(d_2(G(s), s - \varepsilon; G(\varepsilon)))] d\varepsilon \quad (2)$$

The explanations and equations above show that Kim model has two steps to compute the value of American calls. First, using equation (2) for solving the optimal exercise boundary $G(\cdot)$, then equation (1) is solved call option value by the given boundary.

To notice that: If the dividend yield is zero, equation (1) is reduced to the B-S European call formula. It proves that American call option is equivalent to European option without dividend, and American call option will never optimal to exercise early. If the dividend yield is positive, the early exercise premium would be positive even when the interest rate is zero.

¹⁷ When the call is still in the money and still has not been exercised.

3.2.2 Two special cases for calls

1. When both $r = 0, \delta = 0$, then $EEP_{\text{call}} = 0$, which means $V(S, \tau; G(\cdot)) = c(S, \tau)^{18}$, $G(\cdot) = \infty^{19}$, so there is no early exercise would occur.

2. When the call option is perpetual American calls: the critical asset price for perpetual American calls is a time- invariant constant. the equations²⁰ below which are reduced form call option equations(1) and (2):

$$G(\infty) = \frac{\beta K}{\beta - 1}, \quad \text{where } \beta = \frac{-\left(r - \delta - \frac{1}{2}\sigma^2\right) + \sqrt{\left(r - \delta - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2}$$

$$V(S; G(\infty)) = \frac{K}{1-\beta} ((\beta - 1)S/\beta K)^\beta$$

$G(0)$ represents the lower bound, and $G(\infty)$ represents the upper bound.

3.3 The optimal exercise boundary

Early exercise occurs if benefits outweigh the cost associated with early exercise, otherwise it won't happen, and the holder will continue to hold on to the option. However, the price of the underlying assets is changing as the time goes by. A boundary can be found to measure if it is optimal to early exercise, namely, the optimal exercise boundary that is with exercise region (benefits > costs) and free region (benefits < costs).

For calls, the cost associated with early exercise is that one can forgo the interests earned on the exercise price, and may regret early exercise if the asset price decreases substantially later on²¹. The optimal exercise boundary $G(\cdot)$ in Kim model for calls,

¹⁸ Calculated from formula (1)

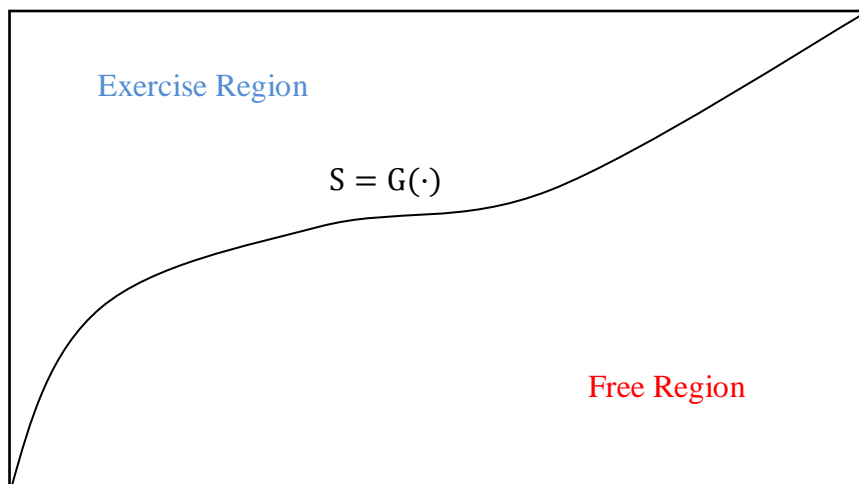
¹⁹ Calculated from formula (2), the value of a European call on a non-dividend paying asset never equals immediate exercise value for a finite asset price.

²⁰ In Joon Kim, 1990, "The Analytic Valuation of American Options," The Review of Financial Studies, Vol.3, No.4, p555-556

²¹ In Joon Kim, 1990, "The Analytic Valuation of American Options," The Review of Financial Studies, Vol.3, No.4, p558

see figure 2.

Figure 2 The optimal exercise boundary region for American calls



The optimal exercise boundary $G(\cdot)$ for an American call has the following properties.²²

Proposition 1: if the asset price follows a stochastic process with continuous sample paths and the interest rate is a positive constant, then $G(\tau)$ is continuous and nondecreasing in τ .

Proposition 2: If the asset price follows a lognormal diffusion process and the interest rate is a positive constant, then

$$\lim_{s \rightarrow 0} G(s) = G(0) = K, \text{ if } \delta \geq r,$$

$$\lim_{s \rightarrow 0} G(s) = G(0) = \left(\frac{r}{\delta}\right)K, \text{ if } \delta < r.$$

Proposition 3: if the interest rate is less than or equal to the dividend yield, an American call which is optimally held to expiration will have a value of zero at T.

3.4 A valuation for American Puts

The valuation of American put options proceeds in the same way as the valuation of

²² In Joon Kim, 1990, "The Analytic Valuation of American Options," The Review of Financial Studies, Vol.3, No.4, p556-559

American calls. As the American calls, we formulate the American put valuation problem as a free-boundary problem.

3.4.1 Formulas of American Puts

$$EEP_{\text{put}} = \int_0^\tau [rKe^{-r(\tau-s)}N(-d_2(S, \tau - s; B(s)) - \delta Se^{-\delta(\tau-s)}N(-d_1(S, \tau - s; B(s))) - \delta B(s)e^{-\delta(\tau-s)}N(-d_1(S, \tau - s; B(s)))] ds \quad (3)$$

$$W(S, \tau; B(\cdot)) = p(S, \tau) + EEP_{\text{put}} \quad (4)$$

The optimal exercise boundary $B(\cdot)$ is implicitly defined by

$$K - B(s) = p(B(s), s) + \int_0^s [rKe^{-r(s-\varepsilon)}N(-d_2(B(s), s - \varepsilon; B(\varepsilon)) - \delta B(s)e^{-\delta(s-\varepsilon)}N(-d_1(B(s), s - \varepsilon; B(\varepsilon)))] d\varepsilon$$

EEP_{put} is the early exercise premium of American call option

The optimal exercise boundary at expiration is given by:

$$\lim_{s \rightarrow 0} B(s) = B(0) = K, \text{ if } \delta \leq r,$$

$$\lim_{s \rightarrow 0} B(s) = B(0) = \left(\frac{r}{\delta}\right)K, \text{ if } \delta > r.$$

3.4.2 Two special cases for puts

1. When interest rate $r = 0$, then $EEP_{\text{call}} = 0$, which means $V(S, \tau; G(\cdot)) = c(S, \tau)^{23}, G(\cdot) = \infty^{24}$, then there is no early exercise would occur.

²³ Calculated from formula (1)

²⁴ Calculated from formula (2), the value of a European call on a non-dividend paying asset never equals immediate exercise value for a finite asset price.

2. When the put option is perpetual American calls: the critical asset price for perpetual American calls is a time- invariant constant. Let's see the equations²⁵ below which are reduced form call option equations(1) and (2):

$$B(\infty) = \frac{\theta K}{\theta - 1}, \quad \text{where } \theta = \frac{-(r - \delta - \frac{1}{2}\sigma^2) - \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}$$

$$W(S; B(\infty)) = \frac{K}{1 - \theta} ((\theta - 1)s/\theta K)^\theta$$

$B(\infty)$ represents the lower bound; $B(\cdot)$ represents the upper bound

3.5 Summary

Kim model is a valuation of American option price by adding the early exercise premium of the European option value. In the model, early exercise premium is derived from the optimal exercise boundary, and the optimal exercise boundary is calculated in an implicit form of integral equation. This algorithm has been applied in Excel VBA in next section.

4. Application in Excel VBA

VBA (Visual Basic for Applications) is an implementation built into Microsoft Office applications. It enables user to build own defined functions and automate processes. VBA (in Excel) has been chosen as application software in this thesis because the other sources used to be compared with Kim model-calculator are all presented in Excel VBA form.

²⁵ In Joon Kim, 1990, "The Analytic Valuation of American Options," The Review of Financial Studies, Vol.3, No.4, p555-556

4.1 Limitations:

Due to the fact of complexity of Kim model, some limitations are made before programming Kim model in Excel VBA:

1. The special cases mentioned in 3.3.2 and 3.4.2 is not considered.
2. Calls and puts with dividend are not considered. Because the calculators of other models in VBA from the source are without dividend, it is easy to compare the results of put option by using different models with the same situation. And results of call option without dividend will be the same as value given by B-S formulas.

4.3 Method in VBA

The important point to evaluate the American option use Kim model is to determine the optimal boundary. Once the optimal exercise boundary has been decided, an accurate value of American options can be calculated. Calls without dividend in Kim model is the same as B-S model that is applied in Excel VBA. Puts without dividend in Kim model is applied in Excel VBA using the following formulas.

The implicit form of the integral equation for optimal exercise boundary $B(s)$:

$$K - B(s) = p(B(s), s) + \int_0^s [rKe^{-r(s-\varepsilon)}N(-d_2(B(s), s - \varepsilon; B(\varepsilon))) - \delta B(s)e^{-\delta(s-\varepsilon)}N(-d_1(B(s), s - \varepsilon; B(\varepsilon)))]d\varepsilon$$

When $\delta = 0$,

$$K - B(\tau) = p(B(\tau), \tau) + \int_0^\tau rKe^{-r(\tau-\varepsilon)}N(-d_2(B(\tau), \tau - \varepsilon; B(\varepsilon)))d\varepsilon \quad (5)$$

The B-S pricing formula for European put option:

$$p(S, \tau) = Ke^{-r\tau}N(-d_2(S, \tau; K)) - SN(-d_1(S, \tau; K))$$

When $S = B(\tau)$, we get

$$p(B(\tau), \tau) = Ke^{-r\tau}N(-d_2(B(\tau), \tau; K)) - SN(-d_1(B(\tau), \tau; K)) \quad (7)$$

Put equation (7) to (5), we can obtain

$$B(\tau) = K \frac{e^{-r\tau}N(-d_2(S, \tau; K)) - 1 + \int_0^\tau rKe^{-r(\tau-\varepsilon)}N(-d_2(B(\tau), \tau - \varepsilon; B(\varepsilon)))d\varepsilon}{N(-d_1(B(\tau), \tau; K)) - 1} \quad (8)$$

Then a function of $B(\cdot)$ can be get from move the right hand side of equation (8) to the left.

$$\begin{aligned} & F(B) \\ & = B(\tau) \\ & - K \frac{e^{-r\tau}N(-d_2(S, \tau; K)) - 1 + \int_0^\tau rKe^{-r(\tau-\varepsilon)}N(-d_2(B(\tau), \tau - \varepsilon; B(\varepsilon)))d\varepsilon}{N(-d_1(B(\tau), \tau; K)) - 1} \quad (9) \end{aligned}$$

Now the optimal exercise boundary can be determined by solving the function (9).

The integrals in Kim model are solved by using Composite Simpson's rule²⁶. In numerical analysis, the Simpson's rule is a numerical approximation for definite integrals, it is approximation formula:

$$\int_a^b f(x)dx \approx \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

In Composite Simpson's rule, the time interval $[a, b]$ is spilt up in "n" subintervals. Simpson's rule is applied to each subinterval, with the results being summed to produce an approximation for integral over entire interval.

The calculation method of Kim model is programmed in Excel VBA, file "Calculator of Kim model.xlsm". All the VBA codes are in Appendix 3.

²⁶ http://en.wikipedia.org/wiki/Simpson's_rule

4.3 Results

With the parameter²⁷ $S = 100, K = 105, T = 0.5$ year, $r = 0.0355, \sigma = 0.58$

Table 4 Calculated values of American options when n increases doubly, at $\tau = T$

n	American calls	American puts
4	14.949709	18.232537
8	14.949709	18.233132
16	14.949709	18.233187
32	14.949709	18.233183
64	14.949709	18.233184
Benchmark	14.949709	18.233184

When the subinterval “n” increases, the American puts are convergence to 18.233184, the calls stay the same all the time. By checking Excel VBA, it is found that when $n \geq 37$, the results of puts are always 18.233184. So that $n = 40$ is chosen to be the parameter.

5. Comparison

5.1 Kim model V.S. Binomial model

Binomial model is chosen to be the primary model that compares with Kim model. Firstly, both models are based on the similar assumptions, such as Geometric Brownian motion theory of stock price behavior; secondly, the binomial model provides an intuition form of pricing the options price. Comparing with Binomial model gives some specific clues of Kim models’ outperformance and limitations. In the following parts, comparison of these two models is in three aspects.

²⁷ The parameter can be randomly chosen; here those parameters are consistent with the comparison source from Jan Röman.

5.1.1 Algorithm

Binomial Method is to discretize²⁸ option's lifetime into numbers of discrete time intervals (or steps), to evaluate each node's option value backwardly in time. The method is to repeat the same calculation for each time step and it's possible to check the option price at every step of the binomial tree for the possibility of early exercise.

The primary idea of Kim model is to obtain the optimal exercise boundary from a implicit integral equation, and then to get the early exercise premium. It provides an algorithm for calculating the optimal exercise boundary and option price for some time interval not a point of time. However, there is no explicit methods to solve the integral equations developed for calculate the early exercise premium except for two special cases mentioned in the previous sections. To determine the optimal exercise boundary and option price, it's necessary to use some complex numerical integration methods to approximate the solution of the integral equations.

5.1.2 Computing results

Changing numbers of steps "n"²⁹ in Binomial model³⁰ changes American puts value, see table below, here with the same parameter that used before: $S = 100, K = 105, T = 0.5 \text{ year}, r = 0.0355, \sigma = 0.58$.

As "n" increases, the value of puts form Binomial Model becomes less³¹, see figure 3. It means the differences between Kim model and Binominal model are going to be smaller.

²⁸ transfer continuous models and equations into discrete counterparts

²⁹ This "n" means the steps in Binomial model, and it is nothing to do with the "n" mentioned in Kim model.

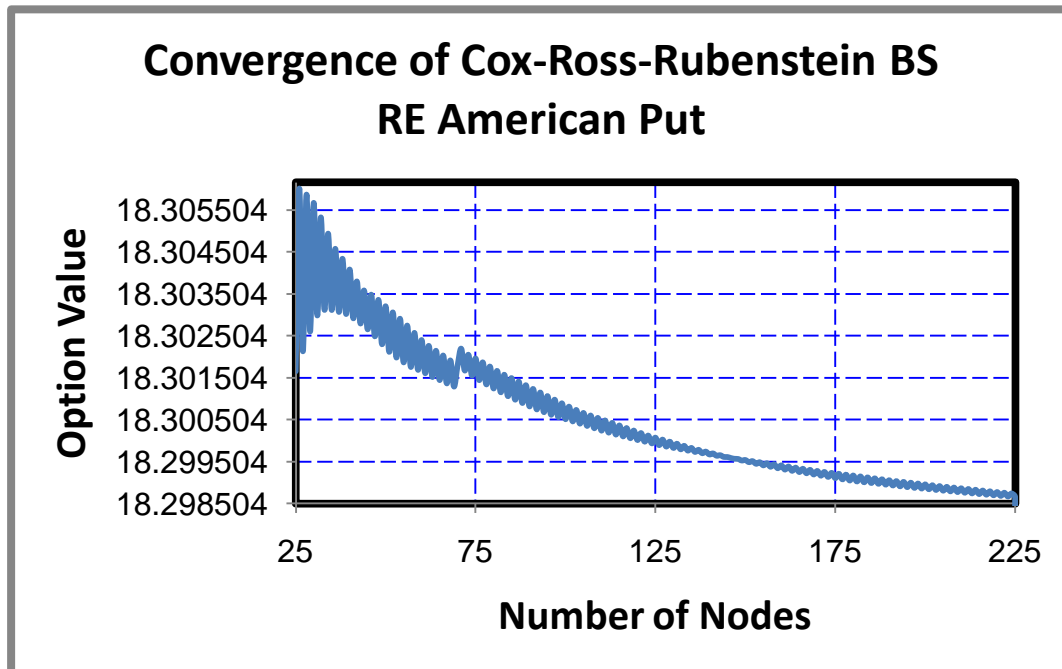
³⁰ Calculator (Excel VBAequityoptions.xlsm) for binomial model is made by Jan R öman, http://janroman.dhis.org/index_eng2.html. The function of Cox-Ross-Rubenstein BS RE is chosen to make the calculation.

³¹ It is not for all the values of puts (see figure 3.), but most of puts value are following this way.

Table 5. Outputs comparison of Kim model and Binomial model

Binomial model	Kim model	Differences = Put _B – Put _K
n = 25, Put _B = 18.301653	Put _K = 18.233184	0.068469
n = 100, Put _B = 18.298463	Put _K = 18.233184	0.065279
n = 400, Put _B = 18.297015	Put _K = 18.233184	0.063831
n = 1600, Put _B = 18.296744	Put _K = 18.233184	0.063560
n = 5000, Put _B = 18.296722	Put _K = 18.233184	0.063538

Figure 3 Convergence of Cox-Ross-Rubenstein BS RE³²



In theory, if “n” is approach unlimited large positive value, the results from Binomial model and Kim model should be almost the same. However, due to the limitation of

³² Excel file “ equityoptions,xlsm”, Jan R öman.

the Excel programming, it is not possible to get the exact same option value at present.

5.1.3 Computing time

Through previous work of programming models, some interesting points have been found out. As “n” becomes larger in binomial model, it takes longer computing time. The reason is that if the binomial tree has n time steps, then there are $\frac{n(n+1)}{2}$ nodes need to be computed, these repeated processes for every node consumes a lot of computing time. However, Kim model do not have this problem, because it deal with the continuous time and use integral method. No matter how the inputs change in Kim model, it will not bring much more burdens on computing process; therefore Kim model takes shorter time comparing with Binomial model that has large “n”.

5.2 Kim Model V.S. Other models

In this part, a brief look for three of the most widely used American models is given:

Baron-Adesi and Whaley: is developed by Giovanni Barone-Adesi and Robert E. Whaley (1987). The model is to price of the option is a European option with adjusted for an early exercise premium.

Bjerk Sund-Stensland: is formulated by Bjerk Sund P. and Stensland G. (1993). It assumes that the exercise is initiated to a corresponding 'flat' boundary, making use of a trigger price.

Roll-Geske Whaley: is derived by Roll, Geske and Whaley (1977). It values American call options on a stock paying a single dividend with specified time to dividend payout.

Table 6. Outputs of options

Models	Calls	Puts
B-S model	14.949709	18.107419
Kim model	14.949709	18.233184
Baron-Adesi and Whaley	14.949709	18.274542
Bjerk Sund-Stensland	14.949709	18.239455
Roll-Geske Whaley	14.949709	# ³³

Apply the models to Excel VBA, file “equityoptions.xlsm”³⁴, with the parameter $S = 100$, $K = 105$, $T = 0.5$ year, $r = 0.0355$, $\sigma = 0.58$, the outputs of option pricing models is shown in the table below. It proves that the price of American calls is the same as the price of European call, when there is no dividend. For puts, all the prices of American type are higher than European one. The price of puts obtained from Kim model has the lowest value among these prices that get from other two American option models.

6. Conclusion

To conclude, Kim model provides a new way that applies integrals to derive the optimal exercise boundary and evaluate the price of American options with continuous dividend. This algorithm calculates the optimal boundary for time intervals instead of the time points. By using this algorithm, Kim model not only can get the relatively accurate results comparing to some other models, and also can get the results quickly. It can be widely used in financial market in the real world.

³³ There is no calculation for puts from excel file “equityoptions.xlsm.”

³⁴ Jan R öman, http://janroman.dhis.org/index_eng2.html

7. References

Literature:

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[ew](#)

http://finance.bi.no/~bernt/gcc_prog/algoritms_v1/algoritms/node24.html

<http://www.hoadley.net/options/bs.htm>

[http://en.wikipedia.org/wiki/Option_\(finance\)](http://en.wikipedia.org/wiki/Option_(finance))

<http://www.wilmott.com/messageview.cfm?catid=4&threadid=78578>

8. Appendixes

Appendix 1

Black – Scholes pricing formulas

The B-S formulas for the price at time 0 of a European call option on non-dividend-paying stock and a European put option on a non-dividend-paying stock are

$$\begin{aligned}c &= S_0 N(d_1) - Ke^{-rT} N(d_2) \\p &= Ke^{-rT} N(-d_1) - S_0 N(-d_2) \\d_1 &= \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\d_2 &= \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}\end{aligned}$$

The variables are:

\ln = natural logarithm

$N(x)$ = standard normal cumulative distribution function

e = the exponential function

(The other variables are defined in the paper, table 1)

Appendix 2

The formula for Binomial model:

$$f = e^{-r\Delta t}[pf_u + (1 - p)f_d] \quad (a)$$

$$f = e^{-2r\Delta t}[p^2f_{uu} + 2p(1 - p)f_{ud} + (1 - p)^2f_{dd}] \quad (b)$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \quad (c)$$

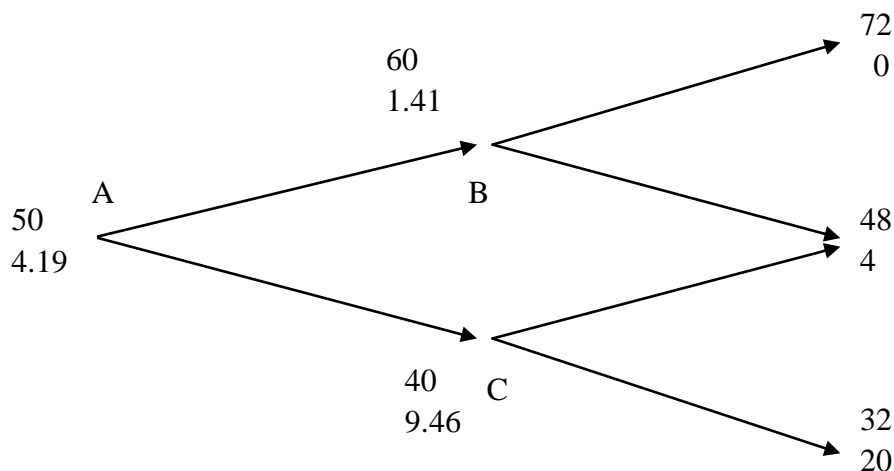
u is the up factor of price, d is the down factor of price; p is the probability of up movement in the price, f is the value of option price.

Example

Consider a 2-year put option with a stock price of \$52 on a stock whose current price is \$50. Suppose there are two time steps of 1 year, and in each time step the stock price either goes up by 20% or moves down by 20%, and the risk-free interest rate is 5%. So that, $u = 1 + 20\% = 1.2$, $d = 1 - 20\% = 0.8$, $\Delta t = 1$ and $r = 0.05$, and equation (C) gives $p = 0.6282$

● When this is a European put option

The possible final prices of stock are: \$72, \$48, \$32, so that the value of put options in the final nodes are $f_{uu} = \max(0, 52 - 72) = 0$, $f_{ud} = 48$, $f_{dd} = 20$. Then equation (a) and (b) give the value of put option $f_u = 1.41$, $f_d = 9.46$ and $f = 4.19$, see the binomial tree below.

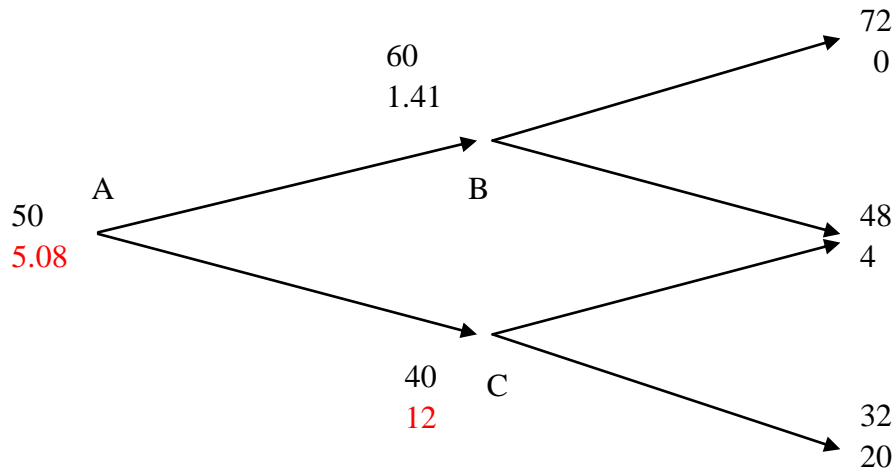


● *When this is a American put option*

The value of the option at the final nodes is the same as the European option.

However, at earlier nodes the value of the options is the greater of

1. The value given by the equation (a),
2. The payoff from early exercise.



The figure above shows the binomial tree for the American put option, it is clear that the values for the option at the final nodes are unchanged. At node B, equation (a) gives the value of the option as 1.41, whereas the payoff from early exercise is negative $-8 = 52 - 60$. Clearly early exercise is not optimal; value of the American put option is 1.41 that same as the European put. At node C, equation gives the value of the option as 9.46, whereas the payoff from early exercise is $12 = 52 - 40$. At initial node A, the value given by equation (a) is

$$e^{-0.05 \times 1} (0.6282 \times 1.41 + 0.3718 \times 12) = 5.0894$$

And the payoff from early exercise is $2 = 52 - 50$. In this case early exercise is not optimal the value of the option is therefore \$5.0894.

Appendix 3

Codes for Kim model

Option Explicit

Sub Kim()

```
Application.DecimalSeparator = "."  
frmkim.underlying.Value = 100  
frmkim.Textstrike.Value = 105  
frmkim.TextMaturity.Value = DateAdd("d", 182, Date)  
frmkim.Textrate.Value = 3.55  
frmkim.nbrn.Value = 40  
frmkim.volatility.Value = 58  
frmkim.Show
```

End Sub

‘Calculates the Normal distribution with $|e(x)| < 1.0e-8$

Function CND(X As Double) As Double

```
Dim sign As Long
```

```
Dim x2, q0, q1, q2 As Double
```

```
If (X < 0) Then
```

```
    X = -X
```

```
    sign = -1
```

```
ElseIf (X > 0) Then
```

```
    sign = 1
```

```
Else ' (x = 0.0)
```

```
    CND = 0.5
```

```
    Exit Function
```

```
End If
```

```
If (X > 20) Then
```

```
    If (sign < 0) Then
```

```
        CND = 0
```

```
    Else
```

```
        CND = 0
```

```
    End If
```

```
    Exit Function
```

```
End If
```

```
X = X * 0.707106781186547
```

```

x2 = X * X
If (X < 0.46875) Then
  q1 = 3209.37758913847 + x2 * (377.485237685302 + x2 * (113.86415415105 _
    + x2 * (3.16112374387057 + x2 * 0.185777706184603)))

  q2 = 2844.23683343917 + x2 * (1282.61652607737 + x2 * (244.024637934444
    -
      + x2 * (23.6012909523441 + x2)))
  CND = 0.5 * (1 + sign * X * q1 / q2)
ElseIf (X < 4) Then
  q1 = X * (8.88314979438838 + X * (0.56418849698867 + X *
    2.15311535474404E-08))
  q1 = X * (881.952221241769 + X * (298.6351381974 + X * (66.1191906371416
    + q1)))
  q1 = 1230.339354798 + X * (2051.07837782607 + X * (1712.04761263407 +
    q1))

  q2 = X * (117.693950891312 + X * (15.7449261107098 + X))
  q2 = X * (3290.79923573346 + X * (1621.38957456669 + X *
    (537.18110186201 + q2)))
  q2 = 1230.33935480375 + X * (3439.36767414372 + X * (4362.61909014325 +
    q2))

  CND = 0.5 * (1 + sign * (1 - Exp(-x2)) * q1 / q2)
Else
  q0 = 1 / x2
  q1 = 6.58749161529838E-04 + q0 * (1.60837851487423E-02 + q0 *
    (0.125781726111229 _
      + q0 * (0.360344899949804 + q0 * (0.305326634961232 + q0 *
        1.63153871373021E-02))))

  q2 = 2.33520497626869E-03 + q0 * (6.05183413124413E-02 + q0 *
    (0.527905102951428 _
      + q0 * (1.87295284992346 + q0 * (2.56852019228982 + q0))))
  CND = 0.5 * (1 + sign * (1 - Exp(-x2)) / X * (0.564189583547756 - q0 * q1 /
    q2)))
End If
End Function

Function nd(X As Double) As Double
  nd = 1 / Sqr(2 * 3.141592654) * Exp(-X * X / 2)
End Function

Function GBlackScholes(mCall As Boolean, S As Double, X As Double, T As Double,
  -

```

```

        r As Double, b As Double, v As Double) As Double
Dim d1 As Double, d2 As Double

If (T <= 0) Then T = 0.000000001
d1 = (Log(S / X) + (b + v * v / 2) * T) / (v * Sqr(T))
d2 = d1 - v * Sqr(T)

If (mCall) Then
    GBlackScholes = S * Exp((b - r) * T) * CND(d1) - X * Exp(-r * T) * CND(d2)
Else
    GBlackScholes = X * Exp(-r * T) * CND(-d2) - S * Exp((b - r) * T) * CND(-d1)
End If
End Function

```

'Calculates the value of an American call Options use kim model

```

Function kimCall(S As Double, X As Double, T As Double, r As Double, v As Double)
As Double

```

```

' Not optimal to early exercise.....
    kimCall = GBlackScholes(True, S, X, T, r, r, v)

```

```

End Function

```

'Use the Simpson's rule to calculate the integrals

```

Function Func(u As Double, Bx As Double, S As Double, tau As Double, r As Double,
v As Double) As Double

```

```

    Dim d1 As Double, d2 As Double

```

'Here u means time, in our Simpson's method we integral time from a to b, in our case is 0 to T, but, if u=a=0, d1 has no definition.

'So that's why we put a=0.00001

```

    d1 = (Log(S / Bx) + (r + v * v / 2) * (u)) / (v * Sqr(u))

```

```

    d2 = d1 - v * Sqr(u)

```

```

    Func = r * Exp(-r * (tau - u)) * CND(-d2)

```

```

End Function

```

```

Function SimpsonInt(a As Double, b As Double, ByVal n As Long, _
    Bx As Double, S As Double, tau As Double, r As Double, v As Double)

```

```

As Double

```

```

' Returns the integral of Func (below) from a to b _
    using Composite Simpson's Rule over n intervals

```

```

Dim i As Double ' index
Dim dH As Double ' step size
Dim dOdd As Double ' sum of Func(i), i = 1, 3, 5, 7, ... n-1, i.e., n/2 values
Dim dEvn As Double ' sum of Func(i), i = 2, 4, 6, ... n-2 i.e., n/2 - 1 values
' 1 + (n/2) + (n/2 - 1) + 1 = n+1 function evaluations

```

```

If n < 1 Then Exit Function
If n And 1 Then n = n + 1 ' n must be even
dH = (b - a) / n

```

```

For i = 1 To n - 1 Step 2
    dOdd = dOdd + Func(a + i * dH, Bx, S, tau, r, v)
Next i

```

```

For i = 2 To n - 2 Step 2
    dEvn = dEvn + Func(a + i * dH, Bx, S, tau, r, v)
Next

```

```

SimpsonInt = (Func(a, Bx, S, tau, r, v) + 4 * dOdd + 2 * dEvn + _
              Func(b, Bx, S, tau, r, v)) * dH / 3 ' weighted sum
End Function

```

'Use function G to derive optimal exercise boundary

```

Function G(S As Double, X As Double, v As Double, r As Double, T As Double,
BxVar As Double, n As Integer) As Double
    Dim d1 As Double, d2 As Double, m As Double

```

```

If (T = 0) Then
    T = 0.000000001
End If

```

```

d1 = (Log(BxVar / X) + (r + v * v / 2) * T) / (v * Sqr(T))
d2 = d1 - v * Sqr(T)
m = SimpsonInt(0.0000001, T, n, BxVar, S, T, r, v)

```

```

G = BxVar - X * (Exp(-r * T) * CND(-d2) - 1 + m) / (CND(-d1) - 1)
End Function

```

'Calculates the value of an American put Options use kim model

```
Function kimPut(S As Double, X As Double, T As Double, r As Double, v As Double,
n As Integer) As Double
```

```
  'If S > G Then ' Never optimal to exercise before maturity
```

```
    Dim Bx As Double, simp As Double
```

```
    'Calculate Bx "optimal strike"
```

```
      Bx = BisectionBx(S, X, T, r, v, n)
```

```
    'Bx here is the "strike" given by the solver.....
```

```
    If S > Bx Then ' Never optimal to exercise before maturity
```

```
      simp = SimpInt(0.0000001, T, n, Bx, S, r, v)
```

```
      kimPut = GBlackScholes(False, S, X, T, r, r, v) + X * simp
```

```
    Else
```

```
      kimPut = X - S
```

```
      Exit Function
```

```
    End If
```

```
End Function
```

```
Function SimpInt(a As Double, T As Double, ByVal n As Long, _
```

```
    Bx As Double, S As Double, _
```

```
    r As Double, v As Double) As Double
```

```
  'Returns the integral of Func (below) from a to b _
```

```
  Using Composite Simpson's Rule over n intervals
```

```
  Dim i As Double ' index
```

```
  Dim dH As Double ' step size
```

```
  Dim dOdd As Double ' sum of Func(i), i = 1, 3, 5, 7, ... n-1, i.e., n/2 values
```

```
  Dim dEvn As Double ' sum of Func(i), i = 2, 4, 6, ... n-2 i.e., n/2 - 1 values
```

```
  '1 + (n/2) + (n/2 - 1) + 1 = n+1 function evaluations
```

```
  If n < 1 Then Exit Function
```

```
  If n And 1 Then n = n + 1 ' n must be even
```

```
  dH = (T - a) / n
```

```
  For i = 1 To n - 1 Step 2
```

```
    dOdd = dOdd + Func(a + i * dH, Bx, S, T, r, v)
```

```
  Next
```

```
  For i = 2 To n - 2 Step 2
```

```
    dEvn = dEvn + Func(a + i * dH, Bx, S, T, r, v)
```

```
  Next
```

```

SimpInt = (Func(a, Bx, S, T, r, v) + 4 * dOdd + 2 * dEvn + _
          Func(T, Bx, S, T, r, v)) * dH / 3 ' weighted sum
End Function

```

'Solve Bx here.....

```

Function BisectionBx(S As Double, X As Double, T As Double, r As Double, v As
Double, n As Integer) As Double

```

```

    Dim counter As Long
    Dim BxLow As Double, BxHigh As Double, EPSILON As Double
    Dim cLow As Double, cHigh As Double, result As Double

```

```

    counter = 0
    BxLow = 0.0000001
    BxHigh = 10 * S
    EPSILON = 0.000000001
    cLow = G(S, X, v, r, T, BxLow, n)
    cHigh = G(S, X, v, r, T, BxHigh, n)
    result = BxLow - cLow * (BxHigh - BxLow) / (cHigh - cLow)

```

```

    Do While (Abs(G(S, X, v, r, T, result, n)) > EPSILON)

```

```

        counter = counter + 1
        If (counter > 100) Then GoTo L1

```

```

        If (G(S, X, v, r, T, result, n) < 0) Then

```

```

            BxLow = result
        Else
            BxHigh = result
        End If

```

```

        cLow = G(S, X, v, r, T, BxLow, n)
        cHigh = G(S, X, v, r, T, BxHigh, n)
        result = BxLow - cLow * (BxHigh - BxLow) / (cHigh - cLow)

```

```

    Loop

```

```

L1:

```

```

    BisectionBx = result

```

```

End Function

```

```

Function Max(X As Double, y As Double) As Double

```

```

    Max = y
    If (X > y) Then
        Max = X
    End If

```

```

End Function

```

```

Function Min(X As Double, y As Double) As Double
    Min = y
    If (X < y) Then
        Min = X
    End If
End Function
Sub DeleteCharts()
    Dim i As Long
    Dim WB As Workbook

    Set WB = ThisWorkbook
    Application.DisplayAlerts = False
    For i = WB.Charts.Count To 1 Step -1
        WB.Sheets(i).delete
    Next i
    For i = WB.Sheets.Count To 1 Step -1
        If WB.Sheets(i).Name <> "Start" Then
            WB.Sheets(i).delete
        End If
    Next i
    Application.DisplayAlerts = True
End Sub

```



```

Private Sub Comcalclate_Click()
Dim S As Double, X As Double, T As Double, r As Double, v As Double, b As
Double
    Dim h As Double, h0 As Double, callval As Double, putval As Double
    Dim gamma As Double, vega As Double, ctheta As Double, ptheta As Double
    Dim cdelta As Double, pdelta As Double, bound As Double
    Dim mTime As String, dTime As String
    Dim n As Integer

    On Error GoTo L0

    dTime = Date
    mTime = frmkim.TextMaturity.Value

    S = frmkim.underlying.Value
    X = frmkim.Textstrike.Value
    T = DateDiff("d", dTime, mTime) / 365
    r = frmkim.Textrate.Value / 100
    n = frmkim.nbrn.Value
    v = frmkim.volatility.Value / 100
    h = 0.0005
    b = r

    callval = kimCall(S, X, T, r, v)
    putval = kimPut(S, X, T, r, v, n)
    h0 = S * h
    cdelta = (kimCall(S + h0, X, T, r, v) - callval) / h0
    pdelta = (kimPut(S + h0, X, T, r, v, n) - putval) / h0
    gamma = kimCall(S + 2 * h0, X, T, r, v) + callval
    gamma = gamma - 2 * kimCall(S + h0, X, T, r, v)
    gamma = gamma / (h0 * h0)
    h0 = v * h
    vega = ((kimCall(S, X, T, r, v + h0) - callval) / h0) / 100
    h0 = T * h
    ctheta = ((kimCall(S, X, T - h0, r, v) - callval) / h0) / 365
    ptheta = ((kimPut(S, X, T - h0, r, v, n) - putval) / h0) / 365
    bound = BisectionBx(S, X, T, r, v, n)

    frmkim.Textdays.Value = DateDiff("d", dTime, mTime)
    frmkim.Textcallvalue.Value = Round(callval, 6)
    frmkim.Textputvalue.Value = Round(putval, 6)
    frmkim.Textcalldelta.Value = Round(cdelta, 6)
    frmkim.Textputdelta.Value = Round(pdelta, 6)
    frmkim.Textcallgamma.Value = Round(gamma, 6)

```

```

frmkim.Textputgamma.Value = Round(gamma, 6)
frmkim.Textcallvega.Value = Round(vega, 6)
frmkim.Textputvega.Value = Round(vega, 6)
frmkim.Textcalltheta.Value = Round(ctheta, 6)
frmkim.Textputtheta.Value = Round(ptheta, 6)
frmkim.Textbound.Value = Round(bound, 6)
Exit Sub
L0:
    MsgBox ("missing data!")
End Sub

Private Sub Comcallprice_Click()
Call Graphkim(1)
End Sub

Private Sub delete_Click()
Call DeleteCharts
End Sub

Private Sub Computprice_Click()
Call Graphkim(0)
End Sub

Private Sub bound_Click()
Call Graphboundary
End Sub

Private Sub Comreset_Click()
    frmkim.underlying.Value = 45
    frmkim.Textstrike.Value = 45
    frmkim.TextMaturity.Value = DateAdd("d", 182, Date)
    frmkim.Textrate.Value = 5
    frmkim.nbrn.Value = 40
    frmkim.volatility.Value = 20
    frmkim.Textdays.Value = ""
    frmkim.Textcallvalue.Value = ""
    frmkim.Textputvalue.Value = ""
    frmkim.Textcalldelta.Value = ""
    frmkim.Textputdelta.Value = ""
    frmkim.Textcallgamma.Value = ""
    frmkim.Textputgamma.Value = ""
    frmkim.Textcallvega.Value = ""
    frmkim.Textputvega.Value = ""
    frmkim.Textcalltheta.Value = ""

```

```

    frmkim.Texputttheta.Value = ""
End Sub

Private Sub Graphkim(mCall As Long)
    Dim S As Double, X As Double, T As Double, r As Double, v As Double, n As
Integer
    Dim Xmin As Double, Xmax As Double, dx As Double
    Dim Ymin As Double, Ymax As Double, dy As Double
    Dim mTime As String, dTime As String
    Dim numpt As Long, i As Long, j As Long, nGraph As Long
    Dim xval(100) As Double
    Dim yval(100) As Double

    On Error GoTo L1

    nGraph = 3
    numpt = 100
    dTime = Date
    mTime = frmkim.TexMaturity.Value

    S = frmkim.underlying.Value
    X = frmkim.Textstrike.Value
    T = DateDiff("d", dTime, mTime) / 365
    r = frmkim.Texrate.Value / 100
    n = frmkim.nbrn.Value
    v = frmkim.volatility.Value / 100
    h = 0.0005
    b = r

    For j = 1 To nGraph
        If j = 1 Then
            Xmin = 0.5 * Min(S, X)
            Xmax = 2 * Max(S, X)
        ElseIf j = 2 Then
            Xmin = 0
            Xmax = T
        Else
            Xmin = 0
            Xmax = 1
        End If
        Ymin = 10000
        Ymax = -10000
        dx = (Xmax - Xmin) / numpt

```

```

For i = 1 To numpt
    xval(i) = Xmin + i * dx
    If (j = 1) Then
        If (mCall = 1) Then
            yval(i) = kimCall(xval(i), X, v, r, T)
        Else
            yval(i) = kimPut(xval(i), X, v, r, T, n)
        End If
    ElseIf j = 2 Then
        If (mCall = 1) Then
            yval(i) = kimCall(S, X, v, r, xval(i))
        Else
            yval(i) = kimPut(S, X, v, r, xval(i), n)
        End If
    Else
        If (mCall = 1) Then
            yval(i) = kimCall(S, X, xval(i), r, T)
        Else
            yval(i) = kimPut(S, X, xval(i), r, T, n)
        End If
    End If
    If (yval(i) < Ymin) Then
        Ymin = yval(i)
    End If
    If (yval(i) > Ymax) Then
        Ymax = yval(i)
    End If
Next i

dy = Ymax - Ymin
Charts.Add

If j = 1 Then
    ActiveChart.Name = "kim Val(ulg)"
ElseIf j = 2 Then
    ActiveChart.Name = "kim Val(T)"
Else
    ActiveChart.Name = "kim Val(vol)"
End If

ActiveChart.ChartType = xlXYScatterSmoothNoMarkers
ActiveChart.ApplyLayout (10)

```

With ActiveChart

```
.HasTitle = True  
' .HasLegend = False  
.ChartArea.Border.LineStyle = xlContinuous  
.PlotArea.Border.LineStyle = xlContinuous  
.ChartArea.Border.Weight = xlThick  
.PlotArea.Border.Weight = xlThick  
.Axes(xlCategory, xlPrimary).HasTitle = True  
.Axes(xlCategory, xlPrimary).AxisTitle.Font.Size = 16  
.Axes(xlValue, xlPrimary).HasTitle = True  
.Axes(xlValue, xlPrimary).AxisTitle.Font.Size = 16  
.Axes(xlValue, xlPrimary).AxisTitle.Characters.Text = "Option value"
```

If j = 1 Then

```
.ChartTitle.Characters.Text = "Option value as function of Underlying  
Price"
```

```
.Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Underlying  
Price"
```

ElseIf j = 2 Then

```
.ChartTitle.Characters.Text = "Option value as function of Time"
```

```
.Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Time"
```

Else

```
.ChartTitle.Characters.Text = "Option value as function of Volatility"
```

```
.Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Volatility"
```

End If

End With

ActiveChart.Axes(xlCategory).Select

With ActiveChart.Axes(xlCategory)

```
.HasMajorGridlines = True
```

```
.MajorGridlines.Border.Color = RGB(0, 0, 255)
```

```
.MajorGridlines.Border.LineStyle = xlDash
```

```
.TickLabelPosition = xlTickLabelPositionLow
```

```
.MinimumScale = Xmin
```

```
.MaximumScale = Xmax
```

End With

ActiveChart.Axes(xlCategory).Select

With Selection.TickLabels.Font

```
.Name = "Arial"
```

```
.FontStyle = "Regular"
```

```
.Size = 12
```

End With

ActiveChart.Axes(xlValue).Select

```

With ActiveChart.Axes(xlValue)
    .HasMajorGridlines = True
    .MajorGridlines.Border.Color = RGB(0, 0, 255)
    .MajorGridlines.Border.LineStyle = xlDash
    If (Abs(Ymin) < 1) Then
        Ymin = 0
        dy = 0
    End If
    .MinimumScale = Round(Ymin - dy * 0.02, 0)
    .MaximumScale = Ymax + dy * 0.02
End With
With Selection.TickLabels.Font
    .Name = "Arial"
    .FontStyle = "Regular"
    .Size = 12
End With

ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection(1).Name = ""
ActiveChart.SeriesCollection(1).XValues = xval
ActiveChart.SeriesCollection(1).Values = yval
ActiveChart.SeriesCollection(1).Select
Selection.MarkerStyle = -4142
' ActiveChart.HasLegend = True
' ActiveChart.Legend.Delete
ActiveChart.HasLegend = False
Next j
Exit Sub
L1:
MsgBox ("Restarted the this file & push putprice first!")
End Sub
Private Sub Graphboundary()

    Dim S As Double, X As Double, T As Double, r As Double, v As Double, n As
Integer
    Dim Xmin As Double, Xmax As Double, dx As Double
    Dim Ymin As Double, Ymax As Double, dy As Double
    Dim mTime As String, dTime As String
    Dim numpt As Long, i As Long, j As Long, nGraph As Long
    Dim xval(100) As Double
    Dim yval(100) As Double

    On Error GoTo L1

```

```
nGraph = 3
numpt = 100
dTime = Date
mTime = frmkim.TextMaturity.Value
```

```
S = frmkim.underlying.Value
X = frmkim.Textstrike.Value
T = DateDiff("d", dTime, mTime) / 365
r = frmkim.Textrate.Value / 100
n = frmkim.nbrn.Value
v = frmkim.volatility.Value / 100
h = 0.0005
b = r
```

```
For j = 1 To nGraph
  If j = 1 Then
    Xmin = 0.5 * Min(S, X)
    Xmax = 2 * Max(S, X)
  ElseIf j = 2 Then
    Xmin = 0
    Xmax = T
  Else
    Xmin = 0
    Xmax = 1
  End If
  Ymin = 10000
  Ymax = -10000
  dx = (Xmax - Xmin) / numpt
```

```
For i = 1 To numpt
  xval(i) = Xmin + i * dx
  If (j = 1) Then
    yval(i) = BisectionBx(xval(i), X, v, r, T, n)
  End If
  If j = 2 Then
    yval(i) = BisectionBx(S, X, v, r, xval(i), n)
  End If
  If (yval(i) < Ymin) Then
    Ymin = yval(i)
  End If
  If (yval(i) > Ymax) Then
    Ymax = yval(i)
  End If
End For
```

```

End If
Next i

dy = Ymax - Ymin
Charts.Add

If j = 1 Then
    ActiveChart.Name = "BOUND Val(ulg)"
ElseIf j = 2 Then
    ActiveChart.Name = "BOUND Val(T)"
Else
    ActiveChart.Name = "BOUND Val(vol)"
End If

ActiveChart.ChartType = xlXYScatterSmoothNoMarkers
' ActiveChart.ApplyLayout (10)

With ActiveChart
    .HasTitle = True
    ' .HasLegend = False
    .ChartArea.Border.LineStyle = xlContinuous
    .PlotArea.Border.LineStyle = xlContinuous
    .ChartArea.Border.Weight = xlThick
    .PlotArea.Border.Weight = xlThick
    .Axes(xlCategory, xlPrimary).HasTitle = True
    .Axes(xlCategory, xlPrimary).AxisTitle.Font.Size = 16
    .Axes(xlValue, xlPrimary).HasTitle = True
    .Axes(xlValue, xlPrimary).AxisTitle.Font.Size = 16
    .Axes(xlValue, xlPrimary).AxisTitle.Characters.Text = "Critical Asset price"

    If j = 1 Then
        .ChartTitle.Characters.Text = "Optimal exercise boundary "
        .Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Underlying
Price"
    ElseIf j = 2 Then
        .ChartTitle.Characters.Text = "Optimal exercise boundary "
        .Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Time to
maturity"
    Else
        .ChartTitle.Characters.Text = "Optimal exercise boundary"
        .Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Volatility"
    End If
End With

```



```

ActiveChart.Axes(xlCategory).Select
With ActiveChart.Axes(xlCategory)
    .HasMajorGridlines = True
    .MajorGridlines.Border.Color = RGB(0, 0, 255)
    .MajorGridlines.Border.LineStyle = xlDash
    .TickLabelPosition = xlTickLabelPositionLow
    .MinimumScale = Xmin
    .MaximumScale = Xmax

```

End With

```

ActiveChart.Axes(xlCategory).Select
With Selection.TickLabels.Font
    .Name = "Arial"
    .FontStyle = "Regular"
    .Size = 12

```

End With

```

ActiveChart.Axes(xlValue).Select
With ActiveChart.Axes(xlValue)
    .HasMajorGridlines = True
    .MajorGridlines.Border.Color = RGB(0, 0, 255)
    .MajorGridlines.Border.LineStyle = xlDash
    If (Abs(Ymin) < 1) Then
        Ymin = 0
        dy = 0
    End If
    .MinimumScale = Round(Ymin - dy * 0.02, 0)
    .MaximumScale = Ymax + dy * 0.02

```

End With

```

With Selection.TickLabels.Font
    .Name = "Arial"
    .FontStyle = "Regular"
    .Size = 12

```

End With

```

ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection(1).Name = ""
ActiveChart.SeriesCollection(1).XValues = xval
ActiveChart.SeriesCollection(1).Values = yval
ActiveChart.SeriesCollection(1).Select
Selection.MarkerStyle = -4142
' ActiveChart.HasLegend = True
' ActiveChart.Legend.Delete
ActiveChart.HasLegend = False

```

```
Next j
Exit Sub
L1:
MsgBox ("Restarted the this file and push putprice/callprice first!")
End Sub
```