

School of Education, Culture and Communication. Division of Applied Mathematics.

Analytical Finance I (MMA707) Seminar

Monte-Carlo application for Value-at-Risk on a portfolio of Options, Futures and Equities

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# Abstract

The purpose of writing this report is to create a Monte-Carlo application for Value-at-Risk on a portfolio of Options, Futures and Equities. We are going to calculate the value at risk for Options and Futures. We are going to calculate payoffs and the discount values of payoffs. We will also calculate the return. We are going to use Excel program to calculate all of the above and sketch the necessary graphs and tables. In the report, we are going to show subsets of Excel calculations. We will also explain and interpret our findings at the end.

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**Introduction**

The most common measure of risk is volatility. However, volatility does not care about the direction of an investment’s movement. For instance a stock can be volatile because it suddenly jumps higher. Investors however, are concerned about the odds of losing money. Value at Risk (VaR) tackles the investors concern by providing answers to the following questions.

* What is my worse case scenario?
* How much can I really lose in a bad month?

As such VaR must have three components.

* The time Horizon to be analyzed i.e. the time period over which a portfolio is held. In the following pages we have assumed our time horizon to be 30 days.
* The confidence interval i.e. the interval estimate in which the VaR would not be expected to exceed the maximum loss. In our illustration we use several just to compare the results.
* The Loss amount. This can also be expressed as a percentage.

Monte Carlo simulation is a method of generating scenarios that mimic a process and then finding the average values of the scenarios to approximate the outcome of the process. In principle it utilizes the law of large numbers i.e the mean value of a sample chosen from a finite population, is equal to the true mean of the population when the sample is considerably large.

In our assignment we use Monte Carlo simulation to generate terminal prices of a stock whose underlying asset is potatoes. Then we calculate the related payoffs and price series. The prices series is then used to determine the return series which is in turn used to calculate the Volatility and Value at Risk.

# Value at Risk for Option and Futures and Equities

We are going to give a methodology for calculating the Value at Risk measure for futures and Options. The methodology is given uses a Monte Carlo Simulator to generate terminal price series, and then calculates the payoffs and prices series. The prices series is used to specify the return series which is used in calculation of the volatility and Value at Risk.

# Construction of a Monte Carlo Simulator for prices of the underlying

In this section, we are going to show the construction of a Monte Carlo simulator to specify the terminal price of the underlying. We are going to refer to the daily prices of the option because are interested in that. The space or gap step length should be for a day. In our explanation, in this case we assume that the option contract will expire after 30 days so we have used 30 intermediate steps to simulate the development of prices of the underlying security for this period of time.

We are going to use the Black-Scholes formula to simulate the prices.

**St=S0\*exp[(r – q - 0.5σ2)t + σtzt] where**

S0 = initial stock price at time zero,
r = risk free rate
σ = volatility
q = dividend yield
t = time
zt = random sample from a normal distribution, with $μ$ = 0 and $σ$ = 1.
Using Excel zt can be obtained in these models by normally scaling the random numbers generated. The formula used in Excel are RAND() function, i.e. NORMINV(RAND()).



# Expand the Monte Carlo Simulator

To calculate the Value at Risk measure, we need a series of returns that also needs time-series price data. To simulate this specific environment we suppose that we have a series of similar option contracts that starts and expires in a one-day roll-forward basis. Suppose that for the original option the beginning was at time 0 and the expiry was at time step 30, the next option will start at time 1 and expire at time step 31, the next will start at time 2 and expire at step 32, and continues. Based on this introduction, we are going to get a time series of daily terminal prices. To generate time-series data for final prices for a period of 180 days we have repeated this process several times, a subset of which is visible below.



# Run scenarios

Above, we generated 180 days final prices series under a scenario. Now we need to repeat the process several times. In our case we will use 1000 simulations in order to generate a data set of time series. When this process has been completed an average terminal price time-series is calculated as shown in the table below. By taking a simple average of the terminal prices at each future date across all the simulated runs. The following table shows this process which have been done in Excel.

|  |  |
| --- | --- |
| **Terminal Prices** | **Date** |
| **Scenario** | **1** | **2** | **3** |  |  | **178** | **179** | **180** |
| **1** | **180.46** | **180.08** | **184.34** | **...** | **...** | **181.54** | **181.05** | **181.84** |
| **2** | **180.84** | **176.70** | **179.33** | **...** | **...** | **181.50** | **182.19** | **180.82** |
| **3** | **179.19** | **182.80** | **182.25** | **...** | **...** | **183.39** | **176.84** | **182.37** |
| **4** | **181.69** | **180.17** | **178.10** | **...** | **...** | **178.85** | **182.43** | **181.44** |
| **..** | **..** | **..** | **..** | **...** | **...** | **..** | **..** | **..** |
| **..** | **..** | **..** | **..** | **...** | **...** | **..** | **..** | **..** |
| **997** | **179.31** | **180.41** | **181.11** | **...** | **...** | **178.62** | **179.42** | **178.33** |
| **998** | **182.78** | **180.14** | **180.32** | **...** | **...** | **178.61** | **181.36** | **178.74** |
| **999** | **185.06** | **183.47** | **180.92** | **...** | **...** | **181.39** | **181.80** | **181.15** |
| **1000** | **180.06** | **179.36** | **178.16** | **...** | **...** | **180.81** | **180.54** | **182.54** |
| **Average Terminal Price** | **180.65** | **180.78** | **180.84** | **...** | **...** | **180.69** | **180.76** | **180.76** |

The average terminal price for date 1 in the table is the average of all the terminal prices generated for this date to 1000 simulation. The average terminal price for date 178 is the average of all the terminal prices generated for this date of 1000 simulation which is 180.76 as shown below.

# Calculations of the Payoffs

We then calculate the intrinsic or payoffs values of the derivatives contract for each data point given in the terminal price data set explained above. In our explanation above, we assumed that we have a futures contract. A European call option and a European put option all with a strike price of 181. The payoffs for these contracts are calculated in Excel as shown below.

Calculations of payoffs are as follows;

Payoff for a long futures = S0 – K
Payoff for a long call option = Max (S0 – K, 0)
Payoff for the long put option = Max (K – S0, 0)

# The table below shows the futures payoff



# Average payoff time series

Since all the payoffs have been calculated we find the average payoff time series by taking a simple average of the payoffs at each future data over all the simulation.

# Calculations of discount values of payoff (price)

Prices at each data point

The discounted values derived are the prices of the futures contract, Equity, the call and put options respectively. For each data point given in the terminal price data set explained above for which we find the payoffs or intrinsic values of the derivatives contract as written above, we will now calculate the discounted values as below

Discount value = Payoff \* e-rT Where
r = risk free rate.
T = time at maturity.
The table below shows futures prices



# Average price time-series

Since all the prices have been calculated we determine the average price time series by taking a simple average of the prices at each future data in all the simulation.

# Calculations of the return series

We will determine the return series since we have the derivatives average price series by taking the natural logarithm of successive prices. The calculations of futures, equity, call and put option contracts are as shown below



# Calculation of the value at risk measure

In this step we calculate the Value at Risk Using the Historical method where we refer to the simulated daily return processes as our historical Returns. As such VaR is calculated by computing a percentile over the range of the return data set under consideration. This is easily calculated in Excel by the function **=PERCENTILE (array, (100-X) %)** where X is the confidence level. VaR is a function of two parameters; **N** the time horizon and **X,** the confidence level. It is the loss over N days that a portfolio manager is X% certain will not be exceeded.

In our explanation, the 30-days holding period Value at Risk at different confidence levels, using the Historical method is calculated as shown below.



Calculating any N-day VaR is given by the formula; **N-day VaR99%=1-day VaR99%\*√N**

Thus our 1-day VaR =3.10%. In the above table we can say that we are 99% confident that we will not lose more that 17% in one month.



# Histograms showing Value at risk and risk management from Excel

Value at Risk

The simplest and the most common concept you are likely to see when it comes to financial risk management is Value at risk for short. It represents a guess estimate for

* How bad can things be when they really are bad?
* On a really bad day or month, what can you loss the most?
* What is the worst thing that can happen?

From our findings we would love to answer these questions as a way of interpreting the results obtained. For example supposing someone has invested in a call option. We will use the histogram below to answer the questions above.

# Diagram showing the call option

The histogram above calculates on a series of daily changes of prices of the call option. With the risk terms we call daily price changes, and daily returns, and these returns could be positive or negative. The table below takes a daily returns series, sorts the series and then slots each return in a given return bucket.

The return is calculated by the use of Ln ($\frac{P2}{P1}$). Where

Ln = natural logarithm,
P2 = New Price,
P1 = Old Price.

The histogram can be drawn from Excel program, the Histogram tool is found in the Data Analysis window in Excel sheet.



When you want to graph the Histogram you select the daily return series by calculating the percentage change in prices from one day to the next and use that series as your input range.  In the Histogram window click New worksheet ply, cumulative percentage and chart output to draw a graph (Histogram) and a table that explains the Histogram as shown below.

Now that we finish choosing all the necessary options in the Histogram window, click ok and Excel will create or graph a Histogram as shown below.

Our Histogram shows in the x-axis which starts from -23.43% which is the lowest to 21.61% which the highest and here is where our focus of risk is.
If we want to know what is the worst thing that can happen, we can easily see that our worst case scenario based on historical returns as shown by the above histogram is a loss of over 23.43% the left corner on the x-axis for a long call option investor or a loss of 21.61% for a short put option investor.

The next question if into risk will be, **over what time?**

Since returns are calculated on a daily basis therefore, the answer to the above question is over any one given day.

The other question is with what odds?

Luckily our excel Histogram output worksheet has a table with these probabilities.

|  |  |  |
| --- | --- | --- |
| *Bin* | *Frequency* | *Cumulative %* |
| -23.43% | 1 | 0.56% |
| -19.68% | 5 | 3.35% |
| -15.93% | 4 | 5.59% |
| -12.17% | 14 | 13.41% |
| -8.42% | 13 | 20.67% |
| -4.66% | 22 | 32.96% |
| -0.91% | 25 | 46.93% |
| 2.84% | 21 | 58.66% |
| 6.60% | 25 | 72.63% |
| 10.35% | 18 | 82.68% |
| 14.11% | 16 | 91.62% |
| 17.86% | 8 | 96.09% |
| 21.61% | 4 | 98.32% |
| More | 3 | 100.00% |

Since we only have percentage returns to work with, we have presented it in percentage.
If we look at the above table, there is a 0.56% chance that a long call option investor will experience a loss of 23.43% on a given trading day i.e. for instance if we should buy a long call option then there exists 13.41% chance that we will get a loss of about 12.17%.

# Methods used in Value at Risk

First we will like to define Value at Risk, value at risk is a probabilistic matric of a market risk used by organizations like banks to control the risk in their trading portfolios. Value at risk indicates the amount of money for a given probability at a given time such that there is a probability of the portfolio not losing more than the amount of money over the given time.

The methods that can be used to calculate Value at risk are Historical Simulation, Monte Carlo Simulation and variance covariance.

Historical Simulation does not require any statistic assumption in particular to volatility. When using Historical Simulation method, one have to consider the availability of a past sequence of daily portfolio returns for a given days. The Historical Simulation technique assumes that the distribution of tomorrow’s portfolio returns is approximated by the empirical distribution of the past day’s observation.

The Variance covariance method assumes that the returns are normally distributed, it applies the normal distribution.

The Monte Carlo Simulation technique applies a generator function and simulates a series of prices and then applies the same process used in Historical Simulation method. Monte Carlo simulation is a method of generating scenarios that mimic a process and then finding the average values of the scenarios to approximate the outcome of the process.

# Conclusion

Putting all this information together we can say that; if one should invest 100 SEK in long call options then the maximum they can lose is 23.43 SEK, in any given day with a probability of 0.56%. Value at Risk is a widely used risk measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, Value at Risk is a threshold value such that the probability that the market to market loss on the portfolio over the given time horizon exceeds this value in a given probability level. Value at Risk and volatility are the most commonly used risk measurements. Value at Risk is easy to calculate and can be used in many fields. Value at Risk has been developed as a risk assessment tool at banks and other organizations in the last decade.

# List of References

1. John, C.H.(2003). *Options, Futures and other Derivatives 5th Edition*. New Jersey: Prentice Hall
2. Lecture Notes.