

20/1/2008]

Mälardalens Högskola

Seminar : Analytical Finance I

MMA707

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**[A MATLAB GUI Program For Calculating Instrument Portfolio's Sensitivities
in the**

Black-Derman-Toy model]

Abstract

This paper is base on calculating instruments portfolio's sensitivity in the Black-Derman-Toy (1990) Model, which is a one-factor model recombining interest rate model that replicates the term structure.

We determine the sensitivity of price to time decay (theta), interest rate to time elapse by implementing BDT Model in Matlab GUI.

This is worthwhile of study because of the risk posed by a portfolios consisting of a mixture of bonds and short-term bills.

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Input Parameter

$T =$ Maturity in years

$r =$ Zero-Coupon Bond Interest rate (%)

$\sigma =$ Zero-Coupon Bond Volatility

Output Parameter

$P =$ Price in Nominal Term.

1 INTRODUCTION

The market for fixed income derivatives has skyrocketed in recent years, reaching a total value of 400 trillion USD, one of the reasons being that more old peoples are willing to lend their money to financial institutions for a certain rate so as to receive a constant stream of cash flow, either monthly or year, while on the other hand young peoples borrowed money from financial banks to buy collateral securities, such as mortgage, thereby paying a monthly amount on the loan plus an interest rate. Also the rate with respect to time will be different as long term loans usually have a higher interest rate than short term loans.

This lending and borrowing rate should be equal in a clearing market scenario, but since we are in stochastic environments they are not equal. They are determined in part by the federal governments and some micro and macro economic factors such as supply/demand and inflation rate, and we are going to study these rates.

We will focus on the correlation between interest rate and a portfolio consisting of only interest rate-linked securities such as bonds, swaps, T-bill, futures, and options.

The correlation of interest rate to our portfolio will be analysed under

Delta = Duration,

Gamma = Convexity and vega which determine the sensitivity of short term rate in shift in the term structure of interest rate.

Before going to BDT model, we give a short introduction to Binomial interest rate models.

We assume one period interest rate develops in a binomial model according to a stochastic process.

At some point in time, the interest rate becomes negative (occurs in Japan in 20th century, due to over saving.), which is an undesirable property of a model which is supposed to describe nominal interest rate.

- *The probability of interest rate going up or down from each node is 0.5*
- *The states probabilities can be used to value calculation: The value of a state-dependent security is the present value of its expected payoffs.*

This then leads us to the description of our model.

2 THE BLACK-DERMAN-TOY MODEL

This is a one factor model which is arbitrary-free and thus consistent with the observed term structure of interest rates. It was first developed in-house by Goldman Sachs in 1980. The short-rate volatility is potentially time dependent, and the continuous process of the short-term interest rate is given by:

$$d\ln(r) = \theta(t)dt + \sigma(t)dV$$

It is the simplest recombining interest rate model which replicates the term structure. The insight from which the BDT is built from is as follows.

At any particular point in time, we know the term structure of interest rate say today

Years to Maturity	Zero-Coupon Bond Rate (%)	Zero-Coupon Bond Volatility
1	9.0	24.0
2	9.5	22.0
3	10.0	20.0
4	10.5	18.0
5	11.0	16.0

Table 1

The rate tree is shown below

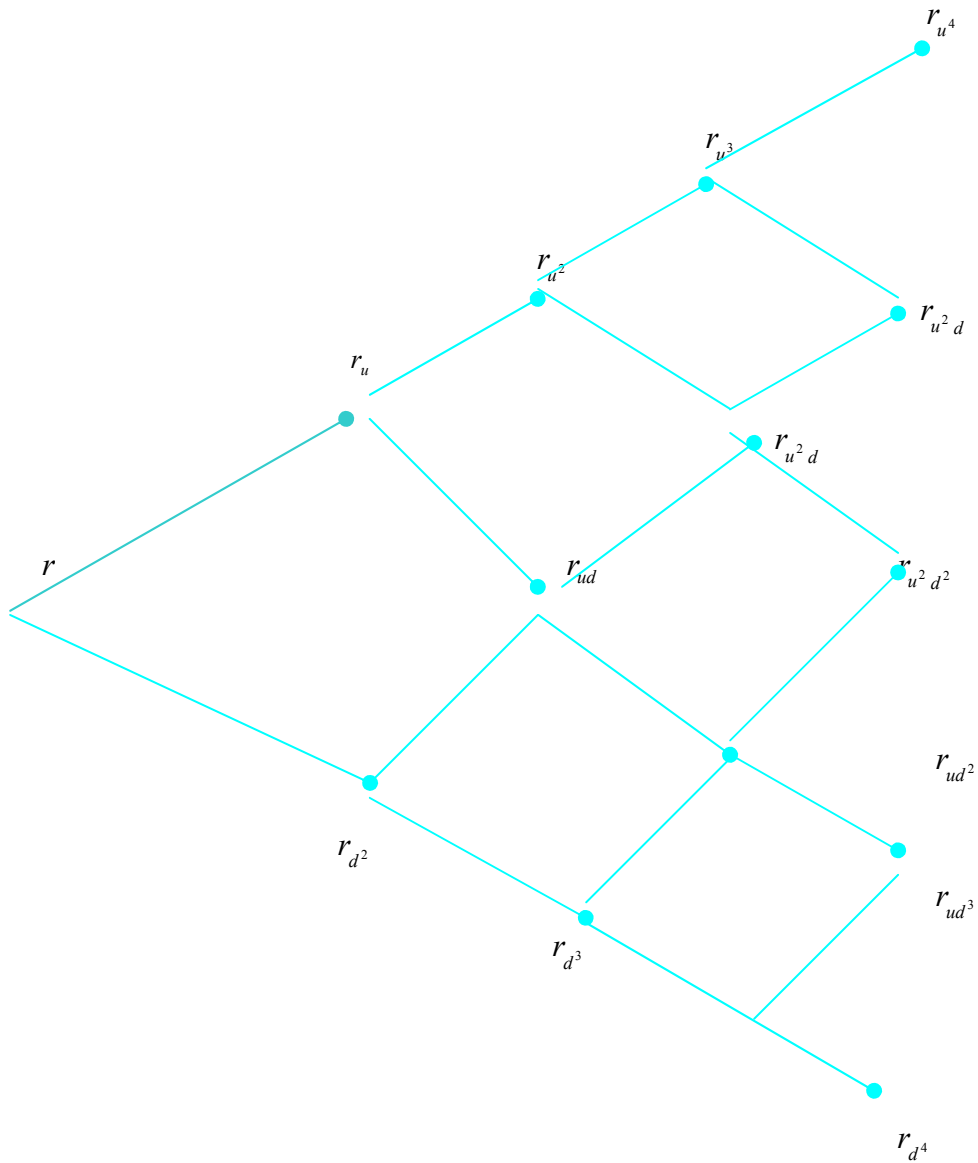


Figure 1: Four-Period Interest Rate Tree

The Interest rate is found by first calculating the prices of the zero-coupon bond with face value \$100 And maturity from one year to five year in to the future.

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

The Formula for the present value of the zero-coupon bond is given by:

$$PV = \frac{F}{(1+r)^N}$$

Where

$PV =$	Present value of zero-coupon bond
$F =$	Face Value (Principal),
$r =$	Risk free Interest rate (%)
$N =$	Time to Maturity

From Table 1

$$PV = \frac{100}{(1.09)} = 91.74$$

$$PV = \frac{100}{(1.095)^2} = 83.40$$

.....

.....

$$PV = \frac{100}{(1.11)^5} = 59.35$$

A one-period price tree is then given by

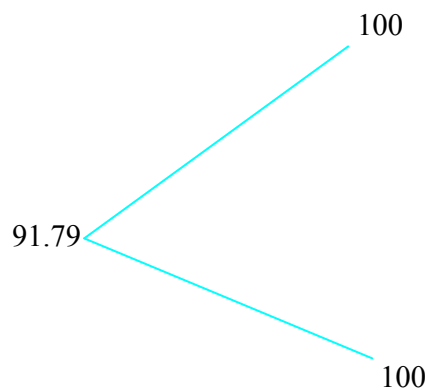


Figure 2: One-Period Price Tree.

We then build a two period price tree

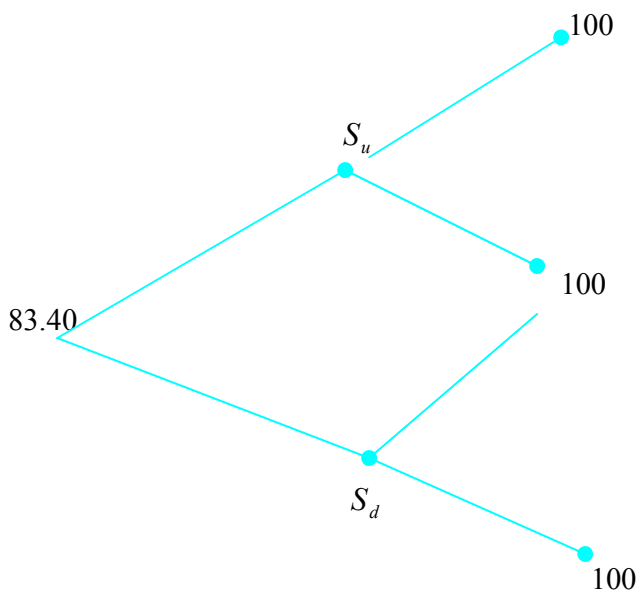


Figure 3: Two-Period Price Tree

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Applying risk-neutral valuation,

$$\frac{0.5\left(\frac{100}{1+r_d}\right) + 0.5\left(\frac{100}{1+r_u}\right)}{(1+0.09)} = 83.40$$

The volatility factor is define by

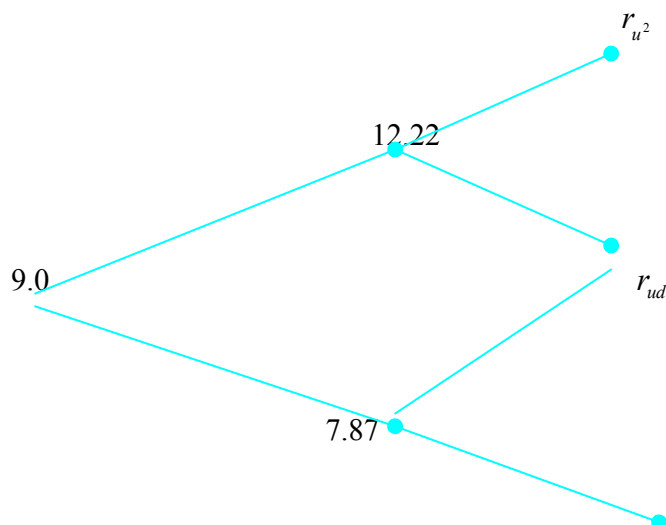
$$Z_n = e^{2\sigma_n\sqrt{\Delta t}}$$

$$r_u = r_d Z_n = r_d e^{0.44}$$

$$r_d = 7.87\%$$

$$r_u = 12.22\%$$

The rate tree for two-period is the given by;



Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

$$r_{d^2}$$

Figure 4: Two-Period Rate Tree

This procedure is done up to the fourth period where the corresponding interest rates and price at each node is determine.

For our model to hold we have made used of the following assumptions,

- Short rates are log-normally distributed
- No tax or transaction cost.
- Expected returns on all securities over one period are equal.

We then implement this model into mat lab GUI to analyse our portfolio sensitivity to interest rate shift.

3 User Guide to BDT Price/Sensitivity Analysis

The figure below has been develop for pricing and analysing the volatility in prices and interest rate as time elapses, and how to use financial toolbox to compute sensitivities of several instruments.

The function *bdtrens* compute the *Delta*, *Gamma*, and *Vega* sensitivity of instruments using interest rate tree created in *bdttree*.

We have develop and simple and user friendly interface for the user to move around with ease. If the user press the load button, the necessary parameters for our analysis, which is inbuild in matlab will be invoked.

By press the Calculate/Sensitivity button, the values of are delta, gamma Vega and price are diplace in the text field and a bar chart appears given us the magnitude of our sensitive parameters to interest rate shift

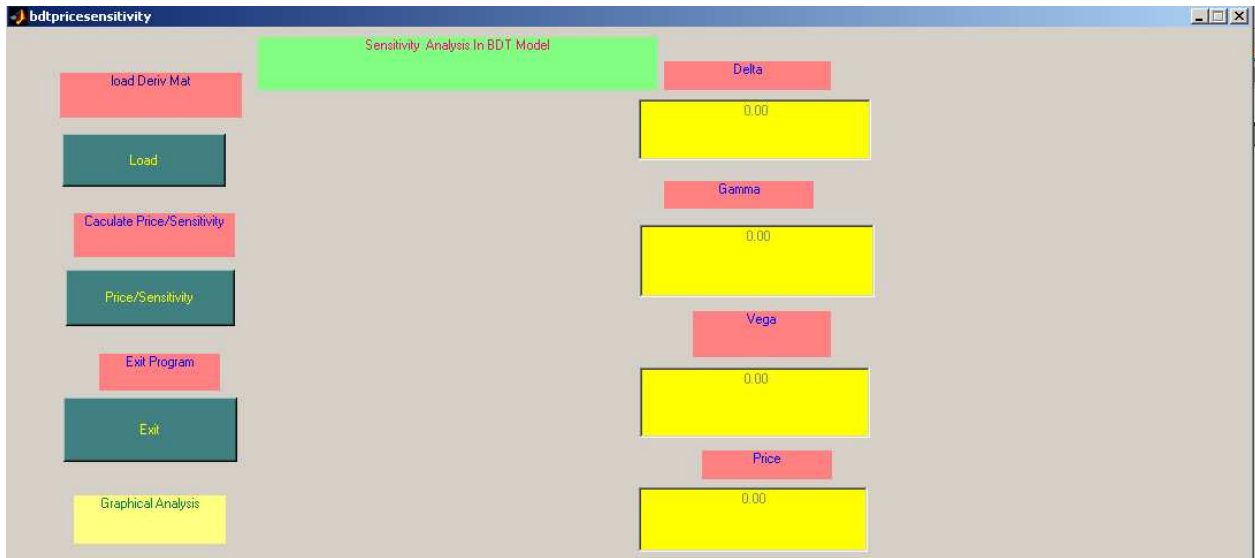


Figure 5: BDT Prefer/Sensitivity Analysis

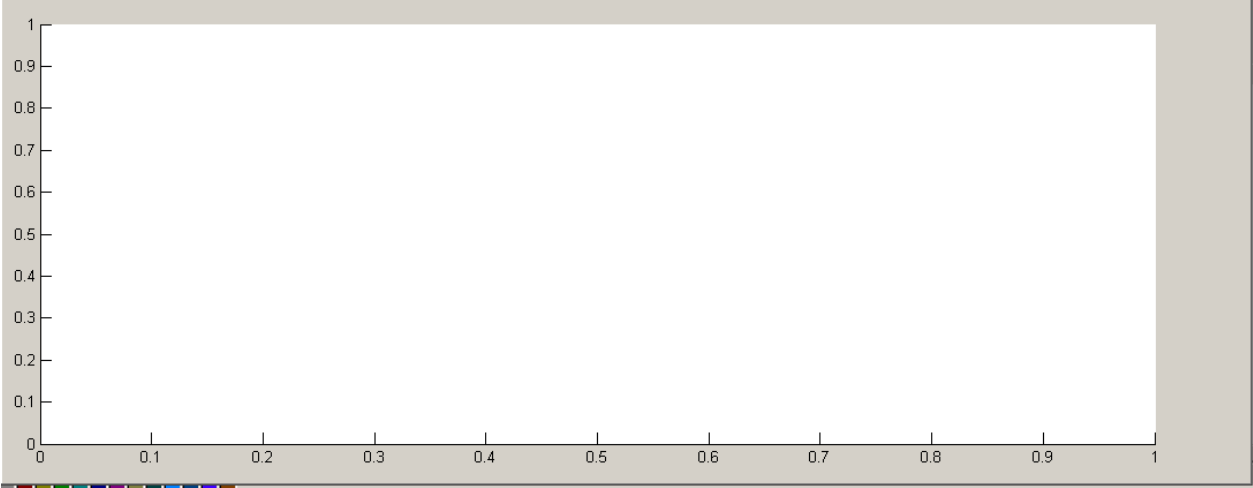


Figure 6: Graph Panel, illustrating sensitivity analysis

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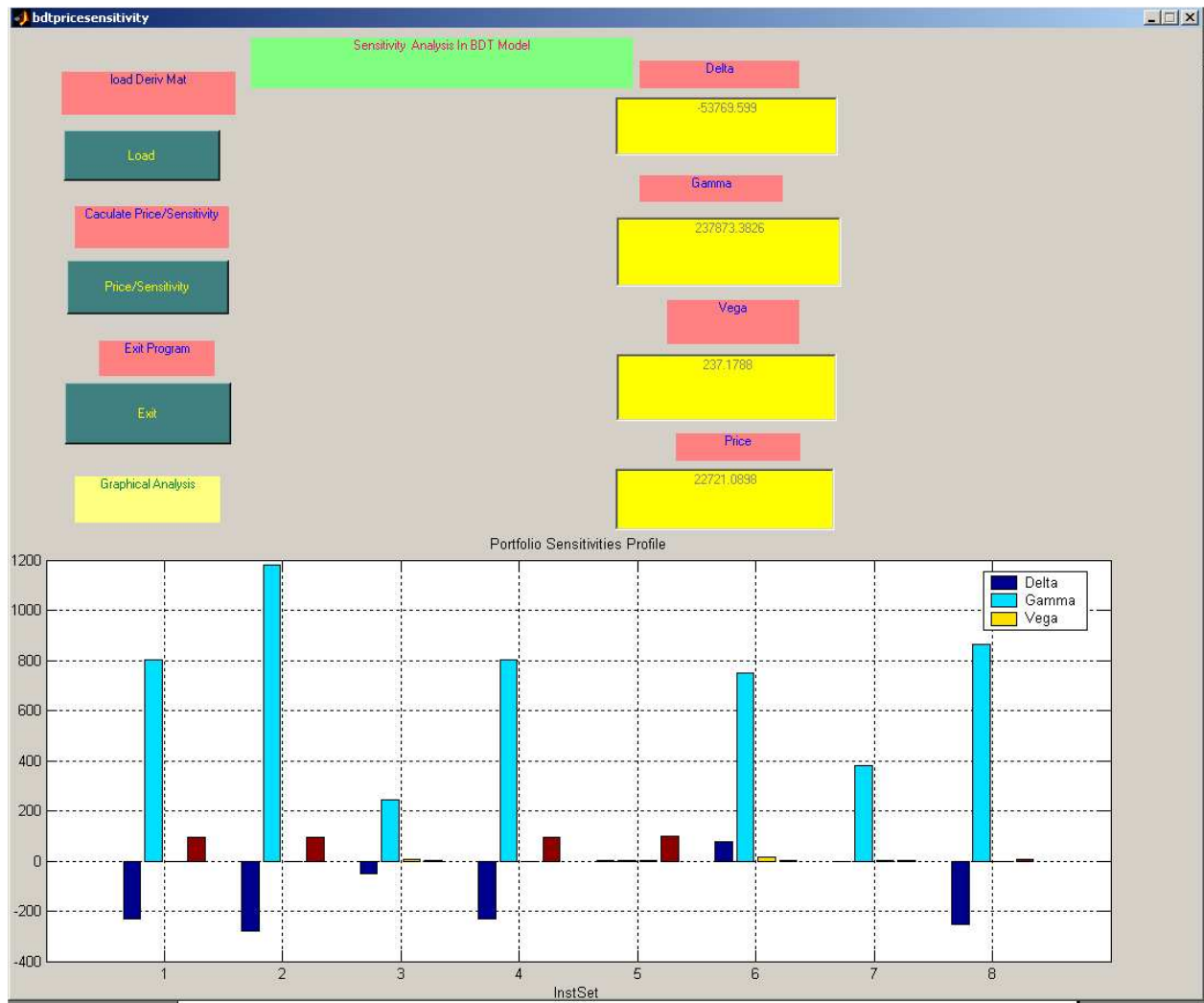


Figure 7: Pricer Illustration the magnitude of the sensitive instruments.

4 Conclusion

From the above analysis, the performance of portfolio's instruments is largely affected by interest rate volatility which fluctuates with the decay of time.

The price fluctuation is also influence by the volatility of the interest rate.

We then proposed that investors should hold a well diversified portfolio's of instruments to wipe out unsystematic risk in the market place and remain only with the interest rate risk(market risk), in which they are pay for.

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

Reference

http://en.wikipedia.org/wiki/Black-Derman-Toy_model Last update 26/09/2007

Matlabweb update 26/09/2007

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

Appendix

```
%author: Mbecho Techago
%
% e-mail: mto04001@student.mdh.se

function varargout = bdtpricesensitivity(varargin)
% BDTPRICESENSITIVITY M-file for bdtpricesensitivity.fig
%   BDTPRICESENSITIVITY, by itself, creates a new BDTPRICESENSITIVITY or raises
the existing
%   singleton*.
%
%   H = BDTPRICESENSITIVITY returns the handle to a new BDTPRICESENSITIVITY or
the handle to
%   the existing singleton*.
%
%   BDTPRICESENSITIVITY('CALLBACK',hObject,eventData,handles,...) calls the local
%   function named CALLBACK in BDTPRICESENSITIVITY.M with the given input
arguments.
%
%   BDTPRICESENSITIVITY('Property','Value',...) creates a new BDTPRICESENSITIVITY
or raises the
%   existing singleton*. Starting from the left, property value pairs are
%   applied to the GUI before bdtpricesensitivity_OpeningFunction gets called. An
%   unrecognized property name or invalid value makes property application
%   stop. All inputs are passed to bdtpricesensitivity_OpeningFcn via varargin.
%
%   *See GUI Options on GUIDE's Tools menu. Choose "GUI allows only one
%   instance to run (singleton)".
%
% See also: GUIDE, GUIDATA, GUIHANDLES

% Edit the above text to modify the response to help bdtpricesensitivity

% Last Modified by GUIDE v2.5 05-Oct-2007 14:21:30

% Begin initialization code - DO NOT EDIT
gui_Singleton = 1;
gui_State = struct('gui_Name',    mfilename, ...
    'gui_Singleton', gui_Singleton, ...
    'gui_OpeningFcn', @bdtpricesensitivity_OpeningFcn, ...
    'gui_OutputFcn', @bdtpricesensitivity_OutputFcn, ...
    'gui_LayoutFcn', [] , ...
    'gui_Callback', []);
if nargin & isstr(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
end
```

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

```
if nargin
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
% End initialization code - DO NOT EDIT

% --- Executes just before bdtpricesensitivity is made visible.
function bdtpricesensitivity_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% varargin   command line arguments to bdtpricesensitivity (see VARARGIN)

% Choose default command line output for bdtpricesensitivity
handles.output = hObject;
handles.s= [];

% Update handles structure
guidata(hObject, handles);

% UIWAIT makes bdtpricesensitivity wait for user response (see UIRESUME)
% uiwait(handles.figure1);

% --- Outputs from this function are returned to the command line.
function varargout = bdtpricesensitivity_OutputFcn(hObject, eventdata, handles)
% varargout  cell array for returning output args (see VARARGOUT);
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
set(handles.edit1,'String','0.00');
set(handles.edit2,'String','0.00');
set(handles.edit3,'String','0.00');
set(handles.edit4,'String','0.00');
% Get default command line output from handles structure
varargout{1} = handles.output;

% --- Executes on button press in pushbutton1.
function pushbutton1_Callback(hObject, eventdata, handles)
% hObject    handle to pushbutton1 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
```

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

```
handles.s =load('deriv.mat');
guidata(hObject,handles)
set(handles.pushbutton2, 'Enable', 'on');
set(handles.pushbutton3, 'Enable', 'on');
set(handles.edit1, 'Enable', 'off');
set(handles.edit2, 'Enable', 'off');
set(handles.edit3, 'Enable', 'off');
set(handles.edit4, 'Enable', 'off');

% --- Executes on button press in pushbutton2.
function pushbutton2_Callback(hObject, eventdata, handles)
% hObject    handle to pushbutton2 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
[Delta, Gamma, Vega, Price] = bdsens(handles.s.BDTree,handles.s.BDTInstSet);
Holdings = instget(handles.s.BDTInstSet, 'FieldName', 'Quantity');
All = [Delta Gamma Vega Price];
%Sensitivity = sum(PortfolioSensitivity);
TargetNumber = Holdings'*All
set(handles.edit1, 'String', num2str(TargetNumber(1)));
set(handles.edit2, 'String', num2str(TargetNumber(2)));
set(handles.edit3, 'String', num2str(TargetNumber(3)));
set(handles.edit4, 'String', num2str(TargetNumber(4)));
bar(All);
grid
xlabel('InstSet')
%ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title ('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', 0)
```

```
% --- Executes on button press in pushbutton3.
function pushbutton3_Callback(hObject, eventdata, handles)
% hObject    handle to pushbutton3 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
display Goodbye
close(gcf);
```

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

```
% --- Executes during object creation, after setting all properties.
function edit1_CreateFcn(hObject, eventdata, handles)
% hObject    handle to edit1 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc
    set(hObject,'BackgroundColor','y');
else
    set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
end

function edit1_Callback(hObject, eventdata, handles)
% hObject    handle to edit1 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of edit1 as text
%       str2double(get(hObject,'String')) returns contents of edit1 as a double

% --- Executes during object creation, after setting all properties.
function edit2_CreateFcn(hObject, eventdata, handles)
% hObject    handle to edit2 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc
    set(hObject,'BackgroundColor','y');
else
    set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
end

function edit2_Callback(hObject, eventdata, handles)
% hObject    handle to edit2 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of edit2 as text
```

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

```
%    str2double(get(hObject,'String')) returns contents of edit2 as a double
```

```
% --- Executes during object creation, after setting all properties.
```

```
function edit3_CreateFcn(hObject, eventdata, handles)
```

```
% hObject    handle to edit3 (see GCBO)
```

```
% eventdata reserved - to be defined in a future version of MATLAB
```

```
% handles    empty - handles not created until after all CreateFcns called
```

```
% Hint: edit controls usually have a white background on Windows.
```

```
%    See ISPC and COMPUTER.
```

```
if ispc
```

```
    set(hObject,'BackgroundColor','y');
```

```
else
```

```
    set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
```

```
end
```

```
function edit3_Callback(hObject, eventdata, handles)
```

```
% hObject    handle to edit3 (see GCBO)
```

```
% eventdata reserved - to be defined in a future version of MATLAB
```

```
% handles    structure with handles and user data (see GUIDATA)
```

```
% Hints: get(hObject,'String') returns contents of edit3 as text
```

```
%    str2double(get(hObject,'String')) returns contents of edit3 as a double
```

```
% --- Executes during object creation, after setting all properties.
```

```
function edit4_CreateFcn(hObject, eventdata, handles)
```

```
% hObject    handle to edit4 (see GCBO)
```

```
% eventdata reserved - to be defined in a future version of MATLAB
```

```
% handles    empty - handles not created until after all CreateFcns called
```

```
% Hint: edit controls usually have a white background on Windows.
```

```
%    See ISPC and COMPUTER.
```

```
if ispc
```

```
    set(hObject,'BackgroundColor','y');
```

```
else
```

```
    set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
```

```
end
```

```
function edit4_Callback(hObject, eventdata, handles)
```

```
% hObject    handle to edit4 (see GCBO)
```

```
% eventdata reserved - to be defined in a future version of MATLAB
```

Calculating Instruments Portfolio's Sensitivity Using Black-Derman-Toy Model In Matlab GUI

`% handles` structure with handles and user data (see GUIDATA)

`% Hints: get(hObject,'String')` returns contents of edit4 as text

`% str2double(get(hObject,'String'))` returns contents of edit4 as a double