Bose Vandermark (Lehman) Method

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Abstract

This paper presnets a flaw with the Black-Scholes model to valuate plain vanilla options with dividends. We show the underlying issue using an approximation with the Binomial model and compare the results given from the Bose-Vandermark Model (Lehman Model) to show a comparison of the option prices.

Contents

Introduction

The Binomial Model & The Cox Ross Rubenstein Model

The binomial model was introduced by John Cox, Stephen Ross, and Mark Rubinstein in the 1979s article 'Option Pricing: A simplified approach'. The model is an estimate of the Black Scholes model and has the advantage of it being more simplistic and flexible. It doesn't take into consideration a technical study of stochastic processes and gives a numerically accurate method to price both European and American options.

The binomial option pricing formula is based on an assumption that the stock price follows a multiplicative binomial process over discrete periods. The rate of return is a martingale, the risk-free martingale measurement and can be calculated as:

$$u = e^{\sigma \cdot \sqrt{dt}}$$

$$d = e^{-\sigma \cdot \sqrt{dt}}$$

As shown in the above formula the binomial model is characterized by u and d which are constants describing the potential price increase or decrease during a time period. Where dt is the time interval between observations of the price and σ the volatility. The probability of an up movement Q is showed in the formula below.

$$q = \frac{e^{N * r \Delta t} - d}{u - d}, \ p = 1 - q$$

The Cox Ross Rubinstein valuation model for a put option is given as:

$$P(q, N, S, K) = e^{-N * r \Delta t} \sum_{i=0}^{N} \binom{N}{i} q^i (1-q)^{N-i} \max(K - u^i d^{N-i} S, 0)$$

Bose-Vandermark (Lehman) Method

Bose –Vandermark(Lehman) method is a modification of stock price adjustment which helps users of this method be in a scope of spot price. This method is very close to dates, which were discovered by digital technique. Several years this method was used by quants in Lehman Brothers and this method helps quants save time in their calculations.

Here stock price difussion process exirity after dividend showed and diveded showed in two forms for the near and far.

$S(t)=(D_0-D_f)e^{rt}+(S_0-D_n)e^{\left(r-\frac{1}{2}*\sigma^2\right)t+\sigma w(t)}$, $t< t_d < T_2$

$S(t)=-D_f e^{rt}++(S_0-D_n)e^{\left(r-\frac{1}{2}*\sigma^2\right)t+\sigma w(t)}$, $t_d < t_d \leq T_2$

where,

$D_n=de^{-rt_d}(T-t_d)T=D_0(T-t_d)/T$

$D_f=de^{-rt_d}t_d/T=D_0t_d/T$

Stock price before dividend calculated standard Black Scholes

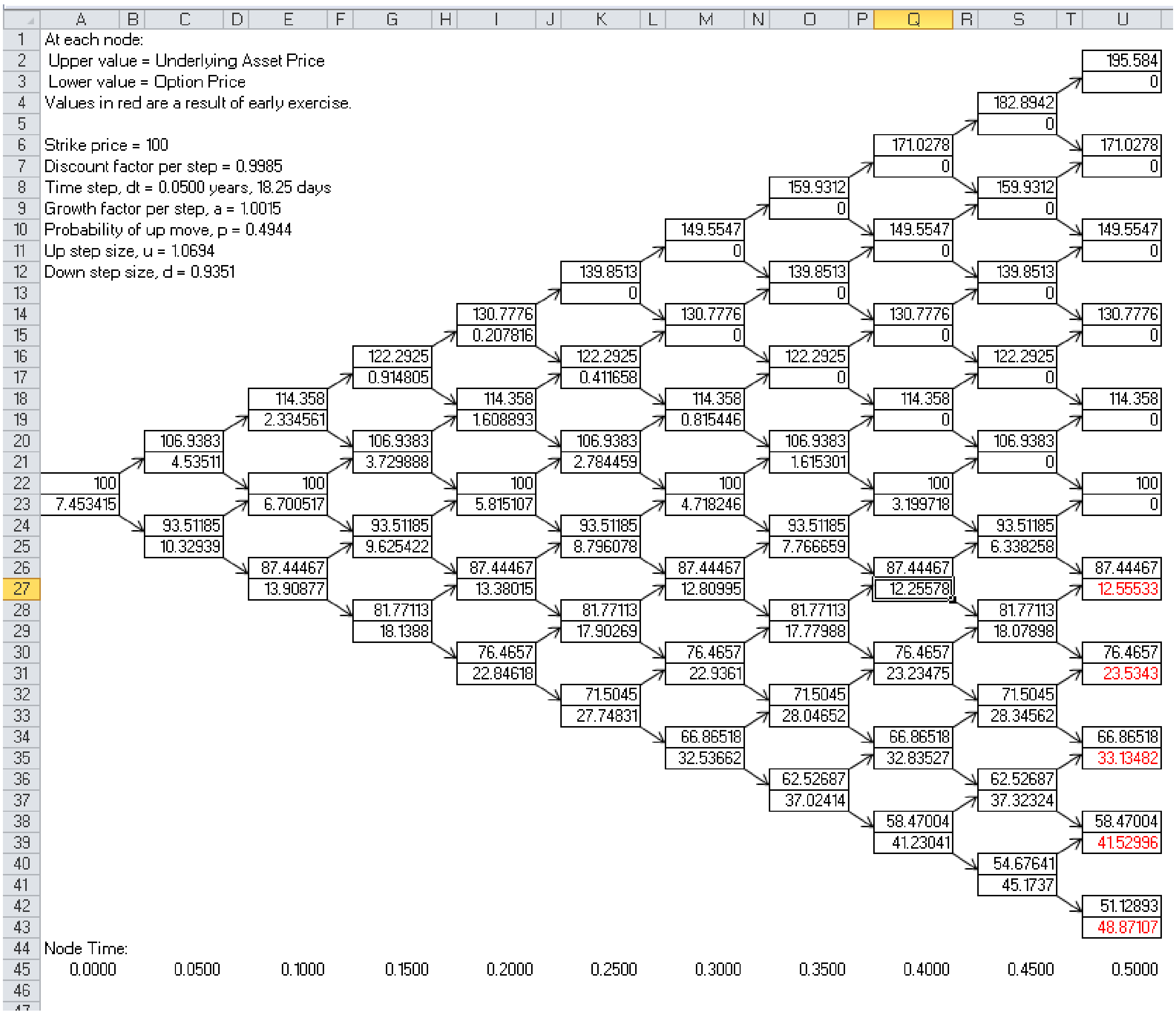
$S(t)=S_0e^{\left(r-\frac{1}{2}*\sigma^2\right)t+w(t)}$, $t < T_1 < t_d$

The Problem: Arbitrage opportunity that occurs when using Cox Ross Rubinstein

Example 1: Let's assume that we have a European put assume we analyze option X_a to see what the effects are when the option price exercises before ($T_a = 0.50 - \varepsilon$) and after ($0.50 + \varepsilon$) a dividend has been payed. The parameters for the option price, dividend, volatility, and interest rate is given below:

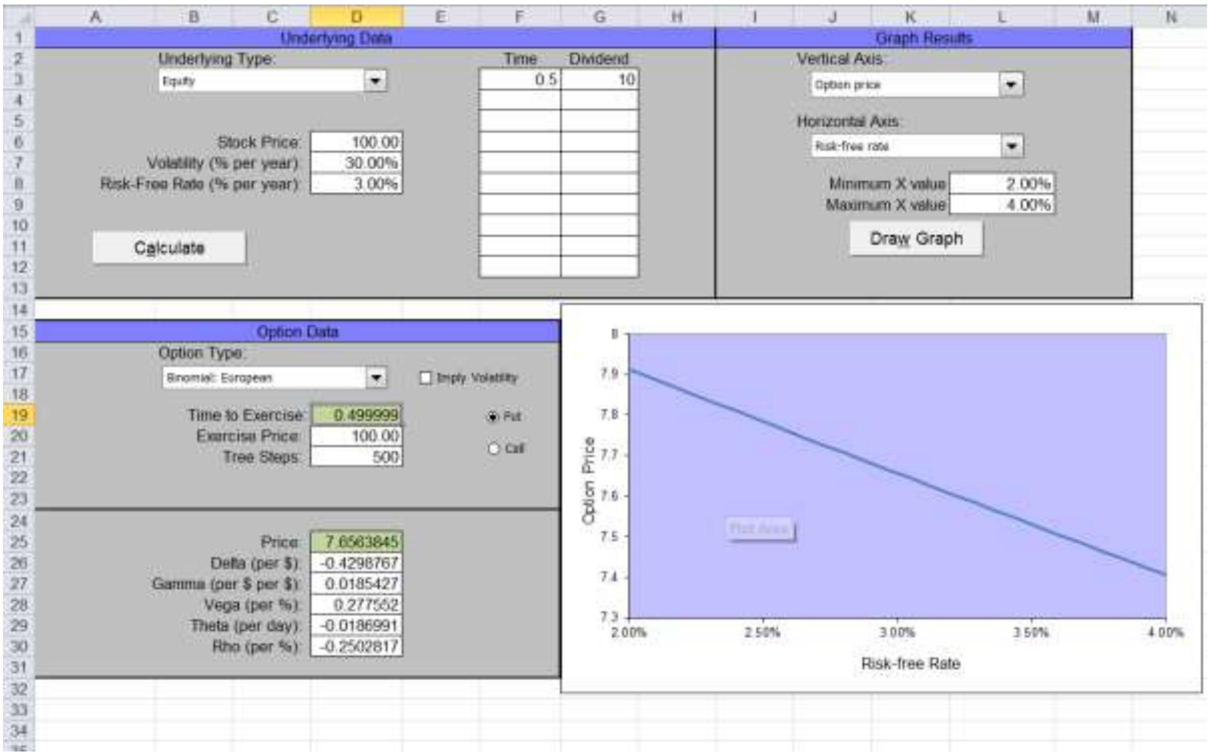
S	=	100.00	Stock price is 100
d	=	10.00	Dividend is 10
t_d	=	0.50	Time to dividend is six months
σ	=	0.30	Volatility is 30%
r	=	0.03	Interest rate is 3%
X_A	=	100.00	Strike price A is 100
T_A	=	$0.50 - \varepsilon$	Option A expires an instant <u>before</u> the dividend
T_B	=	$0.50 + \varepsilon$	Option A expires an instant <u>after</u> the dividend
N	=	500	Steps in the binomial tree

We implement the Cox Ross Rubinstein binomial model which gives us an approximate valuation of the put given if exercise is T_A.



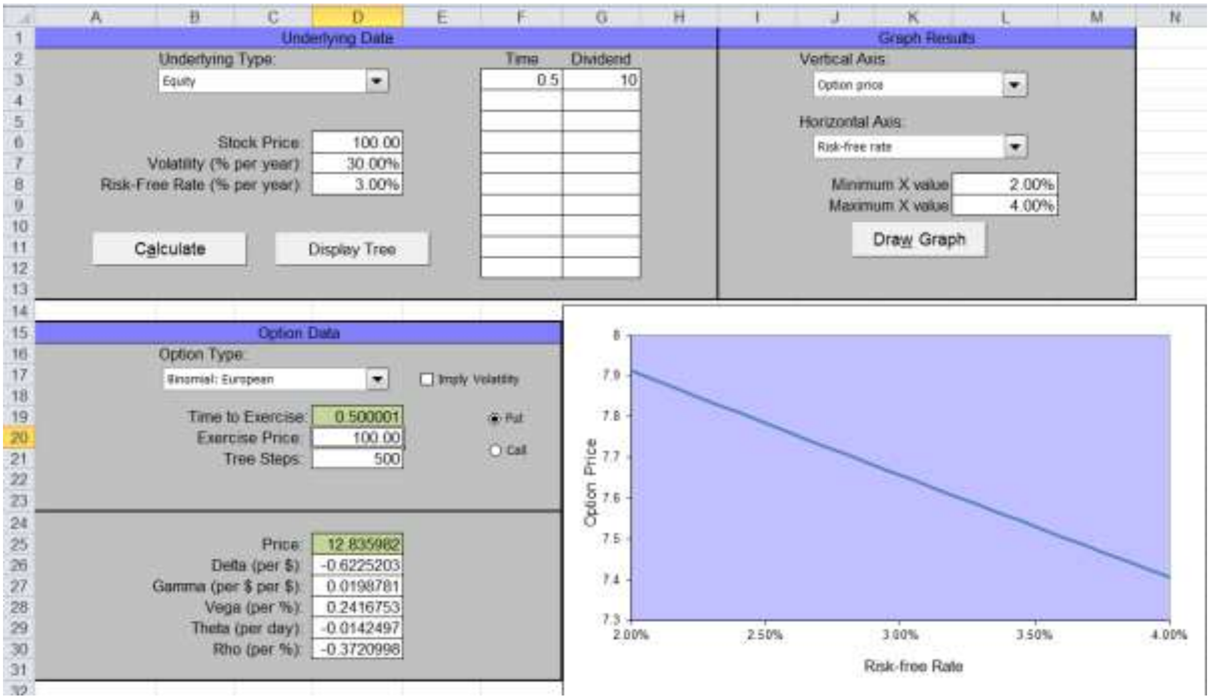
The graph shows only the first 10 steps in the binomial tree.

The valuation of the option is given as 7.4534. So what happens when the same option is exercised just an instant later?



Underlying Data
Underlying Type:
Equity
Time
Dividend
0.5
10
Stock Price:
100.00
Volatility (% per year):
30.00%
Risk-Free Rate (% per year):
3.00%
Calculate
Graph Results
Vertical Axis:
Option price
Horizontal Axis:
Risk-free rate
Minimum X value
2.00%
Maximum X value
4.00%
Draw Graph
Option Data
Option Type:
Binomial: European
Imply Volatility
Time to Exercise:
0.499999
Put
Exercise Price:
100.00
Call
Tree Steps:
500
Price:
7.6563845
Delta (per $):
-0.4298767
Gamma (per $ per $):
0.0185427
Vega (per %):
0.277552
Theta (per day):
-0.0186991
Rho (per %):
-0.2502817
Option Price
8
7.9
7.8
7.7
7.6
7.5
7.4
7.3
2.00%
2.50%
3.00%
3.50%
4.00%
Risk-free Rate

As we see in the graph above the options exercise is 0.49999. In the graph below we will see that the option value jumped up to a price of 12.83598 when exercise time is 0.500001.



The Solution using Bose-Vandermark (Lehman) Model:

To implement the Bose-Vandermark we created a Monte Carlo simulation in Mat Lab to account for the Weiner process.

```
%%%%%%%%%% Option parameters %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
S = 100; % Value of the underlying
K = 100; % Strike (exercise price)
r = 0.03; % Risk free interest rate
sigma = 0.3; % Volatility
T = 0.5; % Time to expiry
d= 10; % Dividend
Td=0.5;  %dividend time

%%%%%%%%%% Monte-Carlo Method Parameters %%%%%%%%%%%%%%%%%%%%%%%%
% randn('state',0) % Repeatable trials on/off
M=1e7; % Number of Monte-Carlo trials

%%%%% Use final values to compute %%%%%%%%%%%%%%%%%%%%%%%%%%
Dn=d*exp(-r*Td)*((T-Td)/T); % near dividend
Df=d*exp(-r*Td)/T; % near dividend
Kd= K-Df;

final_vals= (-Df*exp(r*T))+ (S-Dn)*exp((r-0.5*sigma^2)*T + sigma*sqrt(T)*randn(M,1));
option_values=max(Kd-final_vals,0); % Evaluate the Put option options
present_vals=exp(-r*T)*option_values; % Discount under r-n assumption
int=1.96*std(present_vals)/sqrt(M); % Compute confidence intervals
put_value=mean(present_vals); % Take the average
display([put_value-int put_value+int])
display(put_value)
```

Summary of valuation results:

CCR Put A - Before Expiry: 7.656
CCR Put A - After Expiry: 12.83

Put A Bose-Vandermark before Expiry: 7.809
Put B Bose-Vandermark after Expiry: 7.812 (7.8068 to 7.8199 with 95% confidence)

Arbitrage-Free — Near:	Yes
Arbitrage-Free — Far:	Yes
Expiration Consistency:	Yes
Process Consistency:	No, but the daily difference is small
Always Positive:	No — stock price can become negative!
B&S Compliant:	Yes
Reasonable:	Probably
Easy to Implement:	Hopefully

References

[1] URL: http://www.risk.net/data/Pay_per_view/risk/technical/2002/0902_bos.pdf, September 2002.
[2] Jan R.M.Röman, Lecture notes in Analytical Finance 1,Departamnent Of Matemetics and Pysics Mälardalen University,Sweden,August 29 2013.