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BOOTSTRAPPING A ZERO-SWAP CURVE FOR TENOR 3M USING PYTHON

Seminar Report

MMA708- ANALYTICAL FINANCE I

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1. Introduction

Fundamental to any trading and risk management activity is the ability to value future cash flows of an asset. In modern finance the accepted approach to valuation is the discounted cash flows (DCF) methodology. If $C(t)$ is a cash flow occurring t years from today, according to the DCF model, the value of this cash flow today is

$$V_0=C(t)Z(t)$$

where $Z(t)$ is the present value (PV) factor or discount factor.

This report is based on the usage of the Python Programming Language to bootstrap such scenarios.

2. The Yield Curve

2.1. Valuation

To value any asset, the necessary information is the cash flows, their payment dates, and the corresponding discount factors to PV these cash flows. The cash flows and their payment dates can be directly obtained from the contract specification but the discount factor requires the knowledge of the *yield curve.*

All investors will have a specific risk/reward profile that they are comfortable with, and a bond's yield relative to its perceived risk will influence the decision to buy (or sell) it.

Much of the analysis and pricing activity that takes place in the bond markets revolves around the yield curve. The yield curve plays a central role in the pricing, trading and risk management activities of all financial products ranging from cash instruments to exotic structured derivative products.

The yield curve describes the relationship between a particular yield-to-maturity and a bond's maturity. Plotting the yields of bonds along the term structure will give us our yield curve.
Strictly speaking, the yield curve describes the term structure of interest rates in any market, i.e. the relationship between the market yield and maturity of instruments with similar credit risk. The market yield curve can be described by a number of alternative but equivalent ways: discount curve, par-coupon curve, zero-coupon or spot curve and forward rate curve.

2.2. Discount Curve

The discount curve reflects the discount factor applicable at different dates in the future and represents the information about the market in the most primitive fashion.
This is the most primitive way to represent the yield curve and is primarily used for valuation of cash flows. An example of discount curve for the German bond market based on the closing prices on 29 October 1998 is shown in *Figure 2.1*

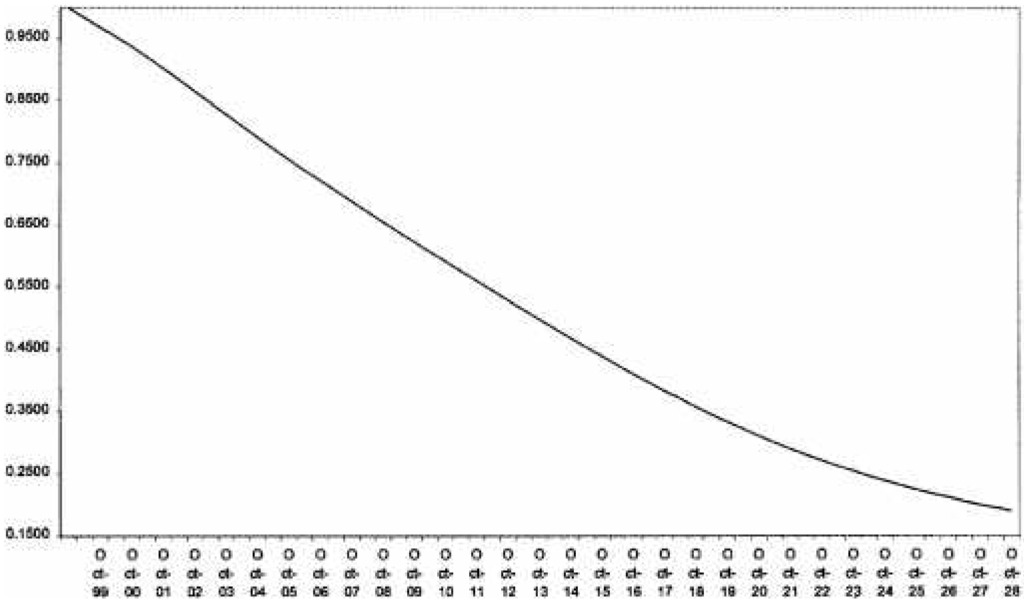


Figure 2.1 Discount curve

2.3. The Par, Spot curve and the Forward Rate curve

The par, spot, and forward curves that can be derived from the discount curve is useful for developing yield curve trading ideas.

Par, spot and forward rates have a close mathematical relationship. The spot interest rates are also called zero-coupon rates, because they are the interest rates that would be applicable to a zero-coupon bond. It indicates the yield of a zero coupon bond for different maturity.

A par yield is the yield-to-maturity on a bond that is trading at par. This means that the yield is equal to the bond's coupon level. A zero-coupon bond is a bond which has no coupons, and therefore only one cash flow, the redemption payment on maturity. It is therefore a discount instrument, as it is issued at a discount to par and redeemed at par. The yield on a zero-coupon bond can be viewed as a true yield, at the time that is it purchased, if the paper is held to maturity. This is because no reinvestment of coupons is involved and so there are no interim cash flows vulnerable to a change in interest rates. Zero-coupon yields are the key determinant of value in the capital markets, and they are calculated and quoted for every major currency.
Zero-coupon rates can be used to value any cash flow that occurs at a future date.

The forward par curve or the forward rate curve can also be constructed. Both these curves show the future evolution of the interest rates as seen from today's market yield curve.

The forward rate curve shows the anticipated market interest rate for a specific tenor at different points in the future while the forward curve presents the evolution of the entire par curve at a future date.
Figure 2.2 shows the par, spot, forward curves German government market on 29 October 1998.

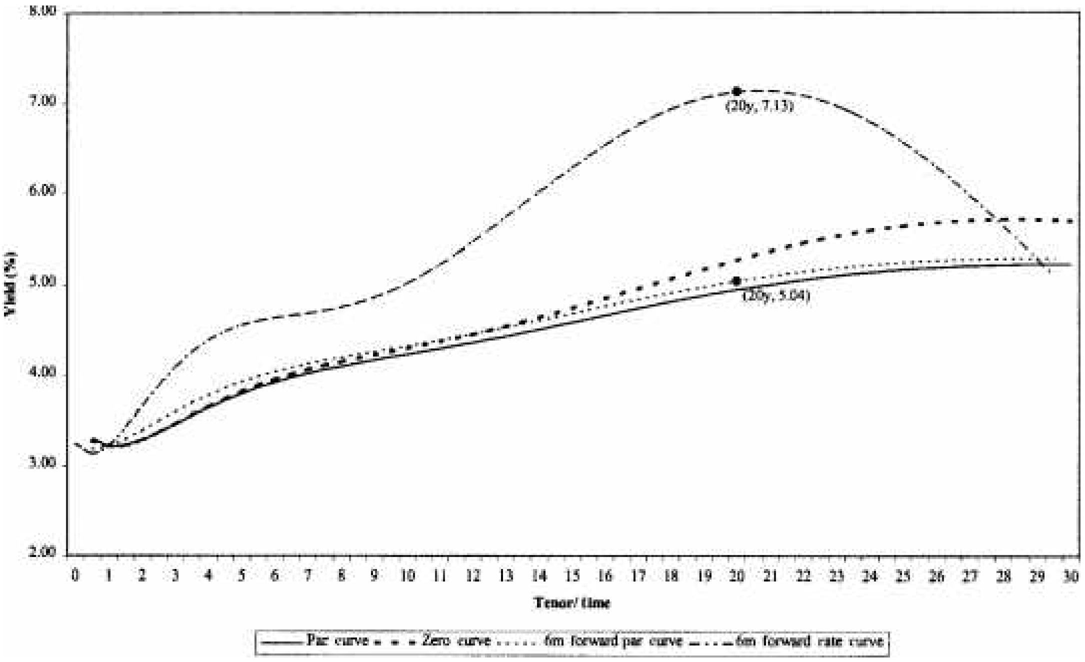


Figure 2.2 Par-, zero-, and forward yield curves.

2.4. Discount factors

Since discount factor curve forms the fundamental building block for pricing and trading in both the cash and derivative markets we will begin by focusing on the methodology for constructing the discount curve from market data. Armed with the knowledge of discount curve we will then devote our attention to developing other representation of market yield curve. The process for building the yield curve can be summarized in *Figure 2.3*

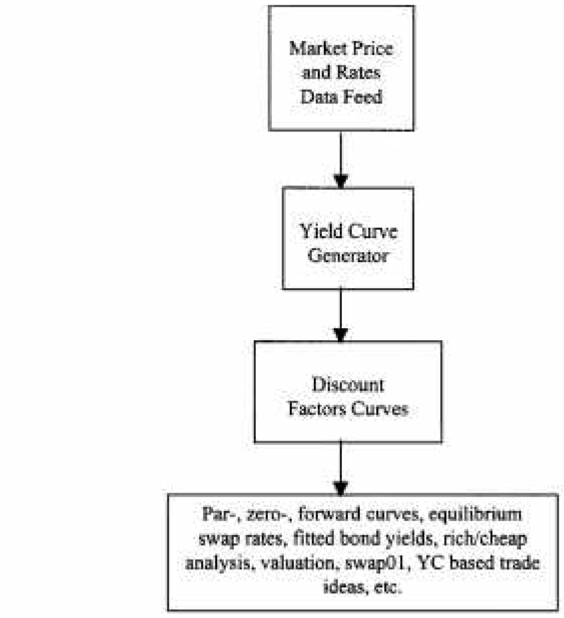


Figure 2.3 Yield curve modeling process.

3. Bootstrapping a swap curve

Market participant also refers to the swap curve as the LIBOR curve. The swap market yield curve is built by splicing together the rates from market instruments that represent the most liquid instruments or dominant instruments in their tenors.
At the very short end, the yield curve uses the cash deposit rates, where available the International Money Market (IMM) futures contracts are used for intermediate tenors and finally par swap rates are used for longer tenors.
A methodology for building the yield curve from these market rates, is referred to as *bootstrapping or zero coupon stripping*.

The LIBOR curve can be built using the following combinations of market rates:

- Cash deposit, futures and swaps
- Cash deposit and swaps

The reason for the popularity of the bootstrapping approach is its ability to produce a no-arbitrage yield curve, meaning that the discount factor obtained from bootstrapping can recover market rates that has been used in their construction.
The downside to this approach is the fact that the forward rate curve obtained from this process is not a smooth curve. While there exists methodologies to obtain smooth forward curves with the help of various fitting algorithms they are not always preferred as they may violate the no-arbitrage constraint or have unacceptable behavior in risk calculations.

3.1. Extracting discount factors from deposit rates

The first part of the yield curve is built using the cash deposit rates quoted in the market. The interest on the deposit rate accrue on a simple interest rate basis and as such is the simplest instrument to use in generating discount curve. It is calculated using the following fundamental relationship in finance:

Present value = Future value × discount factor

To find the discount factors, first, we can calculate the overnight discount factor. i.e.

$$D_{O/N} = \frac{1}{1 + r_{O/N}^{par} \cdot \frac{d_{O/N}}{360}}$$

This will enable derive the spot or zero rate for the overnight i.e.

$$Z_{O/N} = -100 \frac{\ln\left(D_{O/N}\right)}{\frac{d_{O/N}}{365}}$$

Next, we use the tomorrow next rate to calculate the discount factor for the spot date. The tomorrow next rate is a forward rate between trade day plus one business day to trade date plus two business day. Therefore, the discount factor for the spot date is given by the following functions:

$$Discont\ \ factor\ the\ tomorrow\ next,$$

$$D_{\mathrm{T}/N} = \frac{D_{O/N}}{1 + r_{\mathrm{T}/N}^{par} \cdot \frac{d_{\mathrm{T}/N}}{360}}$$

$$The\ zero\ rate\ is\ \ Z_{\mathrm{T}/N} = -100 \frac{\ln\left(D_{\mathrm{T}/N}\right)}{1 \frac{d_{\mathrm{T}/N}}{360}}$$

3.2. Extracting discount factors from futures contract

Next we consider the method for extracting the discount factor from the futures contract. The prices for IMM futures contract reflect the effective interest rate for lending or borrowing 3-month LIBOR for a specific time period in the future. The contracts are quoted on a price basis and are available for the months March, June, September and December. The settlement dates for the contracts vary from exchange to exchange. Typically these contracts settle on the third Wednesday of the month and their prices reflect the effective future interest rate for a 3-month period from the settlement date.

3.3. Extracting discount factors from swap rates

As we go further away from the spot date we either run out of the futures contract or, as is more often the case, the futures contract become unsuitable due to lack of liquidity. Therefore to generate the yield curve we need to use the next most liquid instrument, i.e. the swap rate.

We need to know the swap rate and discount factor. If a swap rate is not available then it has to be interpolated. Similarly, if the discount factors on the swap payment dates are not available then they also have to be interpolated.

4. Applications and Results

4.1. Data Used in Zero Swap-Curve – Tenor 3M

Start date	01-07-14

Deposits

Tenor	Bid	Ask	mid	start date	maturity	period days
O/N	-0.06643	-0.06643	-0.06643	01-07-14	02-07-14	1
T/N	-0.13	-0.03	-0.08	02-07-14	03-07-14	1
1W	-0.019	-0.019	-0.019	03-07-14	10-07-14	7
1M	0.008	0.008	0.008	03-07-14	03-08-14	31
2M	0.041	0.041	0.041	03-07-14	03-09-14	62
3M	0.079	0.079	0.079	03-07-14	03-10-14	92
6M	0.18	0.18	0.18	03-07-14	03-01-15	184

FRA

Tenor	Bid	Ask	mid	start date	maturity	period days
3M-6M	0.23	0.25	0.24	01-10-14	01-01-15	92
6M-9M	0.23	0.25	0.24	01-01-15	01-04-15	90
9M-12M	0.23	0.25	0.24	01-04-15	01-07-15	91
12M-15M	0.241	0.261	0.251	01-07-15	01-10-15	92
15M-18M	0.292	0.312	0.302	01-10-15	01-01-16	92
18M-21M	0.371	0.391	0.381	01-04-16	01-07-16	91
21M-24M	0.456	0.476	0.466	01-10-16	01-01-17	92

Swap 3M

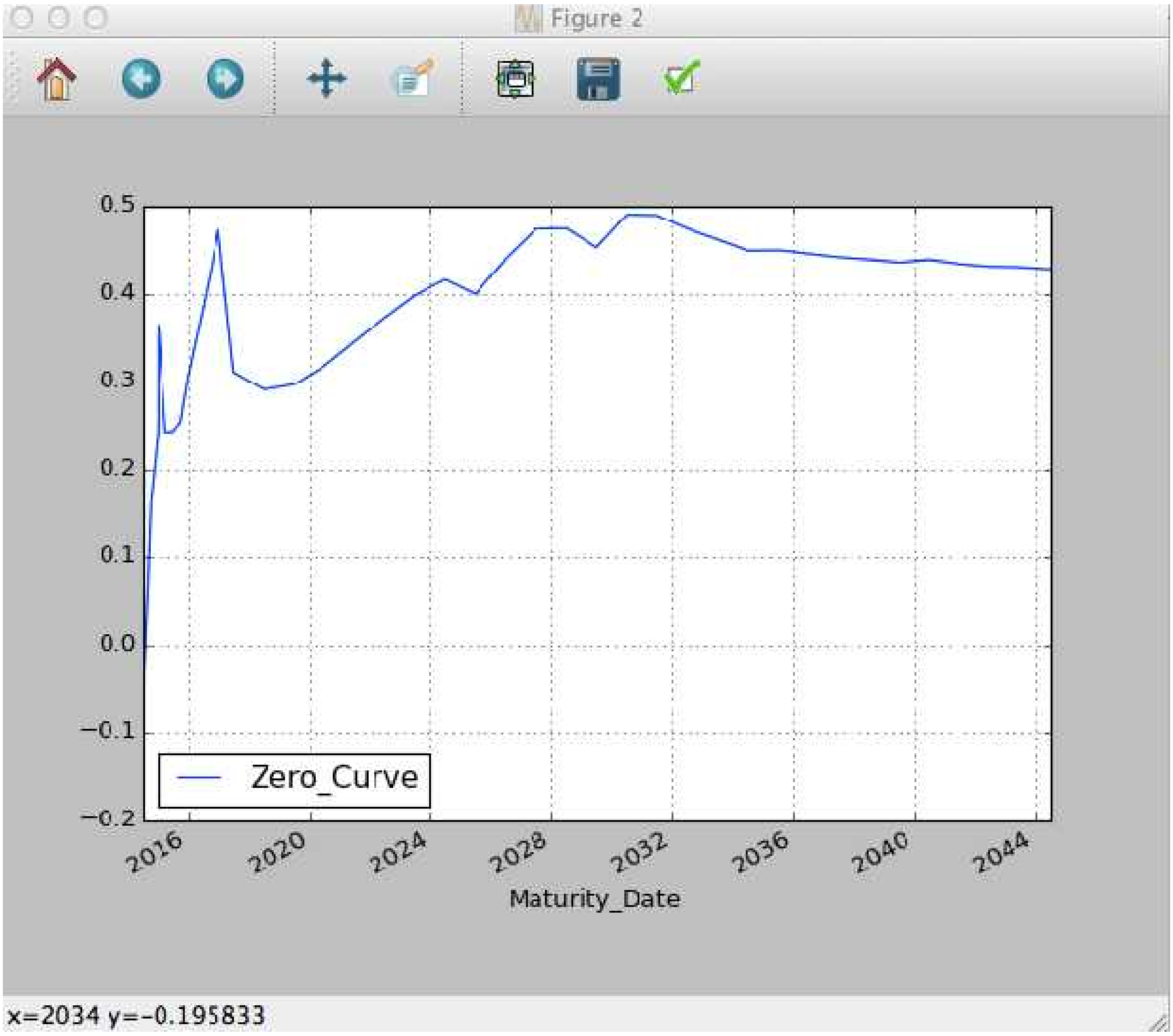
Tenor	Bid	Ask	mid	start date	maturity	period days
2Y	0.076	0.126	0.101	03-07-14	03-07-16	731
3Y	0.116	0.166	0.141	03-07-14	03-07-17	1096
4Y	0.173	0.223	0.198	03-07-14	03-07-18	1461
5Y	0.251	0.301	0.276	03-07-14	03-07-19	1826
6Y	0.353	0.403	0.378	03-07-14	03-07-20	2192
7Y	0.473	0.523	0.498	03-07-14	03-07-21	2557
8Y	0.6	0.65	0.625	03-07-14	03-07-22	2922
9Y	0.73	0.78	0.755	03-07-14	03-07-23	3287
10Y	0.848	0.898	0.873	03-07-14	03-07-24	3653
12Y	1.054	1.104	1.079	03-07-14	03-07-26	4383
15Y	1.281	1.331	1.306	03-07-14	03-07-29	5479
20Y	1.5	1.55	1.525	03-07-14	03-07-34	7305
25Y	1.597	1.647	1.622	03-07-14	03-07-39	9131
30Y	1.643	1.693	1.668	03-07-14	03-07-44	10958

4.2. Python Programming Illustration for the Zero Swap-Curve – Tenor 3M

For the above data, the following was the output using Python:

```
Python 2.7.8 |Anaconda 2.1.0 (x86_64)| (default, Aug 21 2014, 15:21:46)
[GCC 4.2.1 (Apple Inc. build 5577)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
Anaconda is brought to you by Continuum Analytics.
Please check out: http://continuum.io/thanks and https://binstar.org
>>> runfile('/Users/FADE/Documents/Bootstrapping/Swap Curve.py', wdir=r'/Users/FADE/Documents/Bootstr
apping')
Zero Cash : -0.134705526347
Zero Cash : -0.162222582715
Zero Cash :  0.0385279201165
Zero Cash : 0.0162221068652
Zero Cash : 0.0831330189122
Zero Cash : 0.160162111669
Zero Cash : 0.364664611347
Zero Forward : 0.243258741609
Zero Forward : 0.24326036252
Zero Forward : 0.243259552061
Zero Forward : 0.254404526646
Zero Forward : 0.306076348169
Zero Forward : 0.33610577048
Zero Forward : 0.472191114419
Zero Swap : 0.385880769948
Zero Swap : 0.311152646558
Zero Swap : 0.291275361526
Zero Swap : 0.297068501529
Zero Swap : 0.318368491381
Zero Swap : 0.346248343128
Zero Swap : 0.37333225672
Zero Swap : 0.398793403953
Zero Swap : 0.416711047824
Zero Swap : 0.399960415104
Zero Swap : 0.440313175864
Zero Swap : 0.47398445738
Zero Swap : 0.474443634762
Zero Swap : 0.45289845488
Zero Swap : 0.490331604542
Zero Swap : 0.489653993487
Zero Swap : 0.47397396081
Zero Swap : 0.461717941005
Zero Swap : 0.449282949363
Zero Swap : 0.449791619764
Zero Swap : 0.445915668756
Zero Swap : 0.441905004682
Zero Swap : 0.439508002178
Zero Swap : 0.436336518819
Zero Swap : 0.43926605124
```

The discount, zero and forward rate curve output is also as follows:



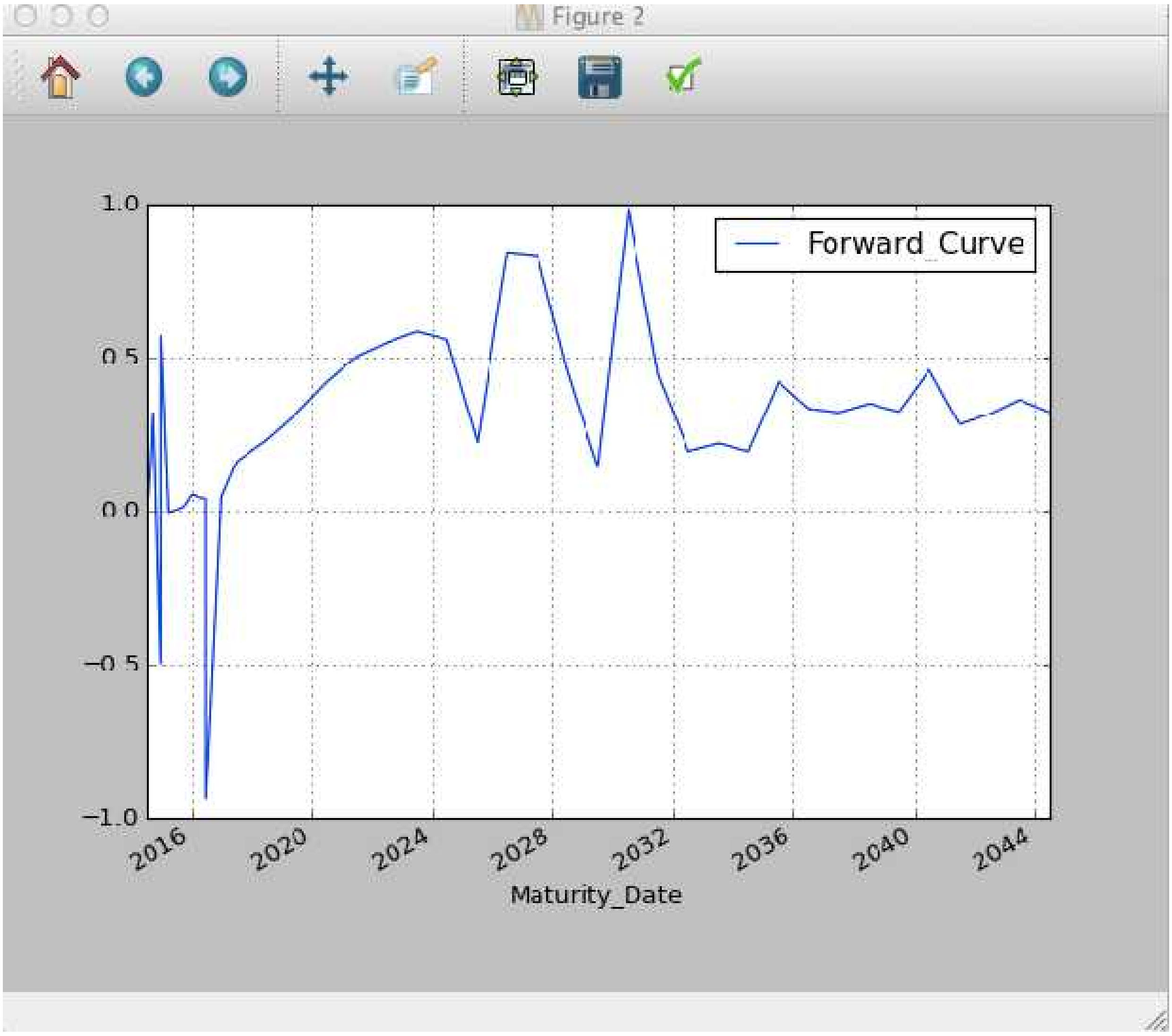
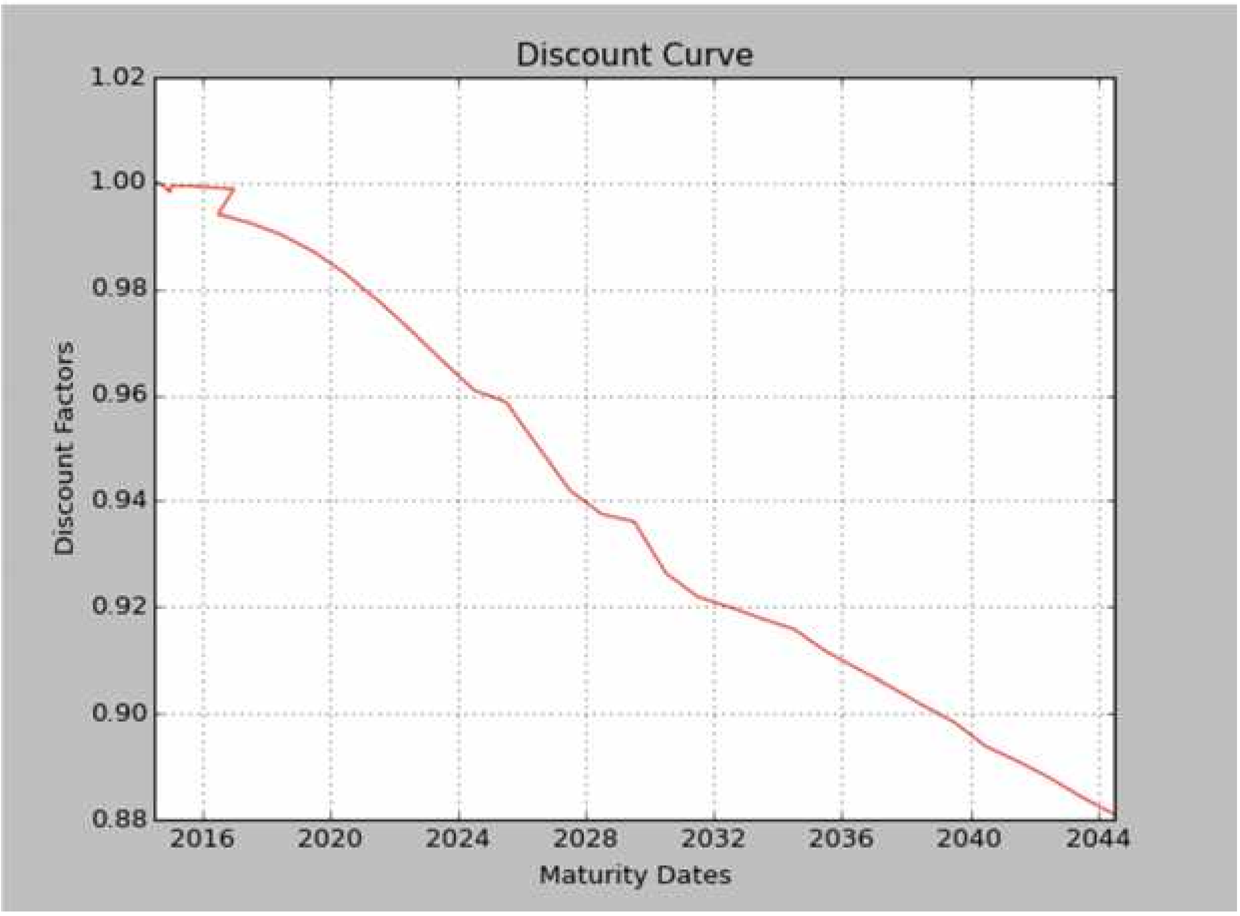


Figure 2
1.0
0.5
0.0
−0.5
−1.0
Forward_Curve
2016
2020
2024
2028
2032
2036
2040
2044
Maturity_Date



Discount Curve
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
Discount Factors
2016
2020
2024
2028
2032
2036
2040
2044
Maturity Dates

5. Conclusions

5.1 Comments

Bootstrapping a yield curve is important when valuing a variety of interest rate products. One of the main purposes of a yield curve is to value and price a portfolio of instruments and financial contracts. Bootstrapping a yield curve can be used to price a whole series of different interest rate instruments. Thus the yield curve can also be used to derive forward rates which are necessary to price instruments like interest rate swaps correctly.

5.2 Appendix

5.2.1 Zero swap curve for tenor 3M

```
import math
import pandas as pd

for (x,w) in zip(C1,M1):
    dc1 = 1 / ((1 + (float(w)/100) * ((float(x) / 360))))
    dc2 = dc1 / ((1 + (float(w)/100) * ((float(x) / 360))))
    dc3 = dc2 / ((1 + (float(w)/100) * ((float(x) / 360))))
    Z1 = -100 * math.log(float(dc1))/ (float(x) / 365)
    print "Zero Cash :",Z1

for (a,b) in zip(F1,M2):
    fr1 = b + ((b/100)/a)* a
    fr2 = math.exp(- float(fr1) * (a / 365))
    fr3 = float(fr2) * (1 / (1 + (b/100) * a/360))
    Z2 = -100 * math.log(float(fr3))/ (float(a) / 365)
    print "Zero Forward :",Z2

for (g,h) in zip(S1,M3):
    Sr = (dc2 - (h/100) * (math.exp((- h/100 + ((h/100)/g) * g * (g/ 365))))) / (1 + (h/100))
    Z3 = -100 * math.log(float(Sr))/ (float(g) / 365)
    print "Zero Swap :",Z3

for (j,k) in zip(M4,N1):
    F1 =( 36500 * k) / j
    print "Forward Cash :",F1

for l,o in zip(M5,N2):
    F2 =( 36500 * o )/ l
    print "Forward ForwardRate :",F2

for s,t in zip(M6,N3):
    F3 =( 36500 * t )/ s
    print "Forward Swap :",F3


df = pd.DataFrame.from_csv('ZeroForward.csv', parse_dates=True)
```

```
df.Zero_Curve.plot(color='g')
df.Forward_Curve.plot(color='r')
df.plot(legend=True)
```

5.2.2 Discount Curve

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.dates as mdates


def graph():
    date, value = np.loadtxt('Discount.csv', delimiter =',', unpack=True, converters =
{0:mdates.strpdate2num('%d-%m-%Y')})
    plt.plot_date(x=date, y=value, linestyle='-', marker="", color='r')
    plt.xlabel("Maturity Dates")
    plt.ylabel("Discount Factors")
    plt.title("Discount Curve")
    plt.grid(True)
    plt.show()

graph()
) :
```

6. References

[1] Röman, Jan, *Lecture Notes For Analytical Finance 2 (2014)*

[2] Lore Marc and Borodovsky Lev, *The Professional's Handbook of Financial Risk Management, Butterworth-Heinemann (2000)*