Mälardalen University

Vasicek

Find the term structure of interest rate

Analytical Finance II

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Date:2011-12-12

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Introduction

The Vasicek model (Vasicek 1977) is one of the earliest no-arbitrage interest rate models which is based upon the idea of *mean reverting* interest rates, which means that if the interest rate is above the long run mean (r > b), then the coefficient (a >0) makes the drift become negative such that the rate will be pulled down in the direction of r or vice versa. The coefficient ”a” is, thus, the speed of adjustment of the interest rate towards its long run normal level.

Vasicek gives an explicit formula for the (zero coupon) yield curve. It also gives explicit formulas for derivatives such as bond options and can be used to create an interest rate tree

Aim

The aim of this report is to use Vasiceks model to find the time structure of the zero coupon bond with the help of Excel /VBA application for Swedish government bond prices. Also we calculated the forward rates, the discount rate and plotted the probability distribution. Finally we will compare the curves we get from Excel.

Derived Formula for Vasicek Model

The yield-based one-factor equilibrium model is given by the dynamic:

$$dr = (b - ar)dt + dv$$

Affine Term structure

The affine term structure is used to describe the bond price P(t,T).

$$P(t,T) = F(r,t,T) = \ e^{(A(t,T) - rB(t,T))}$$

Where in the case of Vasicek:

$\mathrm{B}\ (\mathrm{t},\ \mathrm{T}) = (1/\mathrm{a})(1 - \mathrm{e}^{-\mathrm{a}(\mathrm{T}-\mathrm{t})})$ and

$\mathrm{A}\ (\mathrm{t},\ \mathrm{T}) = -\left(\frac{b}{a}\right)\{T - t - B(t,T)\} +\ \sigma^2/2a^2\ \{T - t - 2B(t,T) + (\frac{1}{2a})(1 - e^{-2a(T-t)})\}$

b/a: long-term equilibrium of mean reverting spot rate process

σ: the standard deviation of the interest rate

r(0): is the zero rate

Valuation of Zero coupon bond

If A(t,T) and B(t,T) are combined in the affine term structure, the following formula for the zero coupon bond is obtained:

$$P(0,T) = \ e^{\left\{\frac{1}{a}\right)(B(t,T))\left(\frac{b}{a}\right) - \left(\frac{1}{2}\right)((\sigma/a)^2) - \ r(0)) - T(\left(\frac{b}{a}\right) - \left(\frac{1}{2}\right)(\sigma/a)^2) - ((\sigma^2)/(4a^3))B(t,T)^2\}}$$

Vasicek Model Graph (probability distribution

The spot rate is normal distributed with mean and variance by:

$$E\big(r(t)\big) = r(0)e^{-at} + \frac{b}{a}(1 - e^{-at})$$

$$Var[r(t)] = \frac{\sigma^2}{2a}(1 - e^{-at})$$

Because the expectation and variance are both time dependent we plotted the general distribution where t goes to infinity the mean is the long term equilibrium a/b and the variance is $\sigma^2 = \frac{\sigma^2}{2a}$.

VBA Program

We downloaded the market data for Swedish government bond at nasdaqomxnordic.com/bonds/Sweden. For the calculation of our Vasicek price we wrote a program on Excel using Microsoft Visual Basic. All input variables are declared as follows:

Variable	parameter
The interest rate	a
The volatility	b
The maturity date	r
The settlement date	st
The Day count basic(4) type(30/360)	dayc
The next coupon date	nc
The face value of the bond	fv
Annual coupon rate	coupon

To make our program easy to read, we wrote a Vasicek price function in which we calculated first A(t,T) and B(t,T) inside the function and then combine these to calculate the price. The function is condensing into the table below:

Function	Formula	Data type
PriceV	Exp(AA - BB * r)	Double

Where the term structure AA and BB are as follows:

Term structure	Formula
AA	-(b / a) * (yr - (1 / a) * (1 - Exp(-a * yr))) + ((sd ^ 2) / (2 * (a ^ 2))) * (yr - (2 / a) * (1 - Exp(-a * yr)) + (1 / (2 * a)) * (1 - Exp(-2 * a * yr)))
BB	(1 / a) * (1 - Exp(-a * yr))

- **Application.WorksheetFunction.YearFrac** is used in the function to return the fraction of the year between the maturity and the settlement date.

Calculation of Vasicek price

The Vasicek price is based on a zero coupon bond. Because the model has to be fitted to bonds with coupons we wrote a code which approaches every coupon as a zero-coupon bond itself and then adds up the Vasicek price of each coupon and of the final payment of the face value.

The formulas used to calculate the Vasicek price are given in the table below. We use the **DO while loop**, to add automatically the price as long as the next coupon date is less than the time to maturity. Since our frequency is yearly, we declare an additional date variable (td) equal to the next coupon date then will add twelve month for the next increment.

Function	Formula	Data type
Vasicek	Vasicek = fv * (1 + coupon) * PriceV(a, b, r, sd, T, st, dayc)	Double
	Loop formula below:	
Vasicek	Vasicek = Vasicek + fv * coupon * PriceV(a, b, r, sd, td, st, dayc)	Double

- all function data type (except for the dates) are declared as double because it provides the greatest and the smallest possible magnitudes for a number and the default value is 0.

Another requirement of the application is that it is able to switch between different day count conventions. In our program this is possible. In the data sheet a date count basis is included. Currently day count 4 is used which stands for 30/360, but it is also possible to use the other three options, by inserting their number:

1. Actual/actual
2. Actual/360
3. Actual/365

To match the Vasicek model we use the excel solver to minimise the squared error between the Vasicek price and the market price. This results in values of a, b, r (0) and volatility.

Results

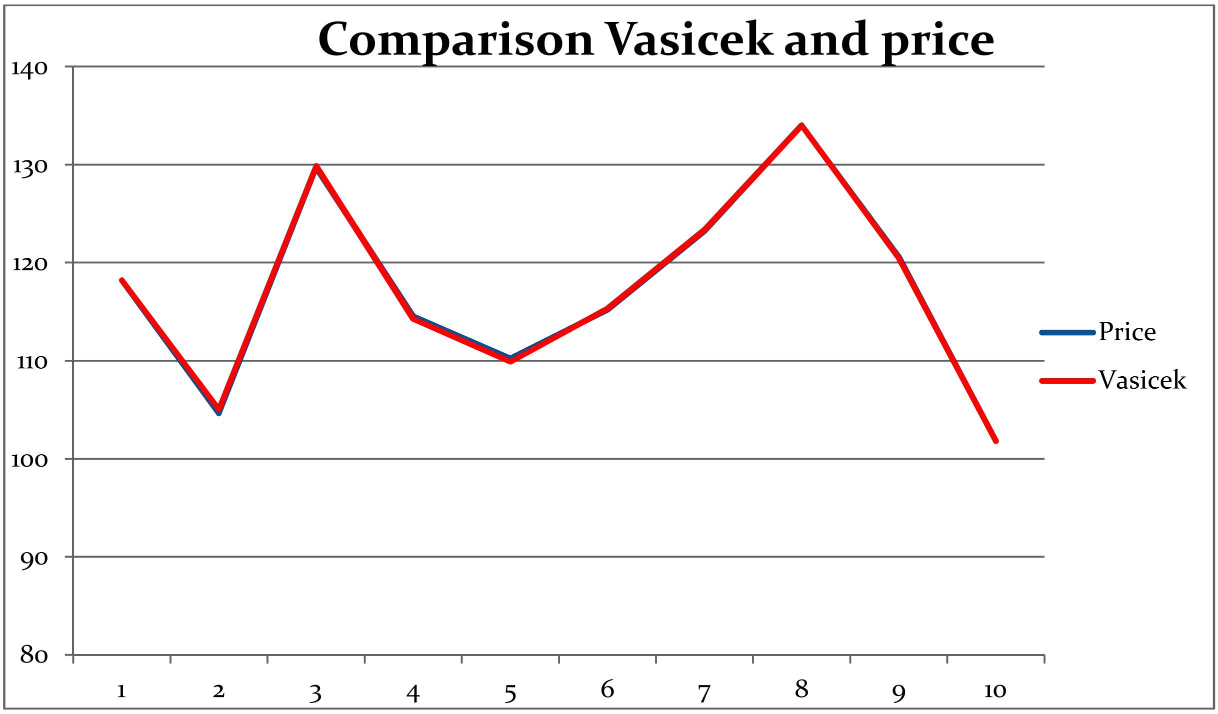
The first result is the term structure of interest rate, which was the main goal of this seminar. This term structure and the forward rate had to be plotted. Also the distribution of the rate and the discount factors had to be plotted.

Term structure of interest rate and forward rate

By using the Excel solver function, which minimizes the sum of the squared error by changing a, b, r(0) and σ, we got the following results:

Variable	Estimated value
a	3,67%
b	0,36%
r(0)	0,40%
σ	1,71%
b/a	9,88%

Most values are as expected. Only the long term equilibrium is a bit high in comparison to the yield on the bonds. The following graph shows the given price and the fitted Vasicek price with the parameters as mentioned above. As can be seen there is a very good fit (at for example point 2 it is actually possible to see the line of the original price).



Since we estimated all the needed variables we can plot the term structure of interest rate by using the following formula:

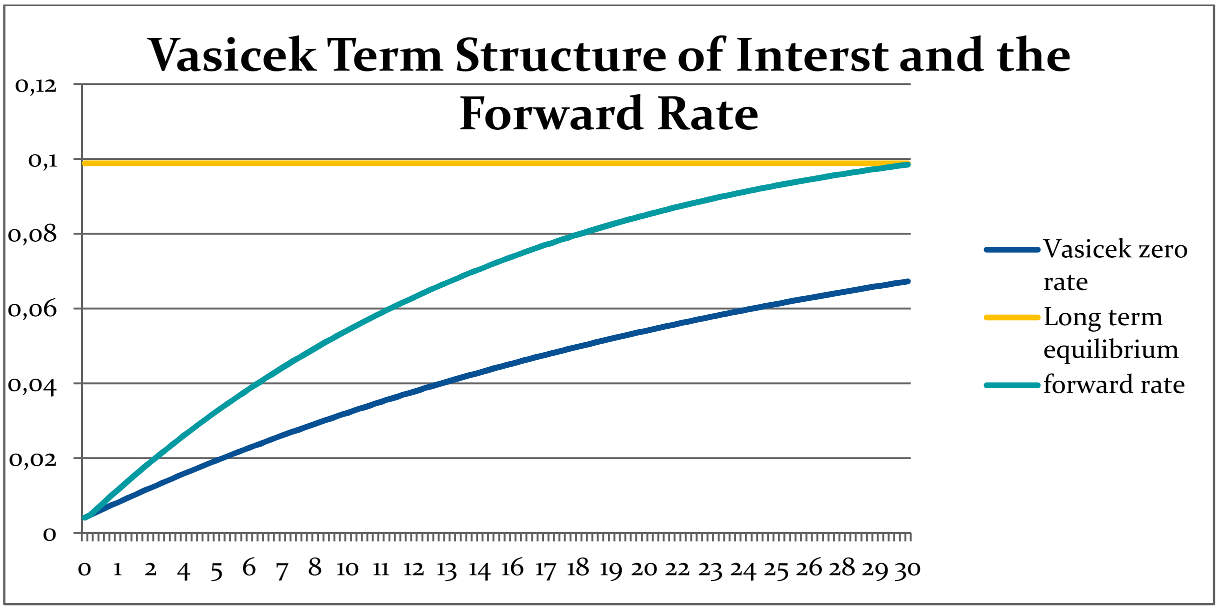
$$E[r(t)] = \frac{b}{a} + \left(r(0) - \frac{b}{a}\right)e^{-at}$$

From the interest rate term structure we calculated the forward rate by the following formula:

$$r_{t_2-t_1}^{forward} = \frac{1}{t_2 - t_1}\left(\frac{\left(1 + r_{t_2}^{spot}\right)^{t_2}}{\left(1 + r_{t_1}^{spot}\right)^{t_1}} - 1\right)$$

Were we know that the first forward rate is equal to the spot rate.

Plotting these two functions and the long term equilibrium on the interval [0, 30], gives the following result:



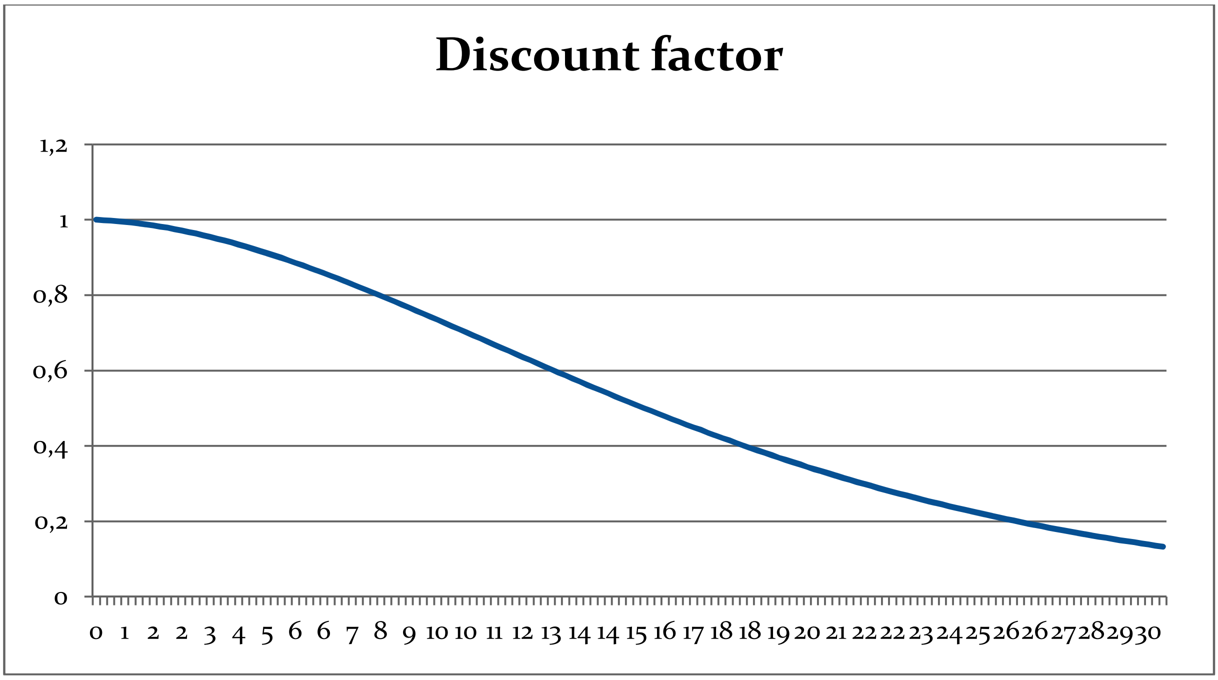
The interest rate and the forward rate will both converge to the long term equilibrium as expected. The forward rate is higher than the spot rate because the spot rate is increasing. Especially when the spot rate increases a lot the forward rate is much higher than the spot rate because the forward rate is based on the difference between two spot rates. If the difference is bigger the forward rate has to be higher.

Discount factors

The discount rates can be easily computed from the term structure. We used the following formula:

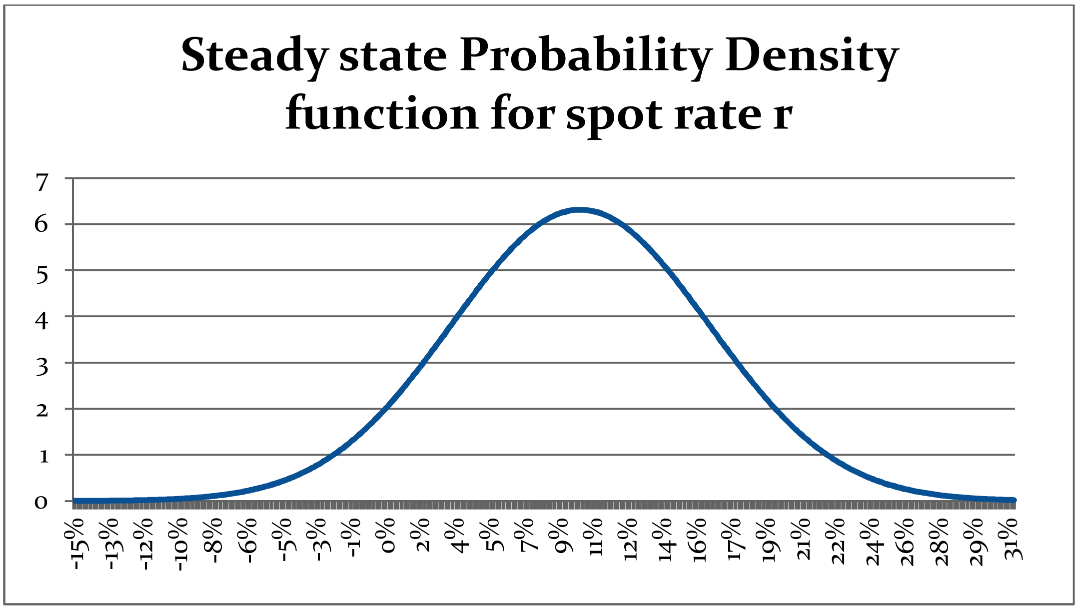
$$discount\ factor_t = e^{-E[r(t)]t}$$

This results in the following plot:



Distribution of interest rate

From the estimated parameters we plotted the probability density function. We know the mean which is the long term equilibrium a/b, we also know the volatility $\sigma^2 = \frac{\sigma^2}{2a}$.



One of the disadvantages of the Vasicek model can be seen from this distribution. The interest rate can also become negative, which is a big disadvantage because this is not according to reality.