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MinLookback Asian Option

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1. Introduction

Asian options are options in which the underlying variable is the average price over a period of time. This type of option contract emerges and gets developed because it tends to cost less than regular American and European options. It is also known that an Asian option can protect an investor from the volatility risk that comes with the market. They are commonly traded on currencies and commodity products which have low trading volumes.

They were originally used in 1987 when Banker's Trust Tokyo office used them for pricing average options on crude oil contracts; and hence the name "Asian" option.

There are two basic forms of Asian option, namely Average Price Option and Average Strike Option. The former indicates to pay the difference between the strike and the average of underlying price, and the latter means taking the average price over specified period as the strike of the option and pays the difference between this strike and underlying market price.

In this investigation we have to calculate the price and the Greeks . Also calculate the Standard Error. Finally, express the value as a Taylor expansion that can be used to re-value the instruments on small changes in the underlying values and volatilities.

2. Asian option

There are two basic forms of Asian option, namely Average Price Option and Average Strike Option:

Average price options: at expiry the option pays the difference between the strike and an average of the underlying price achieved during a specified **averaging period** in the option's term.

Average strike options: the strike of the option is an average of the underlying price over the specified averaging period, and at expiry the option pays the difference between this strike and the underlying market price

Of the two, the average price option is by far the most common.

Types		payoff
European option	Call	$\max(S(T)-K,0)$
	Put	$\max(K-S(T),0)$
Average price option	Call	$\max(A(T)-K,0)$
	Put	$\max(K-A(T),0)$
Average strike option	Call	$\max(S(T)-A(T),0)$
	Put	$\max(A(T)-S(T),0)$

Also there are three terms that must be specified.

- The averaging period. E.g. the last three month, the entire term of option etc.
- The sampling frequency. E.g. daily, weekly, annually etc.
- The averaging method.

In this investigation we use **simple arithmetic average**.

$$A_i = \frac{1}{N} \sum_{j=1}^N S_i(T_j)$$

3. Pricing

3.1 General Pricing Formula

The price of an Asian call option is given by:

$$\Pi(0) = e^{-rT} E^Q [\Pi(T)] = e^{-rT} E^Q \left[\max \left\{ \frac{1}{n} \sum_{i=1}^n S(t_i) - K, 0 \right\} \right]$$

This can be generalized for baskets where the individual weights are given by ϕ_i :

$$\Pi(0) = e^{-rT} E^Q \left[\max \left\{ \frac{1}{n} \sum_{i=1}^n B(t_i) - K, 0 \right\} \right]$$
$$B(t_i) = \sum_{i=1}^d \phi_i S_i(t_i)$$

Similarly, an Asian put option can be valued as:

$$\Pi(0) = e^{-rT} E^Q [\Pi(T)] = e^{-rT} E^Q \left[\max \left\{ K - \frac{1}{n} \sum_{i=1}^n S(t_i), 0 \right\} \right]$$

or

$$\Pi(0) = e^{-rT} E^Q \left[\max \left\{ K - \frac{1}{n} \sum_{i=1}^n B(t_i), 0 \right\} \right]$$
$$B(t_i) = \sum_{i=1}^d \phi_i S_i(t_i)$$

3.2 Quanto Basket Min Lookback Asian

The Monte Carlo calculates the Greeks so we can use these as a Taylor expansion. Also we used **Quanto Basket Min Lookback Asian as an Instrument**.

For this instruments we have to exception, Firstly, the strike is not a pre-specified constant, but is determined by the minimum of the underlying asset over an initial time period. Secondly, the payoff is determined for a normalized underlying asset. Quanto means that the customer doesn't have any currency risk.

3.2.1 Price and payoff

The price and payoff, respectively, are given by

$$\Pi(0) = e^{-rT} E^Q [\Pi(T)] = e^{-rT} E^Q \left[\max \left\{ \frac{A(T) - K}{B(t_0)}, 0 \right\} \right]$$

Where

$$A(T) = \frac{1}{N - M} \sum_{j=M+1}^N B(t_j)$$

$$K = \min_{j=1, \dots, M} B(t_j) \quad j = 1, \dots, N$$

$$B(t_j) = \sum_{i=1}^d \phi_i S_i(t_j)$$

where $A(T)$ is the Asian price, K the min lookback price, $B(t_j)$ price of the basket at t_j , ϕ_i weight of underlying asset i , $S_i(t_j)$ the price of underlying asset i , t_j reset times, N number of reset dates, M the number of lookback dates and d the number of underlying assets.

4. Python code

```
__author__ = 'Lily Zheng'

# call moudles
from numpy import *
from scipy.stats import*
import matplotlib.pyplot as plt

'''
#####
###
#####
###
'''

### clean lines ###
clear = "\n"*1
print clear
'''

#####
###
#####
###
'''

'''
# temp input given parameters
S0 = 100.0
vol = 0.3
r = 0.02
# K, for min lookback asian option, K is not a given number
T = 252
Per1 = 45
fT = float(T)
fPer1 = float(Per1)

'''

### Input given numbers and print them out! ###
# imput
S0 = float(input("Enter Stock Price (S0):"))
vol = float(input("Enter Volatility(vol):"))
r = float(input("Enter Annual Interest Rate(r):"))
T = int(input("Enter The Number of days to Maturity Time(T):"))
Per1 = int(input("Enter The Number of days for the 1st Period(Per1):"))
fT = float(T)
fPer1 = float(Per1)

#####
###

#Print the given parameters
#"\n" make it run, otherwise occurs error!
print "if S0 =", S0, \
      ", Vol =", vol, \
      ", r =", r, \
      ", T =", T, \
      ", Per1 =", Per1
```

```

', ,
#####
###
#####
###
', ,

### Monte-Carlo Simulation! ###

step =200 # The number of simulations
sigma = float(vol/sqrt(252)) #sigma is volatility
dT= float(1.0/fT)
dP = float(1.0/Per1)

Simul1 = zeros((Per1, step)) #Simul1 is MC Simulation for Strike Price
#Zeros is creating a Per1 * Step matrix
for j in xrange(0, step): #define j
    Simul1[0, j] = S0
    P1 = random.normal(0, vol, Per1) #P = Probability
    #print "P1", P1 #get Normal random for Strike

    for i in xrange(1, Per1):
        a = P1[i]
        #epart = exp((r-(pow(sigma, 2)/2)*dt1)+(sqrt(dt1)*sigma*a))
        #Simul1[i, j] = S0*epart
        Simul1[i, j] = Simul1[i-1, j]*(exp((r-vol**2/2)*dP+(sqrt(dP)*vol* a)))
        #Simul1[i, j] = Simul1[i-1, j]*(1+a)
        #print "s1", Simul1

K1 = array([Simul1])
min_val = K1.min()
#print "min_val = {0}".format(min_val)
print "The Strike Price for Asian Option (K) is = {0}".format(min_val)
## Min value here is for all the simulations min val, is this the best way to calculate?
## or you need to find every min value in each path, and then take the mean of them?

#Simulation for Stock price at time t, average of the Stock prices
Simul2 = zeros((T, step))
for q in xrange(0, step):
    Simul2[0, q] = S0
    P2 = random.normal(0, vol, T) #P = Probability
    #print "P2", P2 #get Normal random for Stock price at time t

    for l in xrange(1, T):
        b = P2[l]
        Simul2[l, q] = Simul2[0, q]*(exp((r-vol**2/2)*dT+(sqrt(dT)*vol* b)))
        #print "s2", Simul2

AS_t = average(Simul2)
print "The Stock Price for Asian option at maturity will be:", AS_t

payOffp = max((AS_t-min_val)/S0 , 0)
payOff = max((AS_t-min_val) , 0)
ACPO = ((exp(-r*T))*payOff)*S0 #ACPO = Asian Call option price at time 0

print "Pay Off for the Min Lookback Asian Call Option is (%): ", payOffp
print "Pay Off for the Min Lookback Asian Call Option is: ", payOff

```



```
print "The Asian Call Option's value at Time 0 is:", ACPO # price or value?
```

```
# for the simultion part, I used 2 different parts, will it affect the result?
```

```
, , ,
```

```
#####  
###  
#####  
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```

```
### Plot Monte Carlo Simulation for simulate Strike Price from time 0 to Period 1 ###
```

```
plt.title('MC Simulation with Black-S for Min Lookback Asian Strike Price')  
plt.xlabel("First Period (Per1)")  
plt.ylabel("Stock Prices")  
plt.xlim(0, Per1)  
plt.plot(Simul1)  
plt.show()
```

```
### Plot Monte Carlo Simulation for simulate Stock Price from time 0 to Maturity ###
```

```
plt.title('MC Simulation with Black-S for Min Lookback Asian Stock Price')  
plt.xlabel("Time from Period 1 to Maturity (T)")  
plt.ylabel("Stock Price")  
plt.xlim(Per1, T)  
plt.plot(Simul2)  
plt.show()
```

Output:

Enter Stock Price (S_0):100

Enter Volatility(vol):0.3

Enter Annual Interest Rate(r):0.02

Enter The Number of days to Maturity Time(T):252

Enter The Number of days for the 1st Period(Per1):30

if $S_0 = 100.0$, $Vol = 0.3$, $r = 0.02$, $T = 252$, $Per1 = 30$

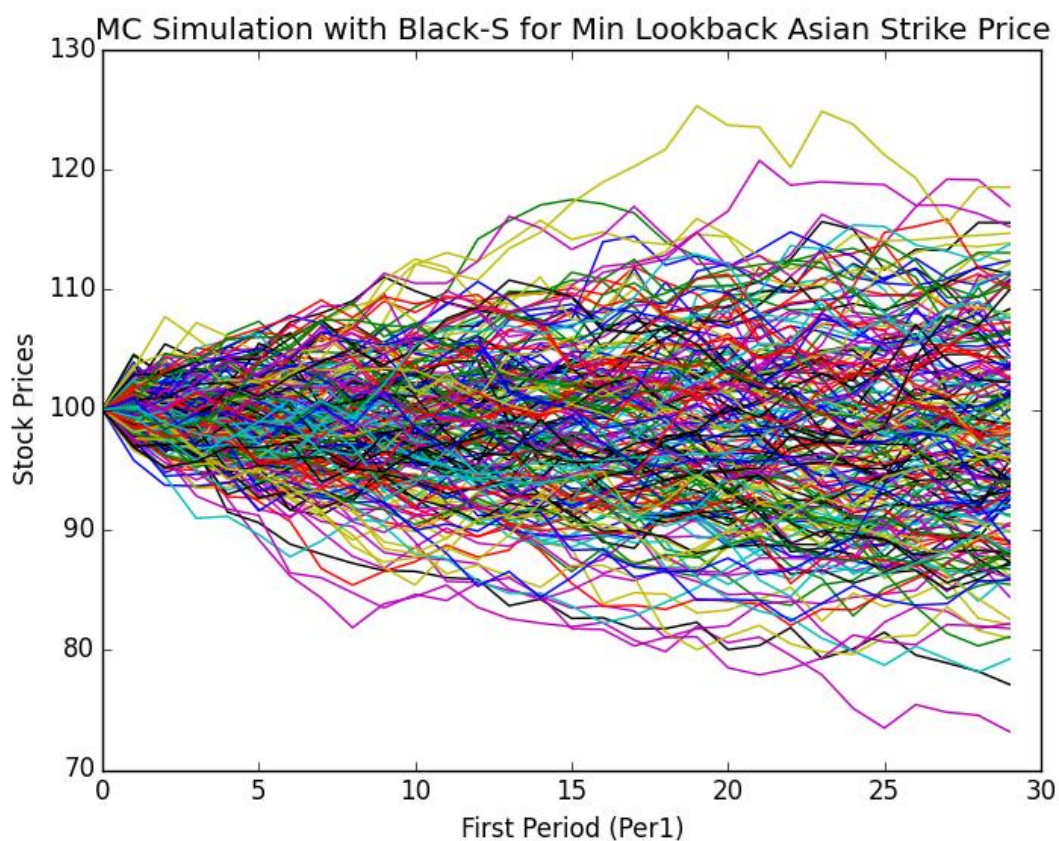
The Strike Price for Asian Option (K) is = 73.1754675265

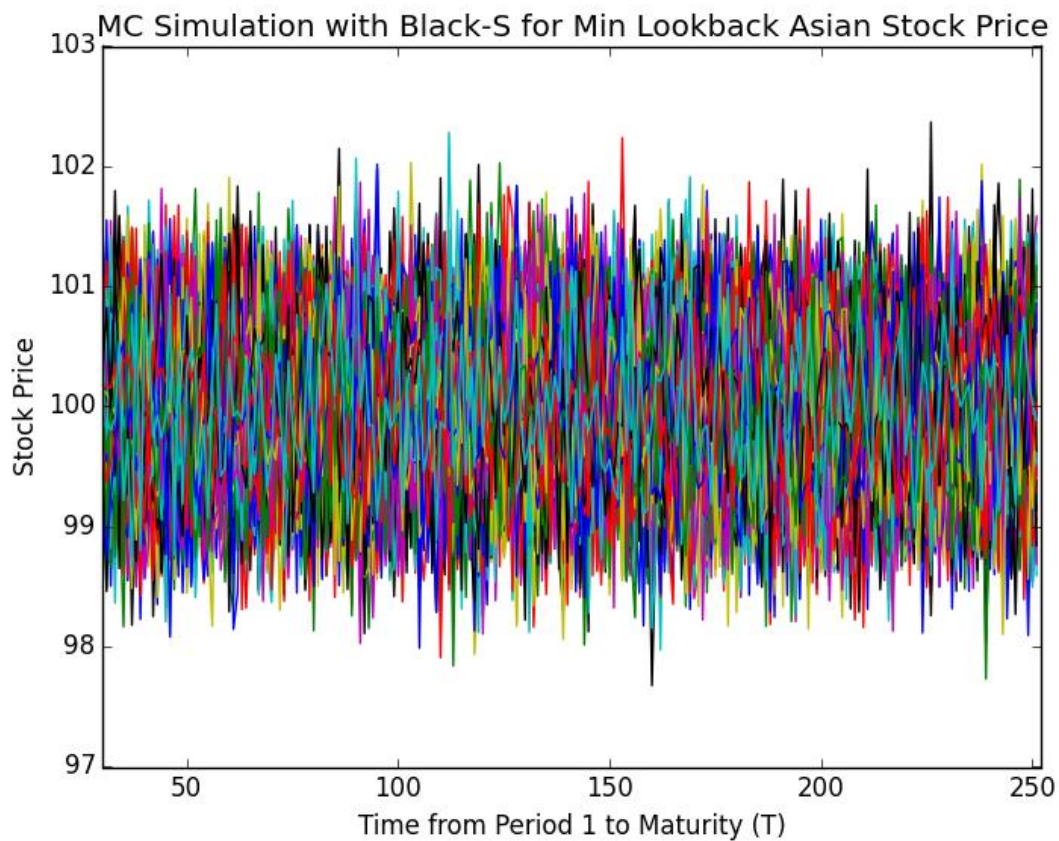
The Stock Price for Asian option at maturity will be: 99.9932430311

Pay Off for the Min Lookback Asian Call Option is(%): 0.268177755046

Pay Off for the Min Lookback Asian Call Option is: 26.8177755046

The Asian Call Option's value at Time 0 is: 17.3611529073





5. References

1. Lecture notes in Analytical Finance 1, Jan R. M. Röman
2. Kijima, M. 2003. Stochastic processes with application to finance, Chapman & Hall/CRC, Boca Raton. ISBN: 1-58488-224-7.
3. John.C.Hull.2007. Options, Futures and Other Derivatives Pearson Education; 6th International edition, ISBN: 0131977059