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Option Pricing Using Monte Carlo Simulation

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Abstract:

Microsoft Excel is widely used to analyse and graph financial data. The purpose of this paper is to show how to customise Excel to our need of solving European options using Monte Carlo Simulations and compare the results obtained with Black-Scholes formula. We illustrate and enhance the power of Excel for financial analysis by discussising the use of Excel to evaluate an option's value.

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Introduction:

Monte Carlo simulation is a very flexible way to model and solve a problem by simulating directly the physical process, and is not necessary to write down the full details that describe the behaviour of the system. A classical example would be evaluating Binomial lattice of more than fifty steps it's not practical, So the interest on Monte Carlo simulation approach can be very helpful in solving large Binomial steps times of options.

Task:

Build an application in Excel to solve option prices (European options) with Monte Carlo Simulation, using constant interest rate and calculate the probability to reach the strike and plot a Histogram.

Solution:

First of all we are going to solve this problem manually using the Black-Scholes formula. Our goal is to try and see if the Monte Carlo simulation will give us the same results as those obtained using the Black-Scholes formula. If the results are similar then we can safely conclude that our application from excel is correct. We are going to illustrate this by a numerical example of a European call and put option.

Example:

So	=	50	the price of the underlying at time t = 0
K	=	50	the Strike price
r	=	0,05	risk free interest rate, which is constant
σ	=	0,3	is the volatility of the prices the options
T	=	0,5	time to maturity.

Black –Scholes:

C (o) = S (o) N (d1) - Ke^{-rT} N (d2) for a call option

P (o) = Ke^{-rT} N (-d2) – S (o) N (-d1) for a put option

Where d1 = $\frac{\log(S(o)/K + (r+\sigma^2/2)T}{\sigma\sqrt{T}}$ the probability to reach strike.

The factor e^{-rT} is call the discounting factor

And d2 = d1 - $\sigma\sqrt{T}$ the probability to reach the strike.

N( ) is the cumulative probability distribution function for a standardized normal distribution with mean zero and standard deviation one

Monte Carlo Simulation for European Options:

The Black and Scholes Formula for European options can be checked by using binomial tree with very large number of time steps. In this paper we will use an alternative way of checking these European options which is the Monte Carlo simulation. Our example is constructed in Excel spreadsheets in the following way:

Cell	C2	=	So	=	50
Cell	D2	=	K	=	50
Cell	E2	=	r	=	0,05
Cell	F2	=	σ	=	0,3
Cell	G2	=	T	=	0,5

Excel spreadsheets for a 1000 simulations:

	A	B	C	D	E	F	G
1	55,47066	0	**So**	**K**	**r**	**σ**	**T**
2	43,49002	-6,34925	50	50	0,05	0,3	0,5
3	62,95399	12,63415	50	50	0,05	0,3	0,5
4	36,00388	-13,6506	50	50	0,05	0,3	0,5
5	79,06593	28,34829	50	50	0,05	0,3	0,5
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995	45,76601	-4,12945	50	50	0,05	0,3	0,5
996	50,61232	0,597204	50	50	0,05	0,3	0,5
997	54,09092	3,989919	50	50	0,05	0,3	0,5
998	66,57384	16,16463	50	50	0,05	0,3	0,5
999	71,46697	20,93695	50	50	0,05	0,3	0,5
1000	45,80668	-4,08979	50	50	0,05	0,3	0,5

Excel work sheet for 1000 simulations.

	H	I	J
1	B & S Price	Simulated Probability	Simulated option price
2	4.818	0,717416761	4,99055537

In excel we used the function NORMSINV this enabled us to find the cumulative inverse function for the standard normal distribution. We also used the function RAND() this Returns an evenly distributed random real number greater than or equal to 0 and less than 1 new random real number is returned every time the worksheet is calculated. Now that we are armed with these two functions we can combine them as NORMSINV(RAND()) and this gives us a random sample from standard normal distribution.

Armed with this knowledge we can calculate the random sample from the set of all stock prices at time T. We formulate Cell A1 for the call option as:

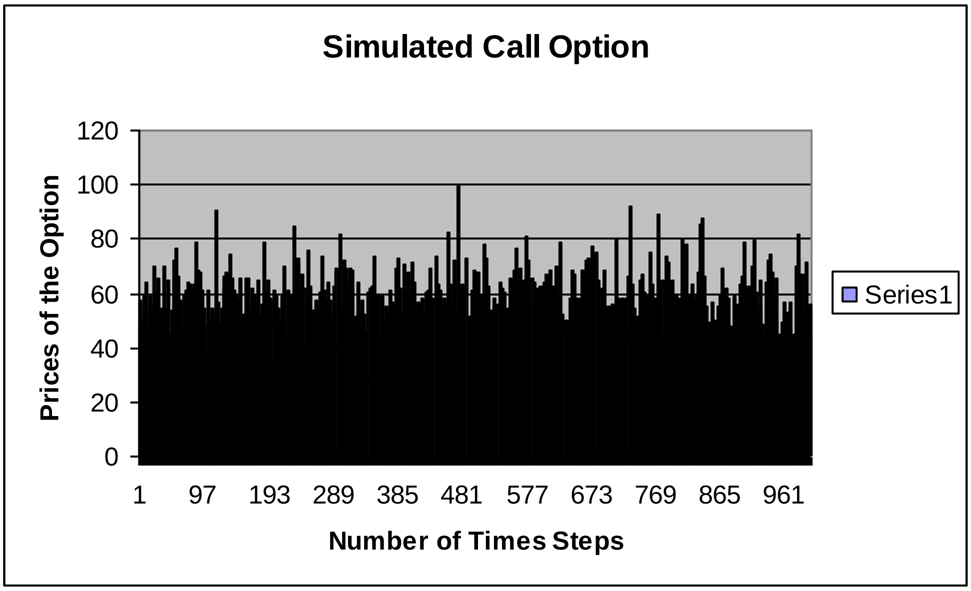
=C2*EXP((E2-F2*F2/2)*G2+F2*NORMSINV(RAND())*SQRT(G2))

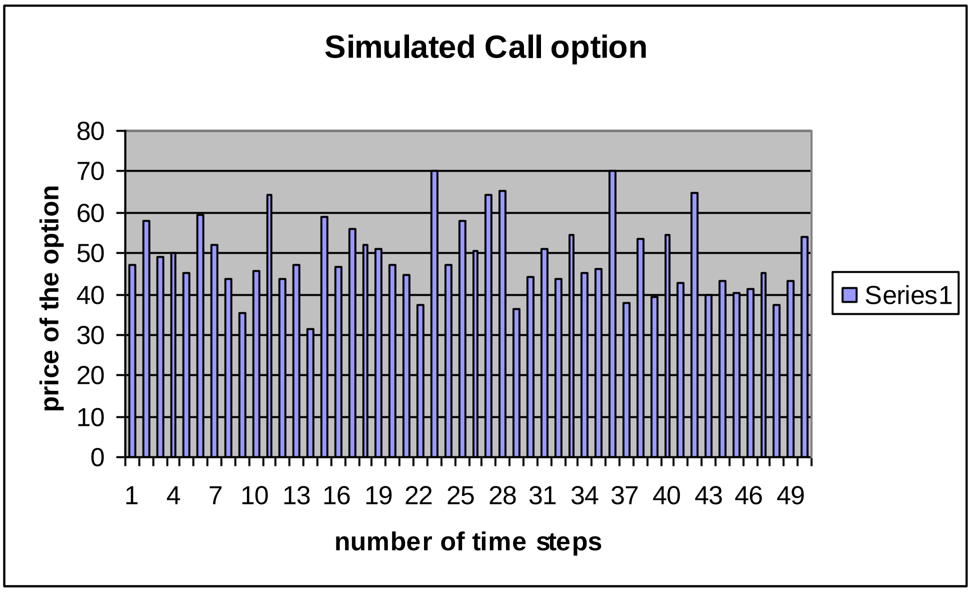
The present value for this payoff of the call option is set in Cell B2 as:

=EXP(-E2*G2)*MAX(A2-D2)

To define the rest of our spread sheet as the first we simple use the drag and select facility as a simple Monte Carlo approach in excel to 1000, 1500 and 2000.

Finding the average of random samples gives us the estimate value of the option and in all the investigated cases we notice that this mean is closely related to the Black-Scholes price. We also further simulated these random samples by entering into cell J2 the formula NORMINV(rand(), mean , sigma), you will generate a simulated value of a normal random variable having a mean and standard deviation sigma. Copying the formula =RAND() into I2 we generate random numbers, and applying the formula NORMINV(I2,mean,sigma) generates different trial values from a normal random variable. When we press the F9 key to recalculate the random numbers the mean remains close to our Black-Scholes option price.


Simulated Call Option
120
100
80
60
40
20
0
1
97
193
289
385
481
577
673
769
865
961
Number of Times Steps
Prices of the Option
Series1


Simulated Call option
80
70
60
50
40
30
20
10
0
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49
number of time steps
price of the option
Series1

Conclusion:

Our work during this paper provided us with a deeper understanding of real life applications of Monte Carlo simulations to finance and how excel can help us enhance those analysis.

Referees:

(i) Lecture notes in Analytical finance 1 by Senior Consulting Lecturer Jan Römans

(ii) Principles of financial Engineering Salih N. Neftci Elsevier academic press 2004

(iii) An Introduction to Mathematics Financial Derivatives academic press 2000

(iv) Stochastic Process with Applications to finance Chapman & Hall/CRC 2002