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Division of Applied Mathematics

Analytical Finance II

Seminar paper on Bootstrapping

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This paper presents an Excel application for a zero coupon curve using bootstrapping. The table below present data extracted from NASDQAOMX Stockholm. It represents data for the Swedish bonds and Treasury Bills with different maturities. The former has coupon payments whereas, the later has none. From this data, we calculate the spot rates and using the arbitrage arguments, we derive the forward rates. We then extend the zero coupon curve to 30 years by using constant forward rate. We will plot the curves and use the curve to value other bonds. The rate shall be found in the intermediate points by interpolation.

Name	Yield	Maturity	Coupon
1012	1.11%	12/15/2010	0.00
1101	1.07%	1/19/2011	0.00
1102	1.10%	2/16/2011	0.00
1045	1.13%	3/15/2011	5.25
1046	1.60%	10/8/2012	5.50
1041	2.19%	5/5/2016	6.75
1049	2.48%	8/12/2015	4.50
1050	2.68%	7/12/2016	3.00
1051	2.83%	8/12/2017	3.75
1052	2.97%	12/1/2019	4.25
1047	3.08%	1/12/2020	5.00
1053	3.67%	3/30/2039	3.50

Table 1: Data for Swedish Fixed Income Derivatives

Bond prices can be calculated using the following related instruments which pay no coupons and also for instruments which pay coupons.

For bonds that pay coupons, we have:

$$\mathrm{P} = \frac{N}{(1+ytm)^t} + \sum_{i=0}^{n} \frac{C}{1+ytm^t}$$

For bonds we no coupons payment, the price can be calculated as follows:

$$\mathrm{P} = \frac{100}{1+ytm\,.\frac{d}{360}}$$

The spot rate

From the table above, we notice that there was only one coupon bond with maturity less than a year. Also, from our lecture notes, we noticed that Swedish bonds have annual frequency. Thus coupons are paid annually. The spot rates for T-bills (no coupons) are the same as their YTM. The spot rates for coupon paying bonds can be deduced from their respective bond prices. We start by stripping the instruments to find the corresponding zero coupon rates.

$$\text{ZCP} = \text{P} - \text{PV (coupon)} * \frac{100}{100+C}$$

Notice that we can subtract the present value of the coupons from the present value of the bond. To calculate the present value of all coupons with payout before the maturity of the bond, we discount it with their respective YTM.
We repeat the process above again by taking the next bond that has the least number of coupons. From this, we get the zero coupon bond prices as shown in the formula above. We then calculate the spot rates from their respective zero coupon prices by using the respective discount factors. However, we interpolate to find the rates in between the two known rate r_1 and r_2. r_0 is the spot rate at time zero . The slope of the line was calculated using the following formula:

$$\frac{\Delta y}{\Delta x} = \left(\frac{r_2 - r_1}{t_2 - t_1}\right)$$

To find r_0

$$\left(\frac{r_1 - r_0}{t_1}\right) = \left(\frac{r_2 - r_1}{t_2 - t_1}\right)$$

$$r_1(t_2 - t_1) - r_0(t_2 - t_1)$$

$$r_0 = \left(\frac{r_1 t_1 - r_1 t_2}{t_1 - t_2}\right)$$

The next interpolation point (rate) is calculated by multiplying the time interval and the slope obtained above and adding the spot rater_0.

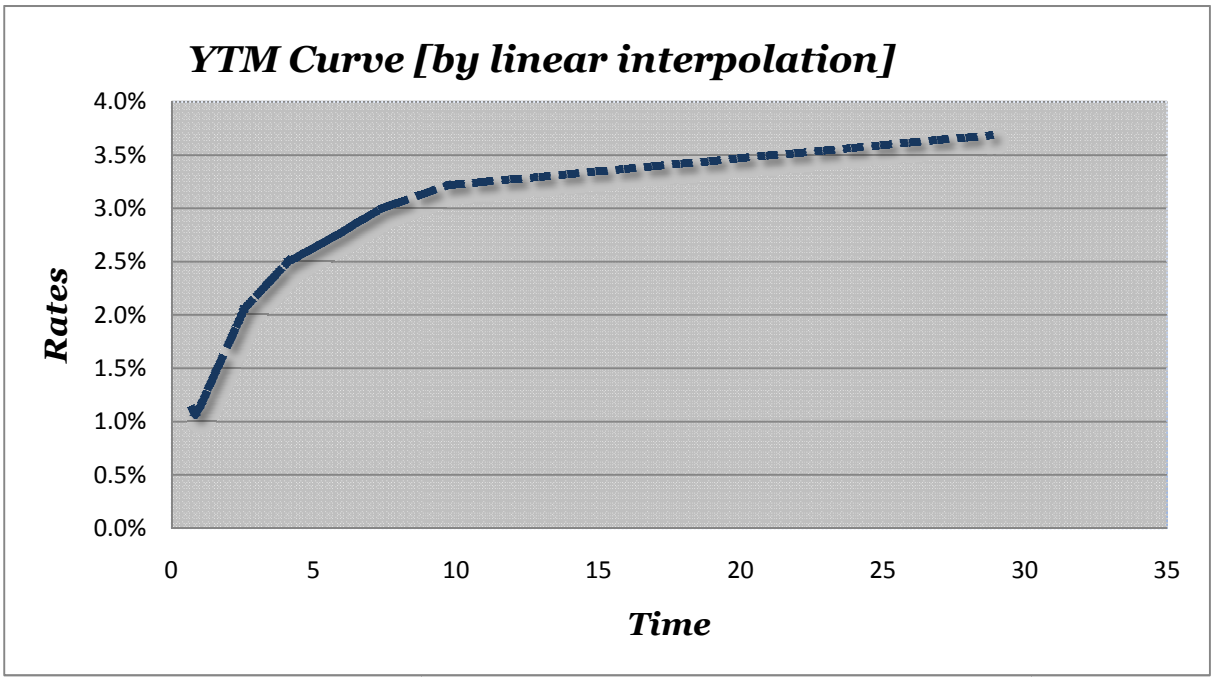
The Forward rate

In order to calculate the forward rates, we use the arbitrage argument from the spot rates as follows:

$$r_{t_2-t_1}^{forward} = \left(\frac{\left(1 + r_2^{spot}\right)^{t_2}}{\left(1 + r_1^{spot}\right)^{t_1}} \right)^{\frac{1}{t_2 - t_1}} - 1$$

The results are as follows:

Name	Yield	Maturity	Time		Coupon	Bond Price	PV(C)	P-C	ZCP	spot	*i*
1012	1.11%	12/15/2010	0.758	15-Dec	0.00	99.164	0.00	99.164	99.164	1.112%	1
1101	1.07%	1/19/2011	0.853	19-Jan	0.00	99.097	0.00	99.097	99.097	1.068%	1
1102	1.10%	2/16/2011	0.928	16-Feb	0.00	98.990	0.00	98.990	98.990	1.100%	1
1045	1.13%	3/15/2011	1.008	15-Mar	5.25	104.069	5.191	98.878	93.946	1.134%	1
1046	1.60%	10/8/2012	2.572	8-Oct	5.50	106.657	5.365	101.292	96.011	2.056%	2
1041	2.19%	5/5/2014	4.147	5-May	6.75	116.899	13.027	103.872	97.304	0.686%	4
1049	2.48%	8/12/2015	5.417	12-Aug	4.50	108.279	16.765	91.514	87.573	2.689%	5
1050	2.68%	7/12/2016	6.333	12-Jul	3.00	100.859	13.744	87.115	84.578	2.831%	6
1051	2.83%	8/12/2017	7.417	12-Aug	3.75	104.563	20.195	84.368	81.319	2.998%	7
1052	2.97%	12/1/2019	9.719	1-Dec	4.25	107.667	29.245	78.421	75.224	3.214%	9
1047	3.08%	12/1/2020	10.719	1-Dec	5.00	113.825	37.953	75.872	72.259	3.303%	10
1053	3.67%	3/30/2039	29.050	30-Mar	3.50	96.822	60.621	36.201	34.977	3.689%	29



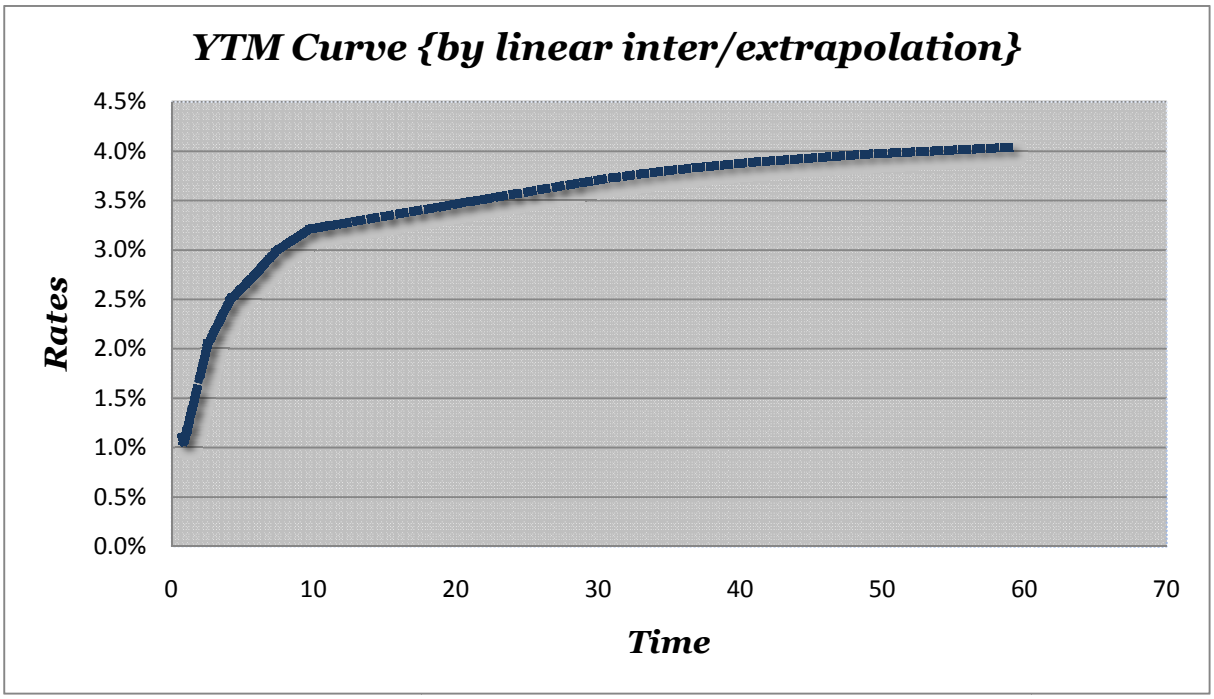


Fig. 2 Yield Curve by Linear Inter/ extrapolation