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Caps and Floors

(MMA708)

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ABSTRACT

The behavior of the floating interest rate is so complicate to predict. The investor who lends or borrows with the floating interest rate will have the risk to manage his asset. The interest rate caps and floors are essential tools to hedge risks. Our report will show their useful and present how to price them.

INTRODUCTION

Caps and floors are one of the most popular interest rate derivatives in the over- the-counter market which are used as tools to manage floating interest rate. They protect against adverse rates risk, while allowing gains from favorable rate movements. Caps and floors are forms of option contracts, conferring potential benefits to the purchaser and potential obligations on the seller. When purchasing a cap or floor, the buyer pays a premium- typically up-front. Though their characteristics are similar to the option contract but they are more difficult to value because of some behaviors of an underlying asset.

To find their values, it is necessary to understand their basic concepts. That is the first part of our paper. We introduce the definitions and then explain how to derive the pricing formula. Next we present the parameters affecting the price, some strategies, the advantages and disadvantages. Next part is using Black's model to price the caps and floors. Then we introduce collars which is the composition of caps and floors. A collar is created by purchasing a cap or floor and selling the other. The premium due for the cap (floor) is partially offset by the premium received for the floor (cap), making the collar an effective way to hedge rate risk at low cost. For more understanding, we illustrate the numerical examples for caps, floors and collars in the next part. The last part, we built an application in Visual Basic Application (VBA) which is a useful tool to value caps, floors and collars. We present it by the simply example to understand how to use this application.

Interest rate derivatives are instruments whose payoffs are dependent in some way on the level of interest rate. But they are more difficult to value than equity and foreign exchange derivatives by a number of reasons.

1. The behavior of an individual interest rate is more complicated than that of a stock price or an exchange rate.
2. For the valuation of many products it is necessary to develop a model describing the behavior of the entire zero-coupon yield curve.
3. The volatilities of different points on the yield curve are different.
4. Interest rates are used for discounting as well as for defining the payoff from the derivative.

(Hull, John C., 2006, p. 611)

There are many interest rate derivatives in the over-the-counter market. One of the most popular derivatives is the interest rate caps and floor.

Notation:

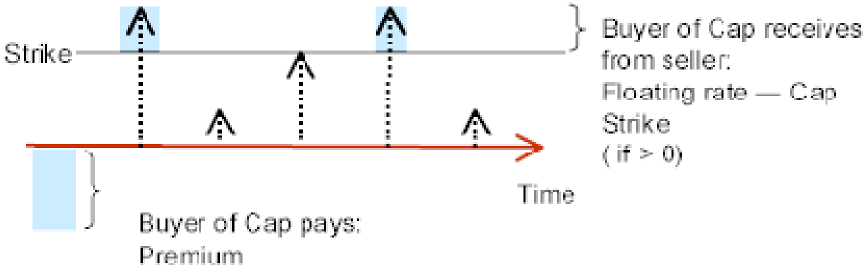
N	:	Nominal amount
δ	:	Tenor of the interest rate derivative
$l(t)$	:	Money market interest rate
K	:	Cap rate or Floor rate (Strike)
F	:	Forward rate
P(t,T)	:	Price at time t of zero-coupon bond paying $1 at time T
T	:	Time to maturity of the interest rate derivative
σ	:	Volatility

CAPS AND CAPLETS

Definition

An interest rate cap is designed to provide insurance against the floating interest rate rising above a certain level. This level is known as the "cap rate". It protects a borrower against the risk of paying very high interest rate. The borrower, who buys the caps, receives payments at the end of each period in which the interest rate exceeds the agreed strike price. More specifically, it is a collection of caplets, each of which is a call option on the LIBOR rate at a specified date in the future.

Bank and institutions will use caps to limit their risk exposure to upward movements in short term floating rate debt. Caps are equally attractive to speculators as considerable profits can be achieved on volatility plays in uncertain interest rate environments.



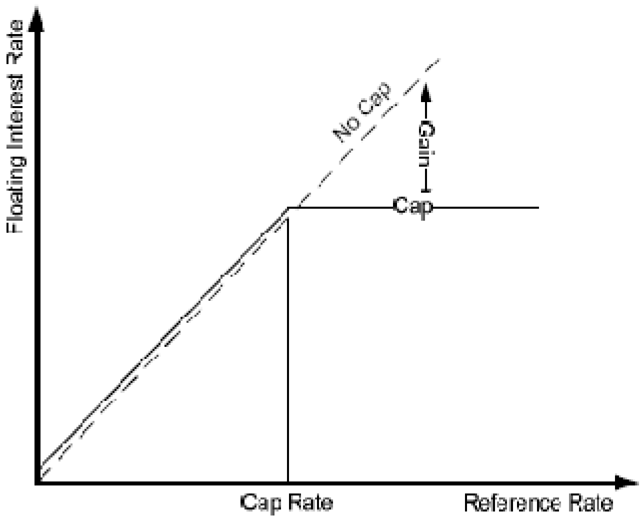
Suppose we have a loan with face value of N and payment dates $t_1 < t_2 < \ldots < t_n$, where $t_{i+1} - t_i = \delta$ for all i. The interest rate ($l(t_i, t_i - \delta)$) to be paid at time t_i is determined by the δ-period money market interest rate prevailing at time $t_i - \delta$. For example, the interest rate to be paid in the next 90-day ($t_{90\text{-}day}$) is determined by the 90-day money market interest rate prevailing at now (t_0). The payment at time t_i is equal to the face value multiplies by the interest rate and the period, which we can write in the mathematic term as $N\delta l(t_i, t_i - \delta)$. Note that the interest rate is set at the beginning of the period, but paid at the end. Define $t_0 = t_1 - \delta$. The date's $t_0, t_1, \ldots t_n$ -1 where the rate for the coming period is determined are called the reset dates of the loan.

If a borrower buy a cap with a face value N, the payment dates t_i where i = 1,…,n as above and the cap rate K yields a time t_i . The cap's payoff is $N\delta\max\{l(t_i, t_{i-\delta}) - K, 0\}$, for i = 1,2,…, n. The period length δ is often referred to as the tenor of the cap. In practice, the tenor is typically either 3, 6, or 12 months. The time distance between payment dates coincides with the "maturity" of the floating interest rate.

A cap can be seen as a portfolio of interest rate options. This cap leads to a payoff at time t_i of

$$C_{t_i}^{i} = N\delta \max\left\{l(t_i, t_{i-\delta}) - K, 0\right\} \qquad (1)$$

This equation is a call option on the LIBOR rate observed at time $t_{i-\delta}$ with the payoff occurring at time t_i. The cap is a portfolio of n such options. LIBOR rate are observed at time $t_{i-\delta}$ for a period δ and the corresponding payoff occur at time $t_1, t_2, t_3, \ldots, t_n$. The n call options underlying the cap are known as caplets. The payoff diagram is shown as in the figure below.



Cap Payoff

From the graph, if there is no cap, the borrower has to pay more and more for the higher floating interest rate. But if there is a cap, that means the borrower has a limit to pay not more than the cap rate.

An interest rate cap can also be characterized as a portfolio of put options on zero-coupon bonds with payoffs on the puts occurring at the time they are calculated. The payoff at time t_i is equivalent to

$$\frac{N\delta}{1+\delta l(t_i,t_{i-\delta})}\max\{l(t_i,t_{i-\delta})-K,0\}$$

or

$$\max\left(N-\frac{N(1+\delta K)}{1+\delta l(t_i,t_{i-\delta})},0\right)$$

The expression

$$\frac{N(1+\delta K)}{1+\delta l(t_i,t_{i-\delta})}$$

is the value of a zero-coupon bond that pays off $N(1+\delta K)$ at time t_i. The expression above is therefore the payoff from a put option, with maturity t_i, on a zero-coupon bond with maturity t_i when the face value of the bond is $N(1+\delta K)$ and the strike price is N. It follows that an interest rate cap can be regarded as a portfolio of European put options on zero-coupon bonds.

From the equation (1) the i'th caplet yields a payoff at time t_i of

$$C^i_{t_i} = N\delta \max\{l(t_i, t_{i-\delta}) - K, 0\}$$

This payoff is the value of the caplet at time t_i but we want to find the caplet's value before time t_i. In the other word, this value is the future value of the caplet so we have to find the present value of this to be the caplet's price. We can obtain its value in the interval between $t_{i-\delta}$ and t_i by a simple discounting of the payoff.

$$C^i_t = p(t, t_i) N\delta \max\{l(t_i, t_{i-\delta}) - K, 0\}, \quad t_{i-\delta} \le t \le t_i$$

In particular,

$$C^i_{t_{i-\delta}} = p(t_i, t_{i-\delta}) N\delta \max\{l(t_i, t_{i-\delta}) - K, 0\}$$

The relation between the price of a zero-coupon bond at time t with maturity at T and the forward rate $l(t, T)$ is

$$p(t,T) = \frac{1}{1 + l(t,T).(T-t)}$$

or

$$l(t,T) = \frac{1}{T-t}\left(\frac{1}{p(t,T)} - 1\right)$$

We then have

$$\begin{aligned} C^i_{t_{i-\delta}} &= p(t_i, t_{i-\delta}) N \max\{1 + \delta l(t_i, t_{i-\delta}) - (1+\delta K), 0\} \\ &= p(t_i, t_{i-\delta}) N \max\left\{\frac{1}{p(t_i, t_{i-\delta})} - (1+\delta K), 0\right\} \\ &= N(1+\delta K)\max\left\{\frac{1}{(1+\delta K)} - p(t_i, t_{i-\delta}), 0\right\} \end{aligned}$$

We can now see that the value at time $t_{i-\delta}$ is identical to the payoff of a European put option expiring at time $t_{i-\delta}$ that has an exercise price of $1/(1 + \delta K)$ and is written on a zero-coupon bond maturing at time *ti*. Accordingly, the value of the *i*'th caplet at an earlier point in time $t \le t_{i-\delta}$ must equal the value of that put option.

If we denote the price of a call option on a zero-coupon bond at time t, with the strike price K, expiry T and where the bond expires at time S with $\pi(t, K, S, T)$, i.e.,

$$\pi(t, K, S, T) = \max\{p(T, S) - K, 0\}$$

The caplet payoff at time t_i is equal to the future value of the nominal amount multiplies by the price of a call option on a zero-coupon bond at time t. The reason that we use the future value of the nominal amount is the borrower has to pay the interest expense based on the nominal amount N at time $t_{i-\delta}$, so if we want to find the interest expense at time t, we have to use the future value of N, that is N(1+δK). We do that in order to the interest expense at time $t_{i-\delta}$ is equal to the interest expense at time t_i when we ignore the time value of money.

We can write in the mathematics term as the following;

$$C_t^i = N(1+\delta K)\pi\left(t, \frac{1}{1+\delta K}, t_{i-\delta}, t_i\right)$$

This payoff is only for one caplet. So we can find the value of the entire cap contract by adding up the value of all caplets corresponding to the remaining payment dates of the cap. Before the first reset date, t_0, none of the cap payments are known, so the value of the cap is given by

$$C_t = \sum_{i=1}^{n} C_t^i = N(1+\delta K)\sum_{i=1}^{n}\pi\left(t, \frac{1}{1+\delta K}, t_{i-\delta}, t_i\right), \qquad t < t_0$$

At all dates after the first reset date, the next payment of the cap will already be known. If we again use the notation $t_{i(t)}$ for the nearest following payment date after time t, the value of the cap at any time t in $[t_0, t_n]$ (exclusive of any payment received exactly at time t) can be written as

$$C_t = Np\left(t, t_{i(t)}\right)\delta \max\left\{l\left(t_{i(t)}, t_{i(t)} - \delta\right) - K, 0\right\} + N\left(1+\delta K\right)\sum_{i=i(t)+1}^{n}\pi\left(t, \frac{1}{1+\delta K}, t_i - \delta, t_i\right), \quad t_0 < t < t_n$$

If $t_{n-1} < t < t_n$, we have $i(t) = n$, and there will be no terms in the sum, which is then considered to be equal to zero. From the results above, cap prices will follow from prices of European puts on zero-coupon bonds. The interest rates and the discount factors appearing in the expressions above are taken from the money market, not from the government bond market. Since caps and most other contracts related to money market rates trade OTC, one should take the default risk of the two parties into account when valuing the cap. The default simply means that the party cannot pay the amounts promised in the contract. Official money market rates and the associated discount function apply to loan and deposit arrangements between large financial institutions, and thus they reflect the default risk of these corporations. If the parties in an OTC transaction have a default risk significantly different from that, the discount rates in the formulas should be adjusted accordingly. However, it is quite complicated to do that in a theoretically correct manner.

The parameters affecting the cap price

Since the cap is a tool to guarantee of a future rate, so the price of the cap will therefore depend on the likelihood that the market will change its view. This likelihood of change is measured by volatility. An instrument expected to be volatile between entry and maturity will have a higher price than a low volatility instrument. The volatility used in calculating the price should be the expected future volatility. This is based on the historic volatility.
As time goes by, the volatility will have less and less impact on the price, as there is less time for the market to change its view. Therefore, in a stable market, the passing of time will lead to the cap falling in value.

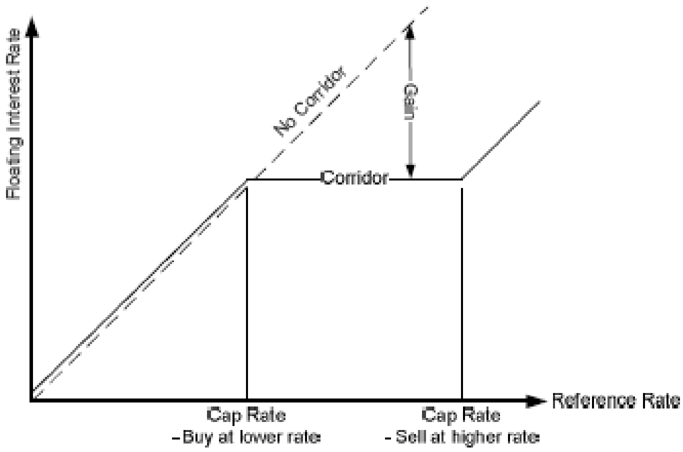
The higher the strike compared to prevailing interest rates the lower the price of the cap. High strike ("*out-of-the-money*") caps will be cheaper than "*at-the-money*" or low strike ("*in-the-money*") because of the reduced probability of the caplets being in the money during the life of the option.

The price of the cap will increase with the length of the tenor as it will include more caplets to maturity.

In the market traders will use volatility to quantify the probability of changes around interest rate trends. Higher volatility will increase the probability of a caplet being in the money and therefore the price of the cap.

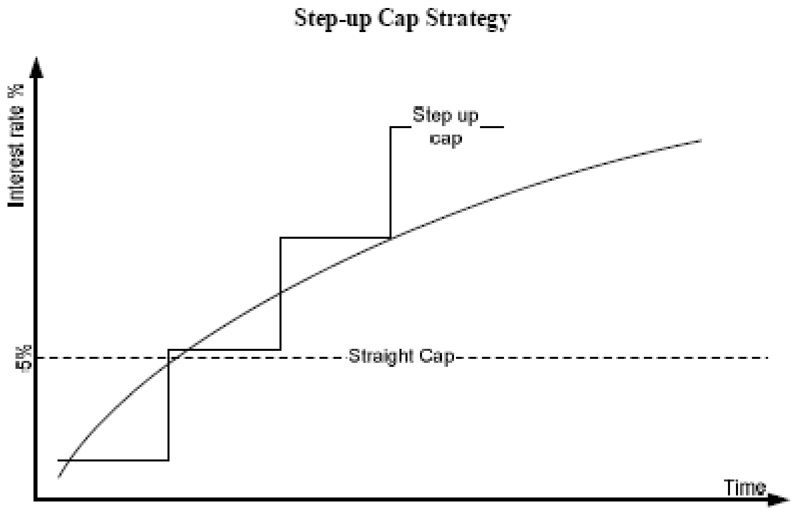
Strategies

Corridor: It is a strategy where the cost of purchasing a cap is offset by the simultaneous sale of another cap with a higher strike. It is possible to offset the entire cost of the cap purchase by increasing the notional amount on the cap sold to match the purchase price. The inherent risk in this strategy is that if short term rates rise through the higher strike the purchaser is no longer protected above this level and will incur considerable risk if the amount of the cap sold is proportionately larger. The payoff diagram is shown as in the figure below



Corridor Payoff

Step-up Cap: In steep yield curve environments the implied forward rates will be much higher than spot rates and the strike for caplets later in the tenor may be deep in the money. The price of a cap, being the sum of the caplets, may prove prohibitively expensive. The step up cap counteracts this by raising the strike of the later caplets to reflect the higher forward rates. This may provide a more attractive combination of risk hedge at a lower price. The payoff diagram is shown as in the figure below



After purchasing the cap, the buyer can make "claims" under the guarantee should LIBOR be above the level agreed on the cap on the settlement dates. A cap is not a continuous guarantee. Claims can only be made on specified settlement dates.

(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

Advantage and disadvantage of caps

Advantages/benefits

Major advantages of caps are that the buyer limits his potential loss to the premium paid, but retains the right to benefit from favorable rate movements. The other one is

- Caps provide investor with protection against unfavorable interest rate moves over the strike rate, while allowing investor unlimited participation in favorable moves down.
- Caps are flexible. The strike rate can be positioned to reflect the level of protection investor seeks. However, the amount of premium payable is also affected by the choice the investor makes.
- The term of the cap is flexible and does not have to match the term of the underlying investment. A cap may be used as a form of short term interest rate protection in times of uncertainty.
- Caps can be cancelled (however there may be a cost to the investor in doing).
- As a cap does not form part of the underlying investment, the protection afforded can apply to any investment with similar commercial terms.
- Purchasers can make considerable profits because interest rate option products are highly geared instruments and, for a relatively small outlay of capital. At the same time, a seller with a decay strategy in mind, where he would like the option's value to decay over time so that it can be bought back cheaper at a later stage or even expire worthless, can make a profit amounting to the option premium, without having to make a capital outlay. (Jan Roman, Lecture in Analytical Finance II, 27 October 2008)
- The cost is limited to the premium paid.

Disadvantages/risks

· The premium is not refundable in any circumstances. This includes situations where the reference rate always exceeds the strike rate and no payments are made.

· Investor will be exposed to interest rate movements if the term of the cap is shorter than that of the underlying investment.

FLOORS AND FLOORLETS (Interest Rate Floor, Westpac Banking Corporation)

Definition

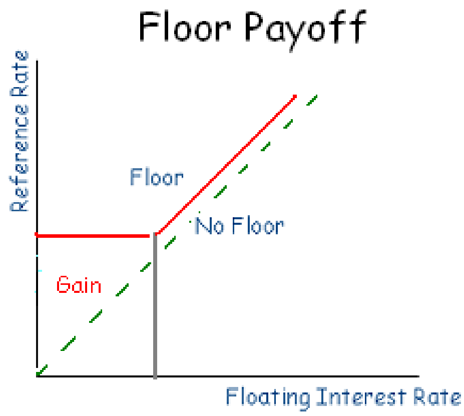
An Interest Rate Floor (Floor) is an interest rate management tool for an investor who has an investment with returns linked to a Floating rate note. A Floor helps an investor to protect himself against a fall in interest rates and maintain the ability to participate in a rise in interest rates. It is an interest management tool that is used with an investment that has returns linked to a market bank bill reference rate such as LIBOR, plus a fixed spread. The underlying investment continues to be governed by the terms and conditions applying to that investment.

A Floor works in conjunction with a variable rate investment. It protects investor against decreasing interest rates by setting a minimum interest rate payable on the investment. This minimum interest rate is known as the strike rate. In exchange for the protection of a Floor, investor pays a premium.

The reference rate to be used is also set at the beginning of the transaction. It provides a benchmark interest rate. It is usually the same as the base rate applying to the underlying investment. The reference rate applies for set periods, called calculation periods.

If the strike rate is more than the reference rate for a calculation period, then the investor will receive an amount based on the difference between these rates. When this amount is used to offset the lower base interest rate applying to the underlying investment, the effective base interest rate for the calculation period becomes the strike rate. If the strike rate is less than the reference rate for a calculation period, then the investor will get or pay nothing for that calculation period and the base interest rate applicable to the underlying investment will be the rate applying under the agreed terms of the investment.

In the real world, the investor can decide to buy Floor with notional less than the size of the underlying investment. Hence, sometime buying the large number of Floor contract may cost than the loss from the change in reference rate. The payoff diagram of full underlying investment in Floor contract is shown below.



In return for the Floor, the investor is required to pays a non-refundable premium. The issuer of the Floor calculates the premium on a transaction by transaction basis at the time the investor establish the Floor. To calculate the premium, there are several factors, including strike rate, notional amount, reference rate, term and the reset dates selected, current market interest rates, and market volatility. The changing in these parameters will affect the value of the Floor, premium. The increasing in the strike rate will make the Floor price go up, also the market volatility.

To calculate the premium or the value of floor we can consider floorlet as a European put on the chosen reference rate with delayed payment of the payoff, the payoff at time $t_i\left(i=1,2,\ldots,n\right)$ is given by:

$$F_{t_i}^{i}=N\delta\max\left\{K-l\left(t_i,t_{i-\delta}\right),0\right\}$$

(Notations of the parameters are the same as caplet)

As call, consider floorlet as a European call on a zero-coupon bond, we will get that the value of the i' th floorlet at time $t_{i-\delta}$ is:

$$F_{t_i}^{i}=N\left(1+\delta K\right)\max\left\{p\left(t_i,t_{i-\delta}\right)-\frac{1}{1+\delta K},0\right\}$$

Then the total value of the floor contact at any time $t<t_0$ is given by:

$$F_t=N\left(1+\delta K\right)\sum_{i=1}^{n}\pi\max\left\{t,\frac{1}{1+\delta K},t_{i-\delta},t_i\right\}$$

and

$$F_t=Np\left(t,t_{i(t)}\right)\delta\max\left\{K-l\left(t_{i(t)},t_{i(t)-\delta}\right),0\right\}+N\left(1+\delta K\right)\sum_{i=i(t)+1}^{n}\pi\max\left\{t,\frac{1}{1+\delta K},t_{i-\delta},t_i\right\}$$

$$t_0\le t\le t_n$$

Advantage and disadvantage of floors

Advantages/benefits

- Floors provide investor with protection against unfavorable interest rate moves below the strike rate, while allowing investor unlimited participation in favorable moves up.
- Floors are flexible. The strike rate can be positioned to reflect the level of protection investor seeks. However, the amount of premium payable is also affected by the choice the investor makes.
- The term of the floor is flexible and does not have to match the term of the underlying investment. A floor may be used as a form of short term interest rate protection in times of uncertainty.
- Floors can be cancelled (however there may be a cost to the investor in doing).
- As a floor does not form part of the underlying investment, the protection afforded can apply to any investment with similar commercial terms.
- The cost is limited to the premium paid.

Disadvantages/risks

· The premium is not refundable in any circumstances. This includes situations where the reference rate always exceeds the strike rate and no payments are made.

· Investor will be exposed to interest rate movements if the term of the floor is shorter than that of the underlying investment.

BLACK'S MODEL FOR VALUATION OF CAPS AND FLOORS

Black-Scholes model is a very popular tool to value the options on foreign exchange, option on indices, and option on future contracts. There have been attempts to extend this model to value the interest rate derivatives.

To value caps and floors, this model assumes the underlying forward rates $l(t_i, t_{i-\delta})$ be lognormal with volatilityσ, then the value of caplet is as follow;

$$C_{t_i}^{i} = N\delta p(0,T)\left[FN(d_1) - KN(d_2)\right]$$

(Hull, John C., 2006, p. 621)

We use the day count convention so $\delta = \frac{t_d}{t_y}$, the value of caplet is

$$C_{t_i}^{i} = \frac{N.\frac{t_d}{t_y}}{1+F.\frac{t_d}{t_y}} e^{-rT}\left[F.N(d_1) - K.N(d_2)\right]$$

(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

The value of floorlet is

$$Floorlet_{t_i}^{i} = N\delta p(0,T)\left[K.N(-d_2) - F.N(-d_1)\right]$$

(Hull, John C., 2006, p. 622)

We use the day count convention so $\delta = \frac{t_d}{t_y}$, the value of floorlet is

$$Floorlet_{t_i}^{i} = \frac{N.\frac{t_d}{t_y}}{1+F.\frac{t_d}{t_y}} e^{-rT}\left[K.N(-d_2) - F.N(-d_1)\right]$$

where $$d_1 = \frac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F/K) - (\sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

Example:

Suppose we have a caplet, with three months to expiry on a 92-day forward rate and a face value of 100 million. The three-month forward rate is 8% (with act/360 as day-count), the strike is 8%, the risk-free interest rate 6%, and the volatility of the forward rate 22% per annum.
(F = 0.08, K = 0.08, T = 0.25, r = 0.06, σ = 0.22)

$$d_1 = \frac{\ln(0.08/0.08) + (0.22^2/2)0.25}{0.22\sqrt{0.25}} = 0.055$$

$$d_2 = 0.055 - (0.22\sqrt{0.25}) = -0.055$$

$$N(d_1) = 0.5219, \qquad N(d_2) = 0.4781$$

$$C_t = \frac{100{,}000{,}000\left(\frac{92}{360}\right)}{1 + 0.08\left(\frac{92}{360}\right)} e^{-0.06(0.25)}\left[0.08(0.5219) - 0.08(0.4781)\right] = 86{,}446.15$$

PUT-CALL PARITY FOR CAPS AND FLOORS

There is a put-call parity relationship between the prices of caps and floors. This is

value of cap = value of floor + value of swap

In this relationship, the cap and floor have the same strike price, R_K. The swap is an agreement to receive LIBOR, which is a floating rate, and pay a fixed rate of R_K with no exchange of payments on the first reset date. All three instruments have the same life and the same frequency of payments.

To see that the result is true, consider a long position in the cap combined with a short position in the floor. The cap provides a cash flow of LIBOR-R_K for periods when LIBOR is greater than R_K. The short floor provides a cash flow of –(R_K -LIBOR)= LIBOR-R_K for periods when LIBOR is less than R_K. There is therefore a cash flow of LIBOR-R_K in all circumstances. This is the cash flow on the swap. It follows that the value of the cap minus the value of the floor must equal the value of the swap.

Note that swaps usually structured so that LIBOR at time zero determines a payment on the first reset date. Caps and floors are usually structured so that there is no payoff on the first reset date. This is why the swap has to be defined as one with no payment on the first reset date.

(Hull, John C., 2006, p. 621)

COLLARS

A collar is an instrument designed to guarantee that the interest rate on the underlying floating-rate note always lies between two levels. It is a combination of a long position in a cap and short position in a floor. Collars can benefit both borrowers and investors. In the case of a borrower, the collar protects against rising rates but limits the benefits of falling rates. In the case of an investor, the collar protects against falling rates but limits the benefits of rising rates. Similar to caps and floors the customer selects the index, the length of time, and strike rates for both the cap and the floor. However, unlike a cap or a floor, an up-front premium may or may not be required, depending upon where the strikes are set. In either scenario, the customer is a buyer of one product, and a seller of the other.

The buyer and the seller agree upon the term (tenor), the cap and floor strike rates, the notional amount, the amortization, the start date, and the settlement frequency. If at any time during the tenor of the collar, the index moves above the cap strike rate or below the floor strike rate, one party will owe the other a payment.

A typical collar can be seen as a portfolio of a long position in a cap with a cap rate K_c and a short position in a floor with a floor rate of $K_f < K_c$ (and the same payment dates and underlying floating rate). The payoff of a collar at time t_i, $i = 1, 2, ..., n$ is thus

$$L_{t_i}^i = N\delta[\max\{l(t_{i(t)}, t_{i(t)} - \delta) - K_c, 0\} - \max\{K_f - l(t_{i(t)}, t_{i(t)} - \delta), 0\}]$$

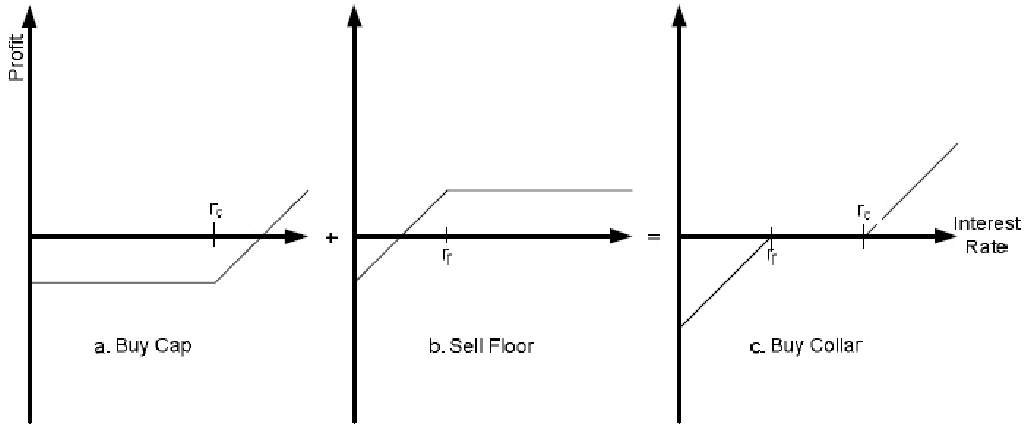
$$= \begin{cases} N\delta[K_f - l(t_{i(t)}, t_{i(t)} - \delta)], & \text{if} \quad l(t_{i(t)}, t_{i(t)} - \delta) \le K_f \\ 0, & \text{if} \quad K_f \le l(t_{i(t)}, t_{i(t)} - \delta) \le K_c \\ N\delta[l(t_{i(t)}, t_{i(t)} - \delta) - K_c] & \text{if} \quad K_c \le l(t_{i(t)}, t_{i(t)} - \delta) \end{cases}$$

The value of a collar with cap rate K_c and floor rate K_f is given by

$$L_t(K_c, K_f) = C_t(K_c) - F_t(K_f)$$

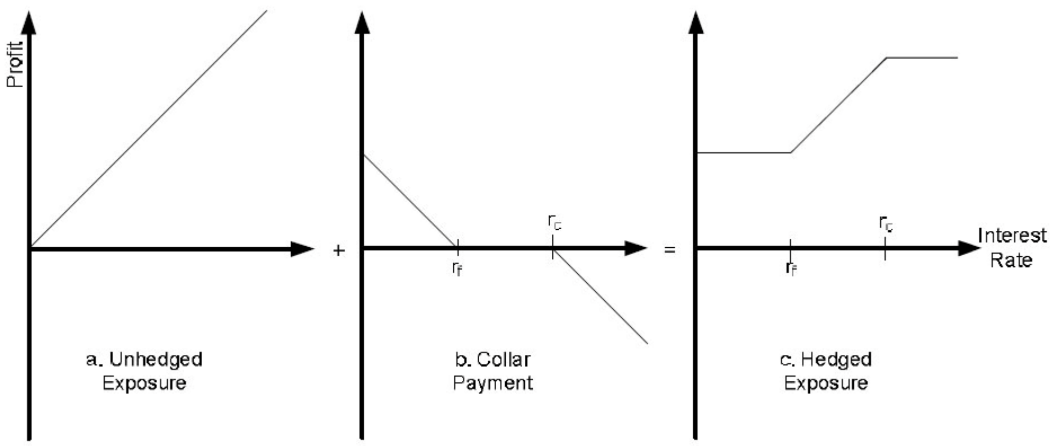
Where the expressions for the values of caps and floors derived earlier can be substituted in. An investor who has borrowed funds on a floating rate basis will by buying a collar ensure that the paid interest rate always lies in the interval between K_f and K_c. Clearly, a collar gives cheaper protection against high interest rates than a cap (with the same cap rate K_c), but on the other hand the full benefits of very low interest rates are sacrificed. In practice, collar is usually constructed, so that the price of the cap is initially equal to the price of the floor. The cost of entering into the collar is then zero.

The figure below illustrates the payoff to buying a one-period zero-cost interest rate collar. If the index interest rate *r* is less than the floor rate *rf* on the interest rate reset date, the floor is in-the-money and the collar buyer (who has sold a floor) must pay the collar counterparty an amount equal to $N \times (r_f - r) \times d_t / 360$. When *r* is greater than *rf* but less than the cap rate *rc*, both the floor and the cap are out-of-the-money and no payments are exchanged. Finally, when the index is above the cap rate the cap is in-the money and the buyer receives $N \times (r - r_c) \times d_t / 360$.



In this section, we will show the hedging uses of interest rate collars. The figure below illustrates the effect that buying a one-period, zero-cost collar has on the exposure to changes in market interest rates faced by a firm with outstanding variable-rate debt. The first panel depicts the firm's inherent or unhedged interest exposure, while the second panel illustrates the effect that buying a collar has on interest expense. Finally, the third panel combines the borrower's inherent exposure with the payoff to buying a collar to display the effect of a change in market interest rates on a hedged borrower's interest expense. Note that changes in market interest rates can only affect the hedged borrower's interest expense when the index rate varies between the floor and cap rates. Outside this range, the borrower's interest expense is completely hedged.

The Effect of Buying an Interest Rate Collar on Interest Expense



(Hull, John C., 2006, p. 621)

(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

Advantage and disadvantage of collars

Advantages/benefits

- Collars provide you with protection against unfavourable interest rate movements above the Cap Rate while allowing you to participate in some interest rate decreases.
- Collars can be structured so that there is no up-front premium payable. While you can also set your own Cap Rate and Floor Rate, a premium may be payable in these circumstances.
- The term of a Collar is flexible and does not have to match the term of the underlying bill facility. A Collar may be used as a form of short-term interest rate protection in times of uncertainty.
- Collars can be cancelled (however there may be a cost in doing so)

Disadvantages/risks

- While a Collar provides you with some ability to participate in interest rate decreases, your interest rate cannot fall to less than the Floor Rate.
- To provide a zero cost structure or a reasonable reduction in premium payable under the Cap, the Floor Rate may need to be set at a high level. This negates the potential to take advantage of favorable market rate movements.
- You will be exposed to interest rate movements if the term of the Collar is shorter than that of the underlying bank bill facility.

(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

EXOTIC CAPS AND FLOORS

There are several contracts trade on the international OTC markets with cash flows similar to plain vanilla contracts, but deviate in one or more aspects. The deviations complicate the pricing methods considerably. Examples of exotic caps and floors are as follows

- A **bounded cap** is like an ordinary cap except that the cap owner will only receive the scheduled payoff if the sum of the payments received so far due to the contract does not exceed a certain pre-specified level. Consequently, the ordinary cap payments $C_{t_i}^{i}$ are to be multiplied with an indicator function. The payoff at the end of a given period will depend not only on the interest rate in the beginning of the period, but also on previous interest rates. As many other exotic instruments, a bounded cap is therefore a path-dependent asset.

- A **dual strike cap** is similar to a cap with a cap rate of $K1$ in periods when the underlying floating rate $l(t+\delta, t)$ stays below a pre-specified level l, and similar to a cap with a cap rate of $K2$, where $K2 > K1$, in periods when the floating rate is above l.

- A **cumulative cap** ensures that the accumulated interest rate payments do not exceed a given level.

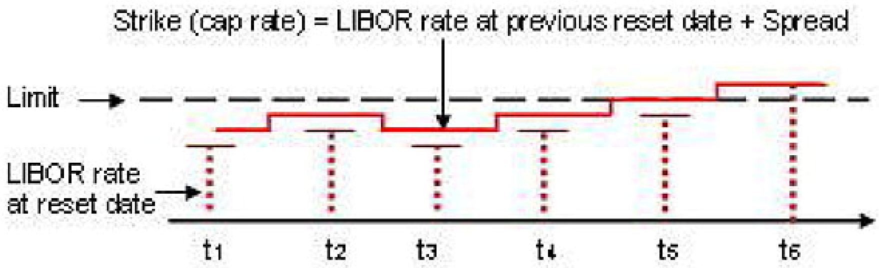
A **knock-out cap** will at any time ti give the standard payoff $C_{t_i}^i$ unless the floating rate $l(t+\delta, t)$ during the period $[ti - \delta, ti]$ has exceeded a certain level. In that case the payoff is zero. Similarly, there are knock-in caps. They are named as: *down and out*,, *down and in*, *up and out*,and *up and in*.

- **Ratchet Cap**

  A Ratchet cap is like a plain vanilla cap except that the strike is given by:

$$K_i = \begin{cases} \min[K, m] & i = 1 \\ \min[K_{i-1} + X, m] & i > 1 \end{cases}$$

  where K are the strikes an m a given limit. In a Ratchet cap there are rules for determine the cap rate for each caplet. The cap rate equals the LIBOR rate at the previous reset date plus a spread. A limit, m is set on the strike level, above which a strike cannot be set.



- **Sticky Cap**

  A Sticky cap is like a plain vanilla cap except that the strike is given by:

$$K_i = \begin{cases} \min[K, m] & i = 1 \\ \min[\min\{K_{i-1}, L_{i-1}\} + X, m] & i > 1 \end{cases}$$

  The sticky cap rate equals the previous capped rate plus a spread. A limit is set on the strike level, above which a strike cannot be set.

- **Flexi Cap**

  A Flexi cap is like an ordinary Cap except that only the a first in-the-money Caplets are exercised.

- **Chooser Cap**

  A chooser cap is like a Flexi Cap except that the contract holder can choose which a Caplets to exercise. Once the reset of a Caplet has taken place, it can no longer be chosen.

- **Momentum Cap**

  A Momentum cap is like a plain vanilla cap except that the strike is given by:

$$K_i = \begin{cases} \min[K, m] & i = 1 \\ \min[K_{i-1} + X, m] & i > 1, L_i - b > L_{i-1} \\ \min[K_{i-1}, m] & i > 1, L_i - b > L_{i-1} \end{cases}$$

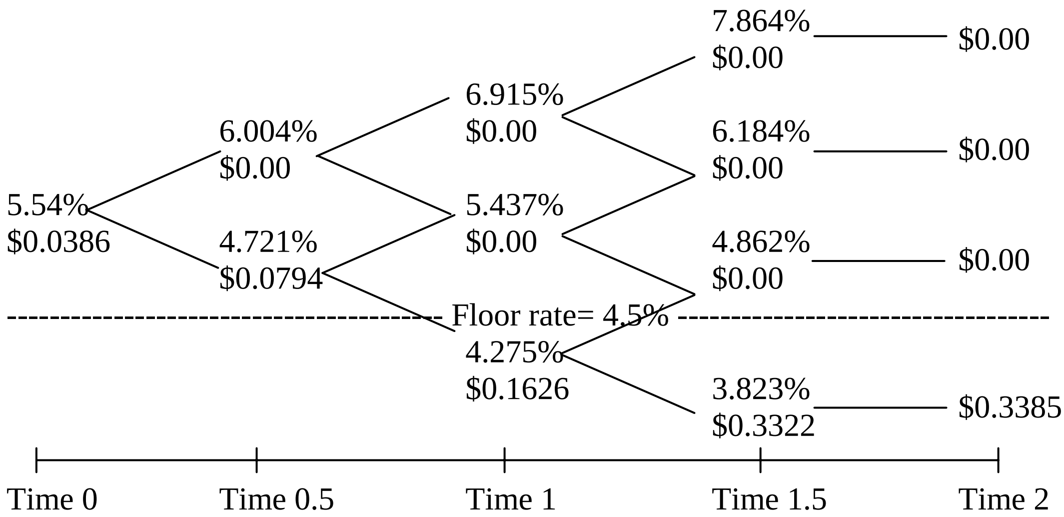
  The sticky cap rate equals the previous capped rate plus a spread. A limit is set on the strike level, above which a strike cannot be set.

(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

NUMERICAL EXAMPLE

Consider a binomial tree for a 2-year semi-annual collar on $100 notional amount with cap rate 6.5% and floor rate 4.5%, indexed to the 6-month rate.

We will first consider the valuation of floor. At time 0, the 6-month rate is 5.54 percent so the floor is out-of the money, and pays 0 at time 0.5. The later payments of the floor depend on the path of interest rates. Suppose rates follow the path in the tree below. The value of the floor is the sum of the values of the 4 puts on the 6-month rates at time 0,0.5,1,1.5. We begin from valuing at time 1.5



As the binomial valuation of put options, we calculate backwards from time 1.5. At time 1.5, the only possible floating interest rate below the floor rate of 4.5 percent is 3.823 percent. And, only when the floating interest rate falling to this level , the collar buyer (who has sold a floor) must pay the collar counterparty of $33.85 at time 2. For the calculation,

$$\$0.3385=\$100*(4.5\%-3.823\%)/2$$

For the value of $33.85, at time 1.5, its present value is,

$$\$0.3322=\$0.3385/(1+3.823\%/2)$$

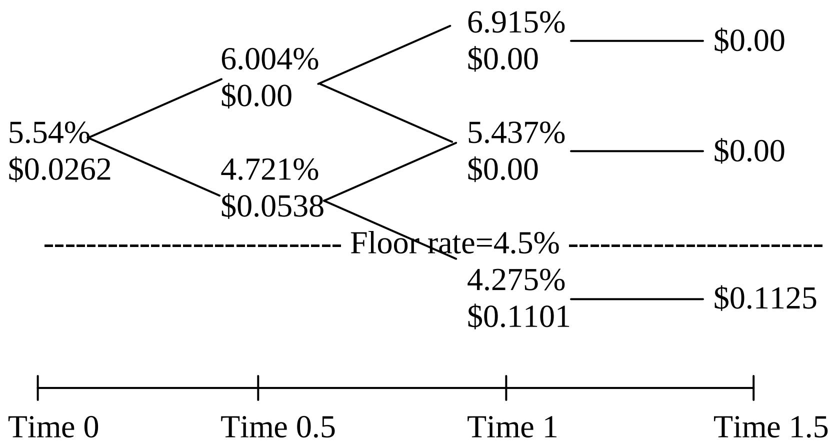
For time 1, 0.5 and 0, assume that in this example, the probabilities to raise and fall are both 50 percent. The calculations for the values at these time spots will be:

$$\$0.1626=0.5*(0+\$0.3322)/(1+4.275\%/2)$$

$$\$0.0794=0.5*(0+\$0.1626)/(1+4.721\%/2)$$

$$\$0.0386=0.5*(0+\$0.1794)/(1+5.54\%/2)$$

Then, we calculate the floorlet due at time 1 as below.



Base on the same calculation method, we simply give the calculation for each node as following:

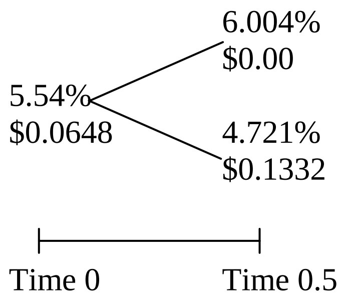
$0.1125=$100*(4.5%-4.275%)/2

$0.1101=$0.1125/(1+4.275%/2)

$0.0538=0.5*(0+$0.1101)/(1+4.721%/2)

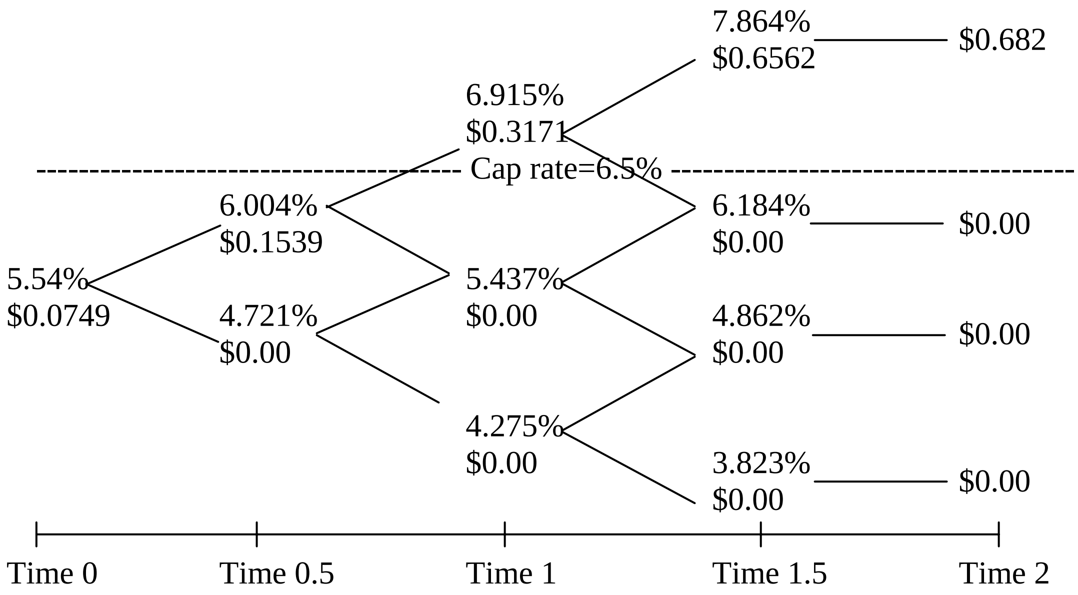
$0.0262=0.5*(0+$0.0538)/(1+5.54%/2)

Because at time 0.5 and 0, the floorlets never get in the money, so the value of the floor will be $0.0648=$0.0386+$0.0262



(Jan Roman, Lecture in Analytical Finance II, 27 October 2008)

For valuation of cap, we use a similar calculation as floor. Suppose that cap rate is 6.5%. The value of the cap is the sum of the values of the 4 calls on the 6-month rates at time 0,0.5,1,1.5. We begin from valuing at time 1.5



At time 1.5, the only possible floating interest rate above the cap rate of 6.5 percent is 7.864 percent. And, only when the floating interest rate rising to this level, the collar buyer receives $68.2 at time 2. For the calculation,

$0.682=$100*(7.864%-6.5%)/2

For the value of $68.2, at time 1.5, its present value is,

$0.6562=$0.682/(1+7.864%/2)

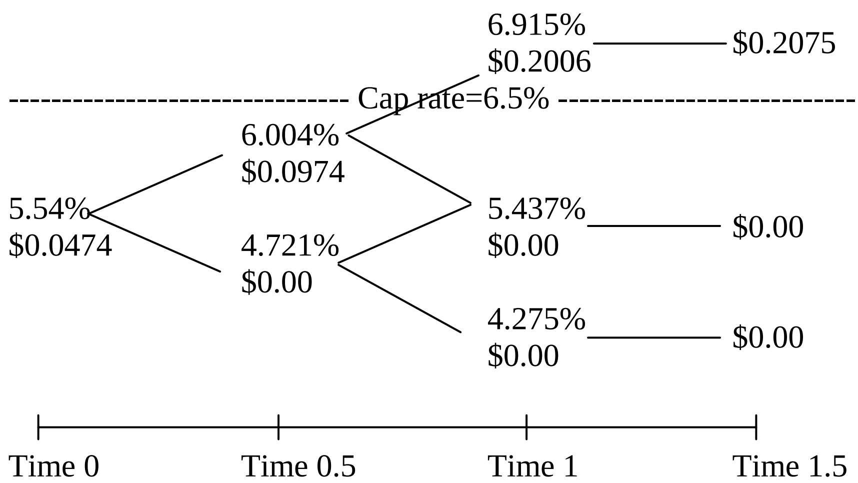
For time 1, 0.5 and 0, values at these time spots will be:

$0.3171=0.5*(0+$0.6562)/(1+6.915%/2)

$0.1539=0.5*(0+$0.3171)/(1+6.004%/2)

$0.0749=0.5*(0+$0.1539)/(1+5.54%/2)

Then, we calculate the caplet due at time 1 as below.



The calculation for each node is as follows:

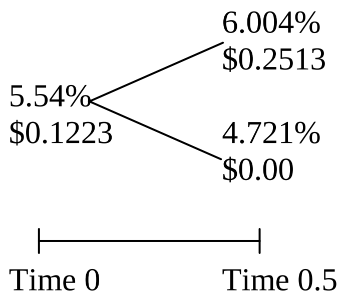
$0.2075=$100*(6.915%-6.5%)/2

$0.2006=$0.2075/(1+6.915%/2)

$0.0974=0.5*(0+$0.2006)/(1+6.004%/2)

$0.0474=0.5*(0+$0.0974)/(1+5.54%/2)

Because at time 0.5 and 0, the caplets never get in the money, so the value of the cap will be $0.1223=$0.0749+$0.0474



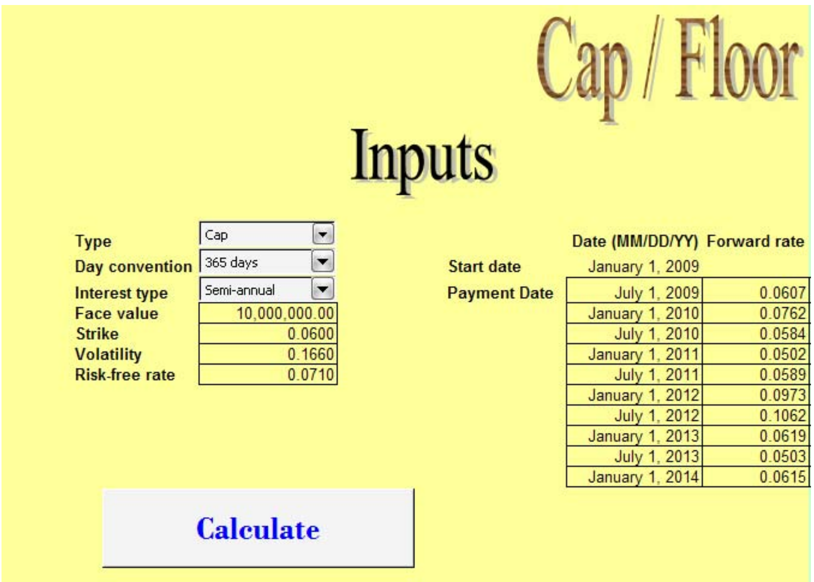
As the value of the collar is equal to the value of cap minus the value of floor, so the value of collar is $0.1223-$0.0648=$0.0575

EXAMPLE: (solved by VBA application)

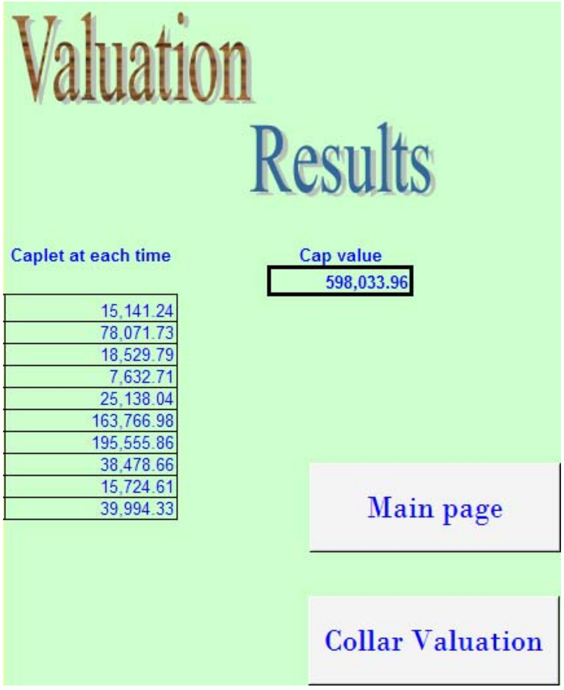
Consider contract that caps the LIBOR interest rate on SEK. 10,000,000 at 6% per annum (with semi-annually compounding) for 5 years. Suppose that the volatility of LIBOR underlying the caps is 16.6% per annum. The risk-free interest rate is assumed to be 7.1% per annum equally for all contract life. The LIBOR is given in the table below.

Date	LIBOR
1-Jul-09	6.070%
1-Jan-10	7.620%
1-Jul-10	5.840%
1-Jan-11	5.020%
1-Jul-11	5.890%
1-Jan-12	9.730%
1-Jul-12	10.620%
1-Jan-13	6.190%
1-Jul-13	5.030%
1-Jan-14	6.150%

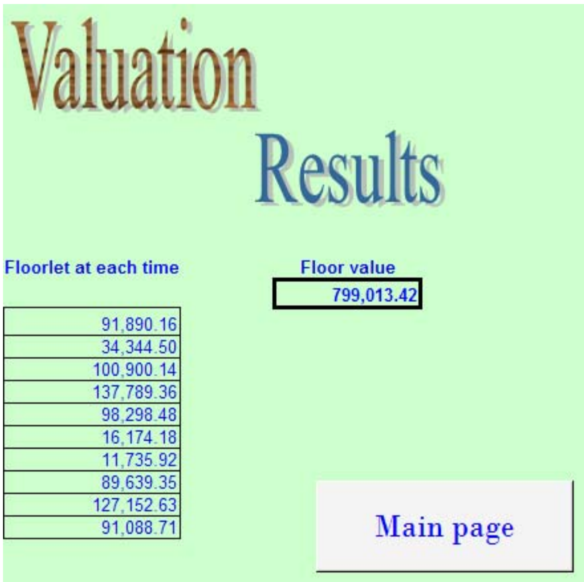
We calculate the caps price by using VBA application. First we have to input all data we have from the problem.



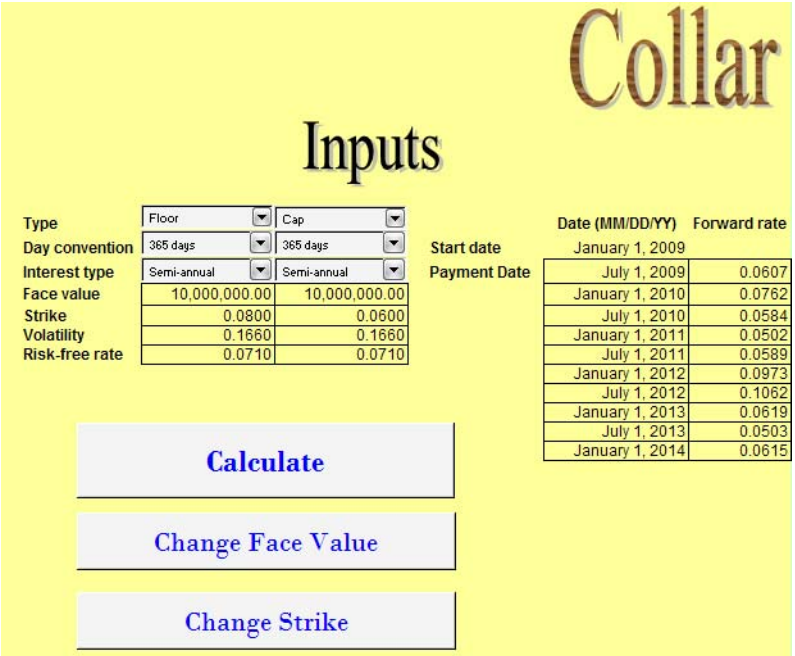
The application will compute and give us results as:



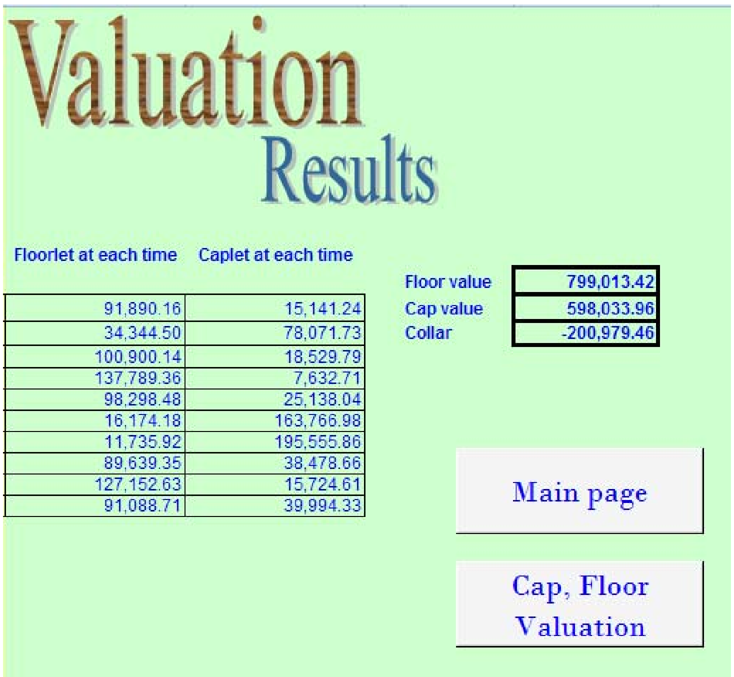
So, we can easy get the answer for this problem that the caps price is equal to SEK. 598,033.96. Then we use the same data to calculate the price for a floor which strike is 8% per annum. Then we will get the answer that the floor price is equal to SEK. 799,013.42.



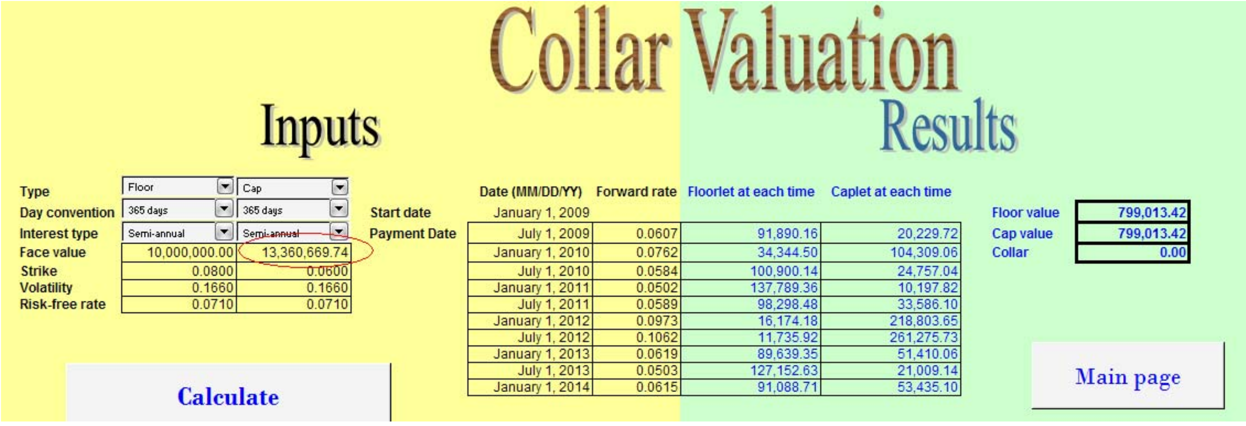
Next, we use the same data to find how much we have to pay if we agree to enter to the collar. In this case we have to input to our VBA application again.



And the output shows that collar price is minus SEK. 200,979.46, because by this given data making the floor value higher than cap value. This cannot satisfy our objective to enter to the collar strategy.



To achieve that, we have two alternative, first is try to change the face value of the floor contract by press the “Change Face Value” button and we get that the collar is now be zero and the face value of the floor contract should be change to SEK.13,360,009.74.


Collar Valuation

Inputs

Type	Floor	Cap
Day convention	365 days	365 days
Interest type	Semi-annual	Semi-annual
Face value	10,000,000.00	13,360,669.74
Strike	0.0800	0.0600
Volatility	0.1660	0.1660
Risk-free rate	0.0710	0.0710

Results

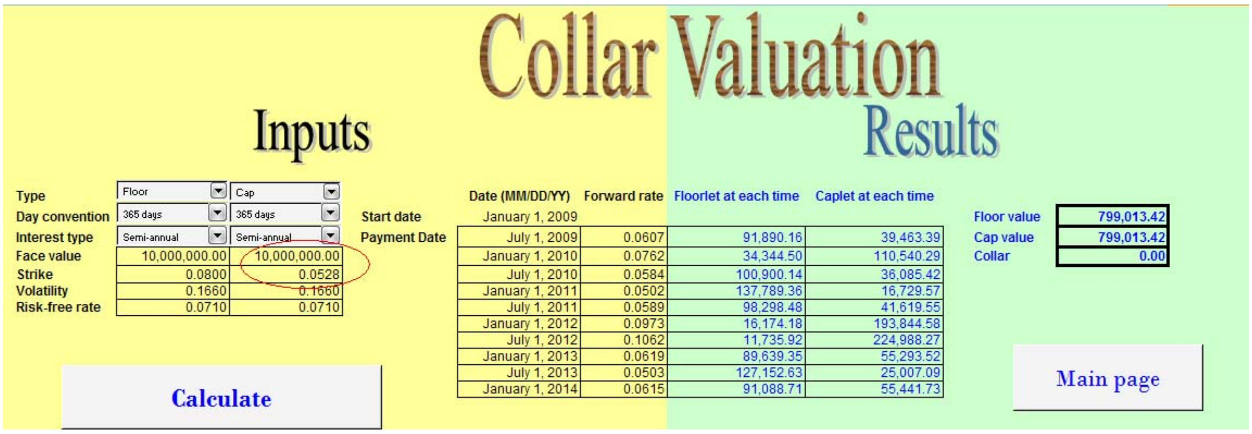
	Date (MM/DD/YY)	Forward rate	Floorlet at each time	Caplet at each time
Start date	January 1, 2009			
Payment Date	July 1, 2009	0.0607	91,890.16	20,229.72
	January 1, 2010	0.0762	34,344.50	104,309.06
	July 1, 2010	0.0584	100,900.14	24,757.04
	January 1, 2011	0.0502	137,789.36	10,197.82
	July 1, 2011	0.0589	98,298.48	33,586.10
	January 1, 2012	0.0973	16,174.18	218,803.65
	July 1, 2012	0.1062	11,735.92	261,275.73
	January 1, 2013	0.0619	89,639.35	51,410.06
	July 1, 2013	0.0503	127,152.63	21,009.14
	January 1, 2014	0.0615	91,088.71	53,435.10

Floor value	799,013.42
Cap value	799,013.42
Collar	0.00

Calculate

Main page

The second alternative is try to change the strike rate of the floor until the floor price make the collar value equal to zero, we do this by pressing the “Change Strike” button. In this case the floor face value will be the same value as the beginning, SEK.10,000,000 but the strike rate will be changed to 5.28% per annum.


Collar Valuation

Inputs

Type	Floor	Cap
Day convention	365 days	365 days
Interest type	Semi-annual	Semi-annual
Face value	10,000,000.00	10,000,000.00
Strike	0.0800	0.0528
Volatility	0.1660	0.1660
Risk-free rate	0.0710	0.0710

Results

	Date (MM/DD/YY)	Forward rate	Floorlet at each time	Caplet at each time
Start date	January 1, 2009			
Payment Date	July 1, 2009	0.0607	91,890.16	39,463.39
	January 1, 2010	0.0762	34,344.50	110,540.29
	July 1, 2010	0.0584	100,900.14	36,085.42
	January 1, 2011	0.0502	137,789.36	16,729.57
	July 1, 2011	0.0589	98,298.48	41,619.55
	January 1, 2012	0.0973	16,174.18	193,844.58
	July 1, 2012	0.1062	11,735.92	224,988.27
	January 1, 2013	0.0619	89,639.35	55,293.52
	July 1, 2013	0.0503	127,152.63	25,007.09
	January 1, 2014	0.0615	91,088.71	55,441.73

Floor value	799,013.42
Cap value	799,013.42
Collar	0.00

Calculate

Main page

CONCLUSION

Interest rate derivatives are very useful instrument to against the variation of the market interest rate. The borrower with floating interest rate can use caps to limit the maximum interest expense which is his future commitment. Whereas the lender with floating interest rate can use floorlets to limit the minimum interest revenue. Collar is one of interest rate derivatives which is a composition of caps and floors. The investor can use them to against rising (falling) rates but limits the benefits of falling (rising) rates. Because caps and floors have a similar characteristics as the call and put option, so the way to price can adapt from the way to price call and put option. Moreover the Black's model can used to find their values also with the lognormal return distribution assumption. The Visual Basic Application can be a very useful tool to make the calculation more simply.

APPENDIX

This application has four main functions. The first one is CalD1 which returns the cumulative probability distribution function for standardized normal distribution of d1. Secondly, CalD2 function is calculated the value of cumulative probability distribution function for standardized normal distribution of d2. Thirdly, Cap_Floor function, which is the most important in this application, returns the value of cap or the value of floor. Finally, MyCollar function is use to compute the collar value.

Additionally, this application requires solver add-in in your Microsoft Excel. Before you use this program, you should install solver in Excel. With a Visual Basic module active, click References on the Tools menu, and then select the Solver.xla check box under Available References.

```
'compute cumulative probability distribution function for standardized normal distribution of d1

Function CalD1(day_Convention As Byte, forward As Double, strike As Double, volatility As Double, start_Date As Date, payment_Date As Date) As Double

Dim d1 As Double, T As Double

    'calculate T

If (day_Convention = 1) Then '360 days

T = ((payment_Date - start_Date) + 1) / 360

Else '365 days

T = ((payment_Date - start_Date) + 1) / 365

End If

If (T <= 0) Then T = 0.000000001

d1 = ((Application.Ln(forward / strike)) + (volatility * volatility / 2) * T) / (volatility * Sqr(T))

CalD1 = Application.NormSDist(d1)

End Function

'compute cumulative probability distribution function for standardized normal distribution of d2

Function CalD2(day_Convention As Byte, forward As Double, strike As Double, volatility As Double, start_Date As Date, payment_Date As Date) As Double
```

```
Dim d2 As Double, T As Double

    'calculate T

If (day_Convention = 1) Then '360 days

T = ((payment_Date - start_Date) + 1) / 360

Else '365 days

T = ((payment_Date - start_Date) + 1) / 365

End If

If (T <= 0) Then T = 0.000000001

d2 = ((Application.Ln(forward / strike)) - (volatility * volatility / 2) * T) / (volatility *
Sqr(T))

CalD2 = Application.NormSDist(d2)

End Function

' Compute Caplet or Floorlet

Function Cap_Floor(optionType As Byte, day_Convention As Byte, rf_Type As Byte,
start_Date As Date, payment_Date As Date, faceValue As Currency, strike As Double,
riskFree As Double, volatility As Double, forward As Double, previous_Payment As Date)
As Double

Dim T As Double, rf As Double, td As Double

    'calculate T

If (day_Convention = 1) Then '360 days

T = ((payment_Date - start_Date) + 1) / 360

Else '365 days

T = ((payment_Date - start_Date) + 1) / 365

End If

If (T <= 0) Then T = 0.000000001



'calculate Rf
```

```
Select Case (rf_Type)
Case Is = 1 'quarterly
    rf = riskFree / 4
Case Is = 2 'semi-annual
    rf = riskFree / 2
Case Is = 3 'annual
    rf = riskFree
End Select
'calculate td
If (day_Convention = 1) Then '360 days
td = ((payment_Date - previous_Payment) + 1) / 360
Else '365 days
td = ((payment_Date - previous_Payment) + 1) / 365
End If
If (td <= 0) Then td = 0.000000001
If (optionType = 1) Then 'Caplet
    Cap_Floor = ((faceValue * td) / (1 + (forward * td))) * Exp(-rf * T) * ((forward * CalD1(day_Convention, forward, strike, volatility, start_Date, payment_Date)) - (strike * CalD2(day_Convention, forward, strike, volatility, start_Date, payment_Date)))
Else 'Floorlet
    Cap_Floor = ((faceValue * td) / (1 + (forward * td))) * Exp(-rf * T) * ((strike * (1 - CalD2(day_Convention, forward, strike, volatility, start_Date, payment_Date)) - (forward * (1 - CalD1(day_Convention, forward, strike, volatility, start_Date, payment_Date)))))
End If
End Function
' Compute Collar
Function MyCollar(cap As Double, floor As Double) As Double
    MyCollar = cap - floor
End Function
```

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