Creating a zero coupon curve

by bootstrapping with cubic splines.

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Abstract

In this report we calculate spot rate curve by bootstrapping and interpolation with cubic splines. For this task, Ms Excel application is developed and presented in later section.

Contents

1. Introduction

The concept behind zero coupon pricing is the evaluation of all individual cash flows as if they were zero coupon bonds. The evaluation is made using a spot rate curve which accurately describes current market conditions. The pricing of liquid, standardized instruments are quite simple - the current market price is used. The zero-coupon pricing methodology becomes important when pricing OTC instruments, for which no market prices are available. It is also needed for pricing standardized instruments, which do not have reliable market prices. In this case, zero coupon pricing will be used to price these instruments consistently alongside the liquid instruments. This is a kind of relative pricing [1].

During such pricing approach, bootstrapping & interpolation method is used for obtaining spot rate curve. In this report we are building spot rate curve using market data and method of interpolation with cubic splines Also, a general Ms Excel application, which performs all calculations, is presented.

2. Problem description

2.1 Bootstrapping

Bootstrapping is a procedure used to calculate the zero-coupon curve from the given market data. Because zero-coupon bonds offered by the market are not available for every time period, the bootstrapping method is used to fill in the missing figures in order to derive the zero-coupon curve. The bootstrap method uses interpolation to determine the spot rates for zero-coupon securities with various maturities.

So it makes sense to construct a curve of zero-coupon instruments from which we can price any yield, whether forward or spot, without the need of more external information.

As was mentioned above, the bootstrap method uses some interpolation method. It depends on interpolation method, which values will we get between given spot rates. Interpolation methods vary significantly and, in fact, it is the topic for a separate textbook. In this paper, interpolation with cubic splines is selected as the interpolation method for bootstrapping, which we will highlight in more details.

More on interpolation methods as well, as development of new technique can be found in [2].

2.2 Cubic splines interpolation.

Spline interpolation is a form of interpolation where the interpolant is a special type of piecewise polynomial called a spline. In case of third-order polynomials, such splines are called cubic.

For a data set $\{x_i\}$ of $n+1$ points, we can construct a cubic spline with n piecewise cubic polynomials between the data points.

$$S(x) = \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \dots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

If S(x) represents the spline function interpolating function *f*, it is required:

- The interpolation property *S(x_i)=f(x_i) for all i*. Actually, in our case, interpolated function is a set of discrete numbers.

- Twice continuously differentiable:
$$\begin{aligned} S_{i-1}(x_i) &= S_i(x_i) \\ S'_{i-1}(x_i) &= S'_i(x_i) \quad i = 1,\dots,n-1 \\ S''_{i-1}(x_i) &= S''_i(x_i) \end{aligned}$$

The assumed form for the cubic polynomial curve fit for each segment is:

$$S_i(x) = a_i(x - x_i) + b_i(x - x_i)^2 + c_i(x - x_i)^3 + d_i$$

Using this representation and required properties, the cubic spline properties are found by solving the system of equations. Finaly, *a,b,c,* and *d* values correspond to the polynomial definition for each segment.

$$a_i = \frac{y_{i+1} - y_i}{h_i} - \frac{2h_i S_i + h_i S_{i+1}}{6}$$
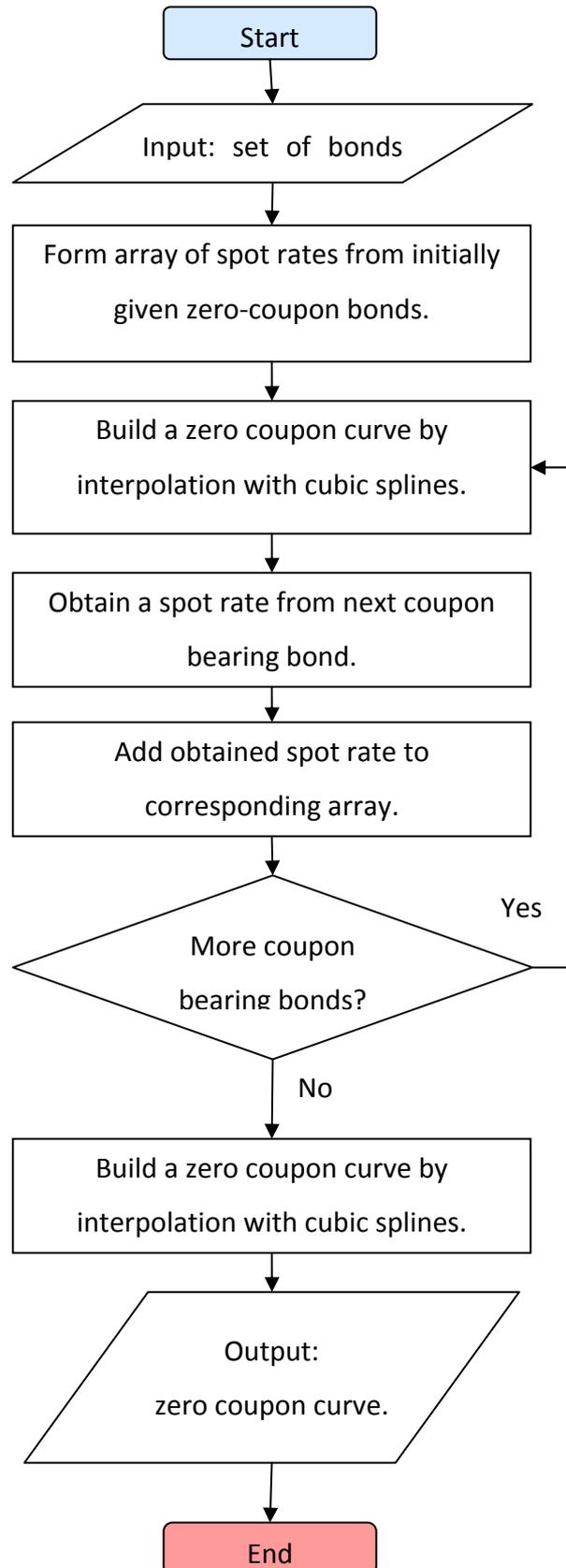$$b_i = S_i / 2$$
$$c_i = (S_{i+1} - S_i)/6h_i$$
$$d_i = y_i$$
$$and \quad h_i = x_{i+1} - x_i$$

More on cubic splline interpolation can be read in [1] and [2].

2.3 Computational algorithm

Our application is built according to the following algorithm.

```
                        ┌─────────────┐
                        │    Start    │
                        └─────────────┘
                               │
                               ▼
                    ╱─────────────────────╲
                   ╱   Input:  set  of  bonds ╲
                  ╱───────────────────────────╱
                               │
                               ▼
              ┌─────────────────────────────────┐
              │  Form array of spot rates from   │
              │   initially given zero-coupon    │
              │             bonds.               │
              └─────────────────────────────────┘
                               │
                               ▼
              ┌─────────────────────────────────┐
              │   Build a zero coupon curve by   │◄──┐
              │ interpolation with cubic splines.│   │
              └─────────────────────────────────┘   │
                               │                     │
                               ▼                     │
              ┌─────────────────────────────────┐    │
              │  Obtain a spot rate from next    │    │
              │      coupon bearing bond.        │    │
              └─────────────────────────────────┘    │
                               │                     │
                               ▼                     │
              ┌─────────────────────────────────┐    │
              │      Add obtained spot rate to   │    │
              │       corresponding array.       │    │
              └─────────────────────────────────┘    │
                               │                     │
                               ▼                     │
                    ╱─────────────────╲     Yes       │
                   ╱   More coupon      ╲─────────────┘
                   ╲   bearing bonds?   ╱
                    ╲─────────────────╱
                               │ No
                               ▼
              ┌─────────────────────────────────┐
              │   Build a zero coupon curve by   │
              │ interpolation with cubic splines.│
              └─────────────────────────────────┘
                               │
                               ▼
                    ╱─────────────────────╲
                   ╱      Output:           ╲
                  ╱     zero coupon curve.    ╱
                 ╱───────────────────────────╱
                               │
                               ▼
                        ┌─────────────┐
                        │     End     │
                        └─────────────┘
```

3. Application overview

Application is built in MS Excel using VBA programming language. It contains two spreadsheets – one for input data and computations, the second one – fro displaying a diagram.

As was mentioned in the previous section, algorithm begins with input data. Input data contains bond names, information about coupons and their frequencies, market prices and time properties. By default it is set to annually.

If there was a transaction, the last price is taken to account, otherwise – average of bid and ask is considered as the price of the asset. Bonds are quoted in clean prices, so in order to go from clean price to the dirty one, we add to the given clean price accrued, since last coupon payment, interest rate .

Nominal amount, by default, is set to 100. It means, that if this field is left empty, value of 100 is assigned to the nominal amount property of the bond.

Time to maturity is calculated with built-in excel function "Yearfrac()" using given bonds expiration date and today's one.

The application understands five types of day count conventions:

- US 30/360
- Actual/actual
- Actual/360
- Actual/365
- European 30/360

By default, day count is set to US 30/360.

	A	B	C	D	E	F	G	H	I	J	K	M	N
1						Input data							
2	Name	Coupon	Frequency	Nominal	Bid	Ask	Last	Exp.	Day count	Maturity	Spot rate		Run
3	KKDKAKT11			100	98,50	99,50	99,96	27.02.2011		0,1889	0,1907%		
4	KIDKAktier11				97,50	98,50	99,00	27.04.2011		0,3556	2,7535%		
5	NBFGLOBAL11			101	99,00		99,05	14.09.2011		0,7361	2,6335%		Diagram
6	DBINF11011	9,00%	2	96	116,29	116,30		04.04.2012		1,2917	4,2954%		
7	DBKlimatek11	1,00%		110	98,64	99,00		08.10.2012		1,8028	6,2428%		☐ Plot forward rate
8	KKALTNRG11			120	98,51	99,50	90,00	20.06.2014		3,5028	8,5160%		
9	KITyskeAkt11	1,00%	12	120	104,01	108,00	120,00	22.06.2015		4,5083	12,1226%		
10	KITyskeAkt12			180	98,89	98,89		27.01.2016		5,1056	12,4099%		
11	KITyskeAkt13	3,00%	2	130	99,86	99,86		27.03.2016		5,2722	13,3091%		
12	KITyskeAkt14			250			99,60	08.11.2017		6,8861	14,2680%		
13													
14													
15													

Figure 3.1

By pressing "Run" button, computational algorithm, that was overviewed in section 2.3 is launched.

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As the result, in output there is a set of spot rates, recovered from the initial market data.

By pressing button "Diagram", we give instruction to display a diagram on the separate spreadsheet. Red dots on the plot correspond to initial points, from the column "Spot rates". Blue curve is obtained by cubic interpolation.
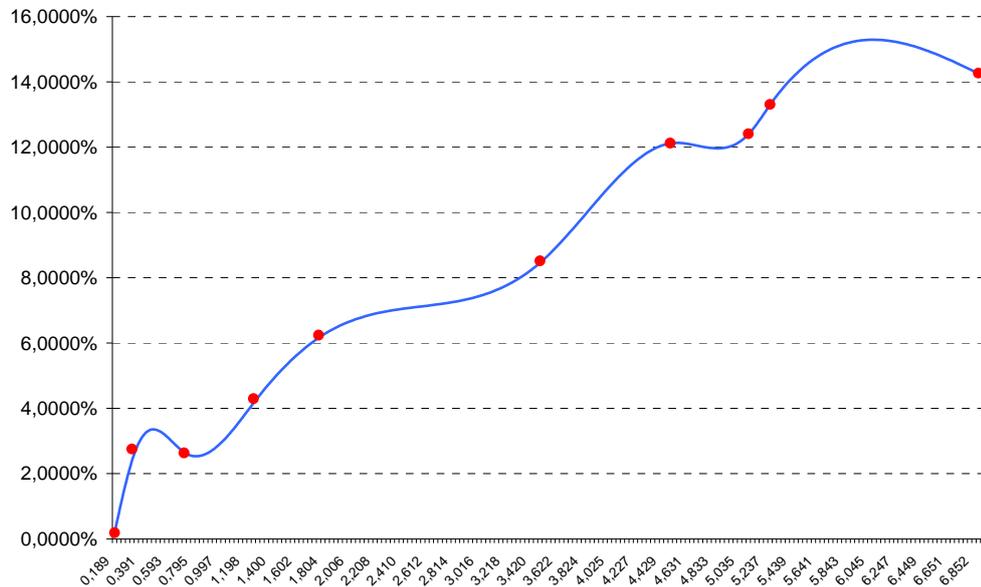

Figure 3.2

Checkbox "Plot forward rate" controls displaying of the forward rate curve, obtained from the spot-rate curve. If checked, forward rate curve is displayed as well.

After bootstrapping & interpolation process is finished, one can start pricing bonds.

Section "Price a bond" contains fields of input information. After filling corresponding fields and pressing button "Prices" theoretical dirty price is given.

	O	P
	Price a bond	
Name		
Coupon		3,00%
Frequency		2
Nominal		100
Start date		09.12.2010
Expiration date		27.03.2011
Time to maturity		0,3000
Difference in time		0,0278
Day count		
Theoretical price:		**102,1904**
	Price	

Figure 3.3

Conclusion

Zero coupon pricing is well developed technique of relative pricing. Spot rates for corresponding cash flows are taken from the spot rate curve This curve is built using spot rates, that are calculated using market data. Then the corresponding set of values of spot rates is interpolated. So, in such way, spot rates curve is obtained. There is a big number of interpolation methods available, modern techniques are discussed in [2]. In this report we have used cubic splines as interpolation method. Developed Ms Excel application can be used for zero curve constructing and further valuation of bonds.

References

[1] Jan R. M. Röman, *Lecture notes in Analytical Finance II,* 2010

[2] Patrick S. Hagan, Graeme West, *Interpolation methods for curve construction,* Applied Mathematical Finance, Vol. 13, No. 2, 89–129, June 2006

[3] Espen Gaarder Haug, *The complete guide to Option Pricing Formulas*, 2nd ed., Mc-Graw-Hill, New York, 2007

[4] Charles O'Neill, *Cubic spline interpolation,* 2002.