Introduction
Barrier Options and Monte Carlo Simulations
The Discretized Black-Scholes Model
Simulation
The Algorithm
Example
Example 1
Others
The End

Barrier Option Pricing

– A Monte Carlo Simulation Approach

Jessica Radeschnig

October 21, 2013

Overview

1 Introduction
- Barrier Options and Monte Carlo Simulations
- The Discretized Black-Scholes Model

2 Simulation
- The Algorithm

3 Example
- Example 1

4 Others
- The End

Barrier Options and Monte Carlo Simulations

The price should be set as to be the value V of the option, where

$$V = \frac{1}{n}\sum_{i=1}^{n} V_i \tag{1}$$

where n is the number of simulations and

$$V_i = \begin{cases} C_i, & \text{for call options} \\ P_i, & \text{for put options} \end{cases}$$

Barrier Options and Monte Carlo Simulations

$$C_i = \mathbf{1} e^{-rT} \max\{S_i(T) - K, 0\}$$
$$P_i = \mathbf{1} e^{-rT} \max\{K - S_i(T), 0\}$$

where r is the risk-free interest rate, T is the time to maturity, $S(T)$ is the stock price at maturity and K is the strike price.

$\mathbf{1}$ is the indicator function:

$$\mathbf{1} = \begin{cases} 1, & \text{if in the contract} \\ 0, & \text{if out of the contract} \end{cases}$$

The Discretized Black-Scholes Model

$$\hat{S}(t_{j+1}) = \hat{S}(t_j)e^{(r-\sigma^2/2)T/m+\sigma\sqrt{T/m}Zj}. \tag{2}$$

where t is the current time and $S(t)$ represents the stock price at that time. T is the time of maturity, r is the annualised risk-free interest rate, σ is the annualized volatility and $W(t)$ is the Brownian Motion with distribution of $\sqrt{T}Z$, where Z is an independently distributed standard normal variable.

The Algorithm

for $i = 1$ to n **do**
  if Knock-In Option
  set $I = 0$
  elseif Knock-Out Option
  set $I = 1$
  for $j = 0$ to $m - 1$ **do**
    if $j = 0$
      generate $Z_i(t_1)$
      calculate $S_i(t_1)$ by (2)
        if Down-and-In Option *and* $S_i(t_1) \leq B$,
          or Up-and-In Option *and* $S_i(t_1) \geq B$
          $I = 1$
        elseif Down-and-Out Option *and* $S_i(t_1) \leq B$,
          or Up-and-Out Option *and* $S_i(t_1) \geq B$
          $I = 0$
        end if

The Algorithm

else $j \neq 0$
  generate $Z_i(t_{j+1})$
  calculate $S_i(t_{j+1})$ by (2)
    if Down-and-In Option *and* $S_i(t_1) \leq B$,
      or Up-and-In Option *and* $S_i(t_1) \geq B$
      $I = 1$
    elseif Down-and-Out Option *and* $S_i(t_1) \leq B$,
        or Up-and-Out Option *and* $S_i(t_1) \geq B$
      $I = 0$
    end if
  end if
end for

The Algorithm

if Call-Option
set $C_i = e^{-rT} max\{S_i(T) - K, 0\} \times I$
elseif Put Option
set $P_i = e^{-rT} max\{K - S_i(T), 0\} \times I$
end if
end for
set V by (1)

Example 1

Coming up....

Conclusions

- Good for solving complicated stochastic differential equations
- Different results each trial
- Increased precision $\Longrightarrow$ increased computational time
- When does benefits outweigh costs?

Introduction
Barrier Options and Monte Carlo Simulations
The Discretized Black-Scholes Model
Simulation
The Algorithm
Example
Example 1
Others
The End

Thank You!