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The project of Analytical Finance I

Modelling Black-Scholes with Stochastic Volatility and Garch Application in EXCEL/VBA

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The Theoritical Background

1. An Overview on Black-Scholes Model

Black-Scholes Model is fairly the best known continuous time model. The model is simple and requires only five inputs: the asset price, the strike price, the time to maturity, the riskfree rate of interest, and the volatility. The simple Black-Scholes assumes that the underlying price is given by the process

$$dX_t^{(i)} = \mu X_t^{(i)} dt + \sigma X_t^{(i)} dW$$

Then the Black-Scholes (BLS) formula for a European Call Option respective a European Put (without dividend yield)investigated in this paper is given by;

$$c(S,t) = S_0 \Phi(d_1) - K e^{-r(T-t)} \Phi(d_1 - \sigma\sqrt{T-t}) \qquad (1)$$

$$p(S,t) = K e^{-r(T-t)} \Phi(d_1 - \sigma\sqrt{T-t}) - S_0 \Phi(-d_1) \qquad (2)$$

where,

$$d_1 = \frac{\log[S_0 / K] + r(T-t)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}$$

S is the current price of the underlying asset, *K* is strike price, *T* is the maturity of the call option, r is the spot rate, σ is the volatility and $\Phi(d)$ is the distribution function. This is important to note that the volatility is constant and is defined as the instantaneous standard deviation of the underlying security.

One of the solutions for volatility of Black-Scholes formula can be implied volatility. It is the volatility of the underlying which when substituted into the Black-Scholes formula gives a theoretical price equal to the market price. Practically by calculating the option price by this method we will see that despite the assumption for a constant volatility, we find that the volatility is not constant. It will form a smile or a frown and it is due to characteristics of the market and dependence of the implied volatility on strike and expiry. Another interpretation of implied volatility can be introducing it as a representation for the market's view of future volatility in a complex way.

2. Garch Model

In order to estimate the volatility for the BLS calculations we use an exponentially weighted average model. This model has different variants which our application follows the simplistic approach on the considered BLS formula for European put and call options. The equation bellow show the Garch(1,1), which relies on a long-run average variance rate,

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

Where three weights are associated to these three long-run average variance rates, and therefore rates must sum to one,

$$\gamma + \alpha + \beta = 1$$

In our Excel/VBA simulation of BLS we determined these three components of weights by the help of the historical data (1995-2004) taken from S&P-500 index on NYSE. The worksheet is also attached for further researches. Note that setting $\omega = \gamma V_L$, the Garch(1,1) model can also expressed as,

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

In our case we considered *sigma* and *u* to be two different random variables generated by Monte Carlo simulations and the weights as illustrated above. The result as a stochastic volatility applied to BLS.

3.1 Monte Carlo Simulation

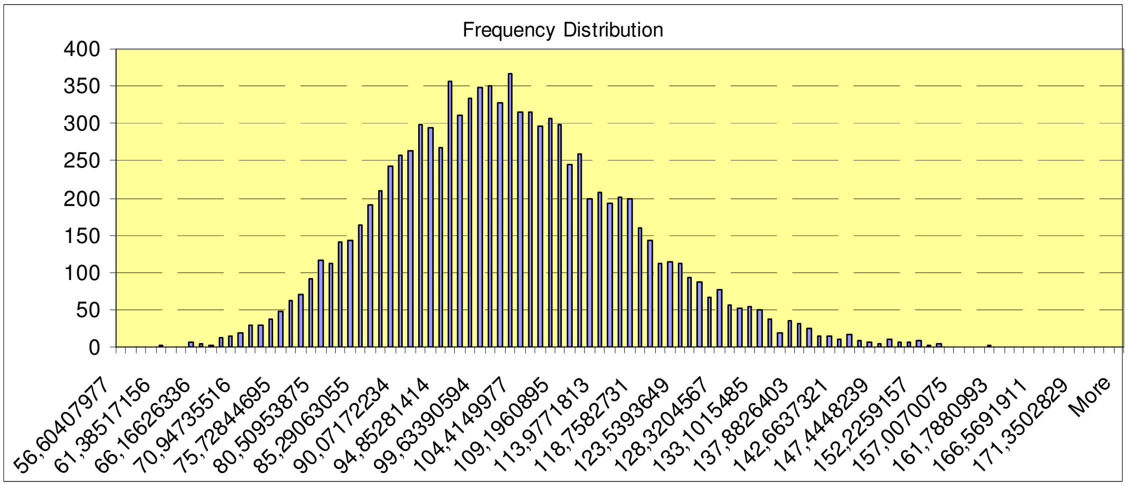
Monte Carlo Simulation is a technique for problem solving, which approximates the probability of certain outcomes by running multiple trial or simulations, using random variables. Thus this computational algorithm will make us able to model the behaviour of uncertainties. The calculation involves a large number of events in a sample which resembles the slight reality, or so called estimation.

3.2 General application of the MCS in excel

For the use of MCS (Monte Carlo Simulation) in excel the expression =RAND() is used which produces a random sample between 0 and 1. For one to produce a sample of a standard normal distribution in excel you type in the cell =NORMSINV(RAND()) , where the NORMSINV stands for the inverse cumulative normal distribution.
In formulas the variable common to most formulas ε , which denotes some random variable. This is also the variable that should be replaced in the excel sheet by =RAND().

4. Histogram

The sub written in VBA sorted the frequencies of outcomes and demonstrated a graphical representation for the distribution. Since BLS and Monte Carlo follows a normally distributed structure, the histogram will be consequently normally distributed. Bellow example;



5. European Call Option

European call option is the basic model for options studied in this paper. It will be followed by this part how the other options introduced here are based on this particular option.
European call option can be exercised only on expiration date, considering this characteristic it is the simplest model to simulate by Monte Carlo simulation. If we let S_t be the price of the underlying asset at the maturity and *K*, be the strike price then if $S_t > K$, the investor have the right to buy the asset for the price *K* and sell it in the market for the price S_t and enjoy the differential. Otherwise the investor abandons the right to exercise and do nothing, which in this case his/her loss will be zero.
The payoff function in the call case of the European option would be:

$$X = \{S_t - K\}_+$$

If we denote the *i*th realization of S_T by s_t^i, then the expected payoff from the call option can be evaluated as

$$E\left[\{S_t - K\}_+\right] \approx \frac{1}{N}\sum_{i=1}^{N}\{s_t^i - K\}_+$$

for sufficiently large *N*.
We should consider the discount rate in our evaluation of European call option. It let us to have a more precise approximation of the differential.

$$X = e^{-rT}E\left[\{S_t - K\}_+\right]$$

Consequently,

$$X = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}\{s_t^i - K\}_+$$

Consequently if one would like to consider a European Put option the order would be in the rivers way as for (2).

References:

Analytical Finance 1 (compendium), by Jan Röman

John C. Hull, Options, Futures, and Other Derivatives. - Prentice Hall, 2002, fifth edition. - ISBN 0-13-009056-5

Paul Wilmott Introduces Quantitative Finance (Paperback) by Paul Wilmott
ISBN: 0471498629