

Bootstrap a EURO zero swap-curve for tenor 3M and 6M

Victor Lopez
Amir Hossein Heydari Zadeh
Esther Torres

June 21, 2015

Analytical Finance II

Agenda

- 1 Methodology
- 2 Formulas
- 3 Algorithm
- 4 Output and Analysis

Introduction

- rates are quoted in market as yield
- day convention is 30/360
- Data that we used are:
 - Over-night rate ($0/N$)
 - tomorrow next rate (T/N)
 - deposit rates for 1W,2W till 6M
 - Forward rate agreement (FRA)
 - Swap from 1Y till 30Y
 - **Do not use shorter Swap since the FRA are more liquid**
 - OIS deposit
 - OIS Swap

Money Market instruments O/N, T/N

- $$D_{O/N} = \frac{1}{1 + r_{O/N}^{par} \frac{d_{O/N}}{360}}$$

$$Z_{O/N} = -100 \frac{\ln(D_{O/N})}{\frac{d_{O/N}}{365}}$$

$r_{O/N}^{par}$ is mid rate from Bid and Ask rates

Note: in calculation of Discount factor we use $Act/360$
and zero rates $Act/365$

Money Market instruments O/N, T/N

- $$D_{O/N} = \frac{1}{1 + r_{O/N}^{par} \frac{d_{O/N}}{360}}$$

$$Z_{O/N} = -100 \frac{\ln(D_{O/N})}{\frac{d_{O/N}}{365}}$$

$r_{O/N}^{par}$ is mid rate from Bid and Ask rates

Note: in calculation of Discount factor we use *Act/360*
and zero rates *Act/365*

- $$D_{T/N} = \frac{D_{O/N}}{1 + r_{T/N}^{par} \frac{d_{T/N}}{360}}$$

$$Z_{T/N} = -100 \frac{\ln(D_{T/N})}{\frac{d_{O/N}}{365}}$$

Money-Market instruments

- $$D_i = \frac{D_{T/N}}{1 + r_i^{par} \frac{d_i}{360}}$$
$$Z_i = -100 \frac{\ln(D_i)}{\frac{d_i}{365}}$$

Forward Rate Agreements

- $$D_{FRA}^i = \frac{D_{FRA}^{i-1}}{1 + r_{FRA}^i \frac{d_{FRA}^i}{360}}$$

$$Z_{FRA}^i = -100 \frac{\ln(D_{FRA}^i)}{\frac{d_{FRA}^i}{365}}$$

Note: use days between two IMM days

Swap

Bootstrap a
EURO zero
swap-curve for
tenor 3M and
6M

Victor Lopez
Amir Hossein
Heydari Zadeh
Esther Torres

Methodology

Formulas

Algorithm

Output and
Analysis

- we have Zero rate up to 2Y and continue the rest of curve with Swap data

Swap

- we have Zero rate up to 2Y and continue the rest of curve with Swap data

$$\bullet D_T = \frac{D_{T/N} - r_T^{par} \sum_{i=1}^{T-1} D_i}{1 + r_T^{par}}$$

$$Z_T = -100 \ln(D_T) \frac{365}{d_T}$$

Algorithm 1 Deposit algorithm

```
1: for all Ask and Bid do  $(Ask + Bid)/2$ 
2:   for  $O/N$  and  $T/N$  do
3:      $(1.0/(1 + (dep.iat[0, 3]/100 * (1.0/360.0))))$ 
4:      $(ONdis/(1.0 + (dep.iat[1, 3]/100 * (1.0/360.0))))$ 
5:      $(-100) * (math.log(ONdis))/(dep.iat[0, 6]/365.0)$ 
6:      $(-100) * (math.log(TNdisc))/(dep.iat[1, 6]/365.0)$ 
7:   end for
8: end for
9: for Other Deposits do
10:    $TNdisc / (1.0 + dep.iat[i, 3]/100 * (dep.iat[i, 6]/ 360.0))$  ▷
    Calculating the discount from 1W until 6M
11:    $(-100) * (math.log(dep.iat[i, 4])) / (dep.iat[i, 6]/365.0)$  ▷
    Calculating the zero rate from 1W until 6M
12: end for
```

Algorithm 2 FRA algorithm

- 1: **for all** Ask and Bid **do** $(Ask + Bid)/2$
 - 2: **end for**
 - 3: $dep.iat[5,4]/(1+fra.iat[0,3]/100*di)$ ▷ Calc the First FRA
Discount ▷ We do not need Stub FRA
 - 4: **for** Other FRA **do**
 - 5: $fra.iat[(i-1),4]/(1+fra.iat[i,3]/100*di)$ ▷ Discount of
Other FRA
 - 6: **end for**
 - 7: **for all** Other FRA **do** ▷ Calc Zero rates
 - 8: $(-100)*(math.log(fra.iat[i,4])/(fra.iat[i,6]/365.0))$
 - 9: **end for**
-

Algorithm 3 Swap algorithm

```
1: for all Ask and Bid do  $(Ask + Bid)/2$ 
2: end for
3:  $(TNdisc-(threem.iat[0,3]/100*fra.iat[2,4]))/(1+threem.iat[0,3]/100)$ 
   ▷ Calc discount of the 2Y Swap
4: for Other discounts do
5:    $sum \leftarrow Pervious\ discounts$ 
6:    $(TNdisc-sum*threem.iat[i,3]/100)/(1+threem.iat[i,3]/100)$ 
7: end for
8: for all
9:   do  $(-100)*(math.log(threem.iat[i,4])/(threem.iat[i,6]/365))$ 
   ▷ Calc all Swap
10: end for
```

Note: these are the Calculation of Swap for give Market Data

Algorithm 4 Extrapolation

```
1: for  $i \leftarrow 1, n$  do
2:   day2=threem.iat[i,6]
3:   day1=threem.iat[i-1,6]
4:   day0=threem.iat[i-2,6]
5:    $time_{interval} = day2 - day1$ 
6:    $year_{diff} = \text{int}(time_{interval}/360)$ 
7:   for  $j \leftarrow 1, year_{diff}$  do
8:      $interpolate_{days} = (day1 + (360 * j))$ 
9:      $interpolate_{zero} = (\text{threem.iat}[i-1,5] + (interpolate_{days} -$ 
      day0)*((threem.iat[i-1,5]-threem.iat[i-2,5])/(day2-day0)))
10:     $approximate = (1/((interpolate_{zero}/100)**(interpolate_{days}/360)))$ 
11:   end for
12: end for
```

Algorithm 5 unzipped Algorithm

```
1: firstReal=6
2: secondReal=8
3: for  $i \leftarrow 1, n$  do
4:   if  $\text{math.isnan}(\text{threem.iat}[i,1]) == \text{False}$  then
5:      $\text{dis}_{col} = \text{threem}['Discount']$ 
6:      $s = \text{sum}(\text{dis}_{col}.\text{loc}[0 : (i - 1)])$ 
7:      $\text{threem.iat}[i,4] = (\text{TNdisc-}$ 
 $s * \text{threem.iat}[i,3] / 100) / (1 + \text{threem.iat}[i,3] / 100)$ 
8:      $\text{threem.iat}[i,5] = (-100) * (\text{math.log}(\text{threem.iat}[i,4]) / (\text{threem.iat}[i,6] / 365))$ 
9:     if  $i > \text{secondReal}$  then
10:       firstReal = secondReal
11:       secondReal = i
12:     end if
13:   else
14:     day2=threem.iat[i,6]
15:     day1=threem.iat[secondReal,6]
16:     day0=threem.iat[firstReal,6]
17:      $\text{time}_{interval} = \text{day2} - \text{day1}$ 
18:      $\text{threem.iat}[i, 5] = (\text{threem.iat}[\text{secondReal}, 5] +$ 
 $(\text{day2} - \text{day0}) * ((\text{threem.iat}[\text{secondReal}, 5] -$ 
 $\text{threem.iat}[\text{firstReal}, 5]) / (\text{day1} - \text{day0})))$ 
19:      $\text{threem.iat}[i, 4] = (1 / ((\text{threem.iat}[i, 5] / 100 + 1) *$ 
 $* (\text{day2} / 365.0)))$ 
20:   end if
21: end for
```

Output of 3M Swap Curve

	Tenor	Zero rate	Days
Victor Lopez	0 O/N	-0.067353	1
Amir Hossein	1 T/N	-0.074232	2
Heydari Zadeh	2 1W	-0.040473	7
Esther Torres	3 1M	0.003162	30
	4 2M	0.039094	60
	5 3M	0.078440	90
	6 6M	0.181593	180
	7 6M-9M	0.188320	270
	8 9M-12M	0.202055	360
	9 12M-15M	0.212525	450
	10 15M-18M	0.228118	540
	11 18M-21M	0.250688	630
	12 21M-24M	0.278376	720
	13 3Y	0.142730	1080
	14 4Y	0.200631	1440
	15 5Y	0.280007	1800
	16 6Y	0.384175	2160
	17 7Y	0.507387	2520
	18 8Y	0.638694	2880
	19 9Y	0.774242	3240
	20 10Y	0.898321	3600
	21 11Y	1.287761	3960
	22 12Y	1.013056	4320
	23 13Y	1.185159	4680

Output 3M Swap Cont

	Tenor	Zero rate	Days
	24 14Y	1.242527	5040
	25 15Y	1.258209	5400
	26 16Y	1.585078	5760
	27 17Y	1.666796	6120
	28 18Y	1.748513	6480
	29 19Y	1.830231	6840
	30 20Y	1.493731	7200
	31 21Y	1.776357	7560
	32 22Y	1.823462	7920
	33 23Y	1.870566	8280
	34 24Y	1.917670	8640
	35 25Y	1.597634	9000
	36 26Y	1.722319	9360
	37 27Y	1.743099	9720
	38 28Y	1.763880	10080
	39 29Y	1.784661	10440
	40 30Y	1.652353	10800

Output 6M Swap Curve

Victor Lopez
Amir Hossein
Heydari Zadeh
Esther Torres

Methodology

Formulas

Algorithm

Output and
Analysis

	Tenor	Zero rate	Days
0	O/N	-0.067353	1
1	T/N	-0.074232	2
2	1W	-0.040473	7
3	1M	0.003162	30
4	2M	0.039094	60
5	3M	0.078440	90
6	6M	0.181593	180
7	6M-9M	0.188320	270
8	9M-12M	0.202055	360
9	12M-15M	0.212525	450
10	15M-18M	0.228118	540
11	18M-21M	0.250688	630
12	21M-24M	0.278376	720
13	3Y	0.263114	1080
14	4Y	0.327148	1440
15	5Y	0.410683	1800
16	6Y	0.518086	2160
17	7Y	0.642584	2520
18	8Y	0.773174	2880
19	9Y	0.902772	3240
20	10Y	1.021871	3600
21	11Y	1.423149	3960
22	12Y	1.229158	4320
23	13Y	1.664338	4680

Output 6M Swap Curve Cont

Victor Lopez
Amir Hossein
Heydari Zadeh
Esther Torres

Methodology

Formulas

Algorithm

Output and
Analysis

	Tenor	Zero rate	Days
24	14Y	1.773133	5040
25	15Y	1.448568	5400
26	16Y	1.741115	5760
27	17Y	1.814252	6120
28	18Y	1.887389	6480
29	19Y	1.960525	6840
30	20Y	1.656956	7200
31	21Y	1.907022	7560
32	22Y	1.948699	7920
33	23Y	1.990377	8280
34	24Y	2.032054	8640
35	25Y	1.740688	9000
36	26Y	1.841167	9360
37	27Y	1.857914	9720
38	28Y	1.874660	10080
39	29Y	1.891407	10440
40	30Y	1.779584	10800

	Tenor	Zero rate	Days
0	O/N	-0.028389	1
1	T/N	-0.028389	2
2	1W	0.483370	9
3	2W	0.480785	16
4	3W	0.534548	23
5	1M	0.565102	32
6	2M	0.593683	62
7	3M	0.622182	92
8	4M	0.656320	122
9	5M	0.689094	152
10	6M	0.716997	182
11	7M	0.750399	212
12	8M	0.781724	242
13	9M	0.810728	272
14	10M	0.839179	302
15	11M	0.870849	332
16	1Y	0.902461	362
17	2Y	1.205173	722
18	3Y	1.444660	1082
19	4Y	0.127399	1440
20	5Y	0.400696	1800
21	6Y	0.717598	2160
22	7Y	1.057199	2520
23	8Y	1.383280	2880

OIS curve

OIS curve Cont

	Tenor	Zero rate	Days
24	9Y	1.676143	3240
25	10Y	1.925703	3600
26	11Y	2.424823	3960
27	12Y	2.316238	4320
28	13Y	2.902040	4680
29	14Y	3.097308	5040
30	15Y	2.680665	5400
31	16Y	2.513867	5760
32	17Y	2.536958	6120
33	18Y	2.560050	6480
34	19Y	2.583141	6840
35	20Y	2.364070	7200
36	21Y	2.329612	7560
37	22Y	2.323868	7920
38	23Y	2.318125	8280
39	24Y	2.312382	8640
40	25Y	2.153204	9000
41	26Y	2.026685	9360
42	27Y	2.005598	9720
43	28Y	1.984512	10080
44	29Y	1.963425	10440
45	30Y	1.935508	10800

OIS with or without Deposits?

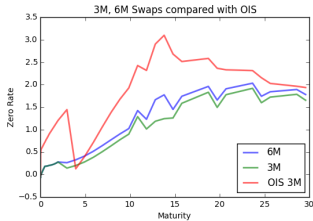


Figure: with Deposits

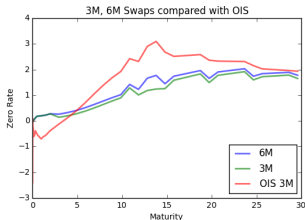


Figure: without Deposits

3M Swap-OIS spread

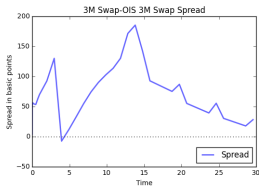


Figure: OIS with Deposits

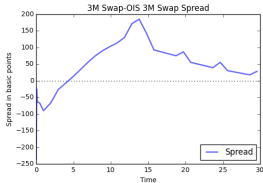


Figure: OIS without Deposits

Model validation using Jan Römans Excel sheet