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Volatility Surface

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Content

1. Implied Volatility

In financial mathematics, implied volatility is the volatility that matches the current price of an option, and represents current and future outlooks of market risk. In other words, it is the volatility that, when used in a given pricing model (such as Black-Scholes), yields a theoretical value for the option equal to the current market price of that option. This is in contrast to the normal definition of volatility, which is backwards-facing and is calculated from historical data (i.e. standard deviation of historical returns).It has been proved, with the aid of call-put parity equation, that the implied volatilities of European call and put options should be equal under the condition that the same strike price and time to maturity.

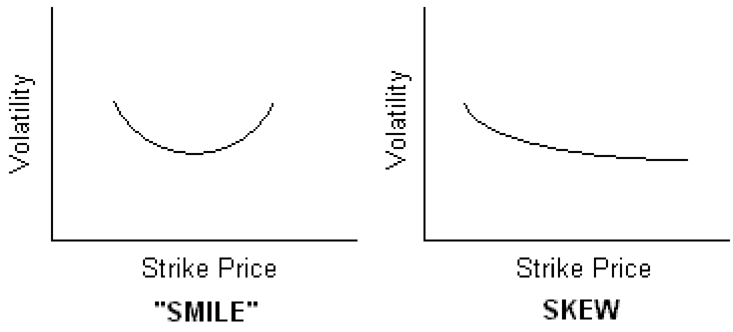
If traders expect the price of a stock to vary a lot, then its implied volatility (call and put options) will trend upwards. Implied volatilities often exceed their historic counterparts prior to a major announcement (such as an earnings announcement or a merger), and tend to the mean afterwards. For example, if the market is enthusiastic about a specific stock (perhaps due to a great earnings report), then a call option will be expensive. Accordingly, a covered call is a good strategy.

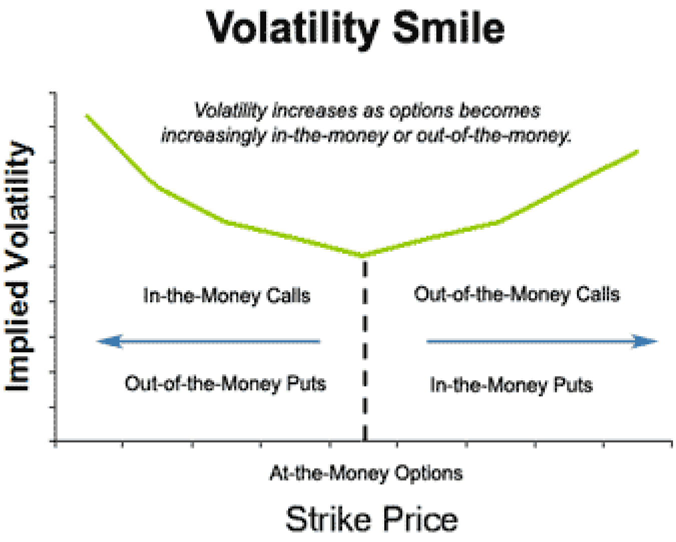
Implied volatility is so significant that options are often quoted in terms of volatility rather than price, particularly between professional traders. Often, the implied volatility of an option is a more useful measure of the option's relative value than its price. The reason is that the price of an option depends most directly on the price of its underlying asset. If an option is held as part of a delta neutral portfolio (that is, a portfolio that is hedged against small moves in the underlying's price), then the next most important factor in determining the value of the option will be its implied volatility.

For instance, a call option is trading at $1.50 with the underlying trading at $42.05. The implied volatility of the option is determined to be 18.0%. A short time later, the option is trading at $2.10 with the underlying at $43.34, yielding an implied volatility of 17.2%. Even though the option's price is higher at the second measurement, it is still considered cheaper based on volatility. The reason is that the underlying needed to hedge the call option can be sold for a higher price.

2.**Volatility Smile**

Volatility smile is one of curve shapes formed by charting the implied volatility of options across the various strike prices. The other shape is the volatility skew (see the graphs below). Basically, what the volatility smile shows is that implied volatility is higher as the options go more and more In The Money (ITM) and Out Of The Money (OTM), forming a curve in the shape of a smile, hence termed as Volatility Smile. Graphical characteristic where the implied volatility of options based on the same underlying asset and expiring on the same day displays a U-shape across the various strike prices.





Implied volatility is one of 2 major price determinants of options in the Black-Scholes Model. The other major determinant is, of course, the price of the underlying asset. In fact, implied volatility is the main determinant of extrinsic value while the price of the underlying stock determines the intrinsic value. Implied volatility has been used as a tool by option trading market makers to balance the supply and demand of every options contract since day one. When demand is high, market makers raise the implied volatility of that particular options contract in accordance to the forces of supply and demand, making a higher profit through a higher extrinsic value. When the implied volatilities of a particular options contract returns a volatility smile when charted, it tells you that both in the money and out of the money options are in huge demand and would contain more extrinsic value than would an At the Money option.

It is obvious that the Volatility Smile chart cannot be plotted without first finding out the implied volatility of the options across each strike price using an options pricing model such as the Black-Scholes model. However, the resulting Volatility Smile does laugh at the fallacy of the Black-Scholes model in assuming that implied volatility is constant over time. The Volatility Smile shows that, in reality, implied volatility is different across different strike prices even for the same period of time.

Options traders commonly track the Volatility chart of various stocks for the "Evolution of Volatility" over long periods of time. Evolution of Volatility is where volatility charts move from Volatility Smiles to Volatility Skews and vice versa over time. Such movement tells options traders that speculators are beginning to pour into a particular stock when a Volatility Skew evolves into a Volatility Smile. Such information may also be of particular interest to stock traders as well.

3. Term Structure of Volatility

For options of different maturities, we also see characteristic differences in implied volatility. However, in this case, the dominant effect is related to the market's implied impact of upcoming events. For instance, it is well-observed that realized volatility for stock prices rises significantly on the day that a company reports its earnings. Correspondingly, we see that implied volatility for options will rise during the period prior to the earnings announcement, and then fall again as soon as the stock price absorbs the new information. Options that mature earlier exhibit a larger swing in implied volatility (sometimes called "vol of vol") than options with longer maturities.

Other option markets show other behavior. For instance, options on commodity futures typically show increased implied volatility just prior to the announcement of harvest forecasts. Options on US Treasury Bill futures show increased implied volatility just prior to meetings of the Federal Reserve Board (when changes in short-term interest rates are announced).

The market incorporates many other types of events into the term structure of volatility. For instance, the impact of upcoming results of a drug trial can cause implied volatility swings for pharmaceutical stocks. The anticipated resolution date of patent litigation can impact technology stocks, etc.

Volatility term structures list the relationship between implied volatilities and time to expiration. The term structures provide another method for traders to gauge cheap or expensive options.

4. Implied Volatility Surface

The implied volatility surface simultaneously shows both volatility smile and term structure of volatility. It is often useful to plot implied volatility as a function of moneyness and time to maturity. The result is a three-dimensional curved surface whereby the current market implied volatility (Z-axis "Delta") for all options on the underlying is plotted against moneyness (Y-axis M) and time to maturity (X-axis "DTM").

4.1 Introduction to least square method under the background of volatility surface:

The concept of volatility surfaces implies that the mid volatility can be seen as function of time to expiration, *T*, and moneyness, *S/X*, i.e. (*T*, *X*). If we assume that the volatility surfaces are given by a 3rd degree polynomial surface

$(T, X) = c_0 + c_1 T + c_2 T^2 + c_3 T^3 + c_4 X + c_5 X^2 + c_6 X^3 + c_7 TX + c_8 T^2 X + c_9 TX^2$

The problem is to find the coefficients *ck*. If they are found, then the implied volatility can be calculated for any given time to expiration and strike price. The mid volatilities from the option series with market prices and parity prices provide several points. There are totally 10 unknown coefficients, *ck*, for a 3rd degree polynomial surface.
The minimum number of data points in order to calculate the coefficients are therefore 10. If there are market prices and parity prices corresponding to more than 10 data points ， these add up to an over determined system of linear equations. In this case there is no exact solution but there is a way to mathematically estimate the best approximation to the coefficients. This is called the method of least squares.

The set of data points can be converted into a linear system of equations using the equation above. The linear system of equations can be expressed as a matrix multiplication *A.c = z* were *A* is a matrix containing the times to expiration and the strike prices, *c* is a vector containing the unknown coefficients and z is a vector containing the mid volatilities.

$$A = \begin{bmatrix} 1 & T_1 & T_1^2 & T_1^3 & X_1 & X_1^2 & X_1^3 & T_1X_1 & T_1^2X_1 & T_1X_1^2 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 1 & T_k & T_k^2 & T_k^3 & X_k & X_k^2 & X_k^3 & T_kX_k & T_k^2X_k & T_kX_k^2 \end{bmatrix}$$

$$c = \begin{bmatrix} c_0 \\ \vdots \\ c_9 \end{bmatrix} \qquad z = \begin{bmatrix} \sigma_0 \\ \vdots \\ \sigma_9 \end{bmatrix}$$

We then do the simple algebra:

$$A \cdot c = z \quad \Rightarrow \quad A^T A \cdot c = A^T \cdot z \quad \Rightarrow \quad c = \left(A^T A\right)^{-1} \cdot A^T \cdot z$$

4.2 Introduction to making volatility surface:

We can find a volatility surface by using a least square method as discussed above from implied volatilities. The calibration process is then:

1. Calculate the implied mid volatilities from option prices.
We can use the internet webpage which is enclosed in the Reference part for calculating implied volatilities using Black Scholes model.

2. Calibrate the two volatility surfaces (call and put) to the implied mid volatilities.

3. Apply a spread to extract bid- and ask volatility surfaces. (Use larger spread for less liquid option series.)

5. Case Analysis

To be more specifically, the data of OMXS30[1] is collected and there are other crucial inputs as well. The risk free interest rate is assumed to be 0.5%, the price of underlying assets is always 1063.71, and dates to maturity are 0.0082 year and 0.0849 years (only valid for the last row). Then the Black-Scholes model is available to compute the implied volatilities[2].

Thus their mid volatility can be found from the table below:

Price Type	Bid	Imp. Bid (%)	Ask	Imp. Ask (%)	Strike	Bid	Imp. Bid (%)	Ask	Imp. Ask (%)	Price Type
Market	43.75	0.02	44.25	27.56	1020	0.3	25.09	0.45	27.05	Market
Market	24.75	20.14	26	26.05	1040	1.45	22.05	1.6	23.21	Market
Market	16.5	20.31	17.5	23.55	1050	2.9	20.82	3.15	21.64	Market
Market	9.5	19.47	10	20.79	1060	5.75	19.48	6.5	21.46	Market
Market	4.75	19.34	5	20.02	1070	10.75	18.67	11.5	20.72	Market
Market	1.95	19.11	2.15	19.85	1080	17.75	17.35	18.75	21.13	Market
Market	0.65	18.95	0.85	20.27	1090	26.25		27.25		Parity
Market	0.2	19.29	0.4	21.85	1100	35.75		36.75	22.91	Market
Market	0.04	22.43	0.09	24.73	1120	55.5		56.5	28.74	Market
Market	0.3	14.13	0.4	14.76	1160	93.5		96.25		Parity
Call						Put		OMXS30, Oct.19&Nov.16, 2012		

The average call mid volatility is 20.28188%, while the average put one is 19.395%.In turn, the difference between them is 0.886875%.In this case, the mid volatility of the option series (put option with strike price of 1090 and 1160) with parity prices is given as the mid volatility of the option series with opsposite option type adjusted with the average difference of the call and put mid volatilites of the option series that has market price in both the call and the put options.

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[1] Source: http://www.nasdaqomxnordic.com/

[2]Source: http://www.volatilitytrading.net/black_scholes_calculator.htm

Mid Vol(%).	Bid	Imp. Bid (%)	Ask	Imp. Ask (%)	Strike	Bid	Imp. Bid (%)	Ask	Imp. Ask (%)	Mid Vol(%).
13.79	43.75	0.02	44.25	27.56	1020	0.3	25.09	0.45	27.05	26.07
23.095	24.75	20.14	26	26.05	1040	1.45	22.05	1.6	23.21	22.63
21.93	16.5	20.31	17.5	23.55	1050	2.9	20.82	3.15	21.64	21.23
20.13	9.5	19.47	10	20.79	1060	5.75	19.48	6.5	21.46	20.47
19.68	4.75	19.34	5	20.02	1070	10.75	18.67	11.5	20.72	19.695
19.48	1.95	19.11	2.15	19.85	1080	17.75	17.35	18.75	21.13	19.24
19.61	0.65	18.95	0.85	20.27	1090	26.25		27.25		18.72313
20.57	0.2	19.29	0.4	21.85	1100	35.75		36.75	22.91	11.455
23.58	0.04	22.43	0.09	24.73	1120	55.5		56.5	28.74	14.37
14.445	0.3	14.13	0.4	14.76	1160	93.5		96.25		13.55813
Call						Put		OMXS30,Oct.19&Nov.16,2012		

Nextly,in order to compute the coefficients vector c, A, the matrix containing the times to maturity and the strike price, and Z, a vector including the mid volatilites of call options , are laid out as follow:

A=[1,0.0082,0.0082^2,0.0082^3,0.958908,0.958908^2,0.958908^3,0.0082*0.958908,0.0082^2*0.958908,0.0082*0.958908^2;

1,0.0082,0.0082^2,0.0082^3,0.97771,0.97771^2,0.97771^3,0.0082*0.97771,0.0082*0.0082*0.97771,0.0082*0.97771^2;

1,0.0082,0.0082^2,0.0082^3,0.987111,0.987111^2,0.987111^3,0.0082*0.987111,0.0082*0.0082*0.987111,0.0082*0.987111^2;

1,0.0082,0.0082^2,0.0082^3,0.996512,0.996512^2,0.996512^3,0.0082*0.996512,0.0082*0.0082*0.996512,0.0082*0.996512^2;

1,0.0082,0.0082^2,0.0082^3,1.005913,1.005913^2,1.005913^3,0.0082*1.005913,0.0082*0.0082*1.005913,0.0082*1.005913^2;

1,0.0082,0.0082^2,0.0082^3,1.015314,1.015314^2,1.015314^3,0.0082*1.015314,0.0082*0.0082*1.015314,0.0082*1.015314^2;

1,0.0082,0.0082^2,0.0082^3,1.024715,1.024715^2,1.024715^3,0.0082*1.024715,0.0082*0.0082*1.024715,0.0082*1.024715^2;

1,0.0082,0.0082^2,0.0082^3,1.034116,1.034116^2,1.034116^3,0.0082*1.034116,0.0082*0.0082*1.034116,0.0082*1.034116^2;

1,0.0082,0.0082^2,0.0082^3,1.052919,1.052919^2,1.052919^3,0.0082*1.052919,0.0082*0.0082*1.052919,0.0082*1.052919^2;

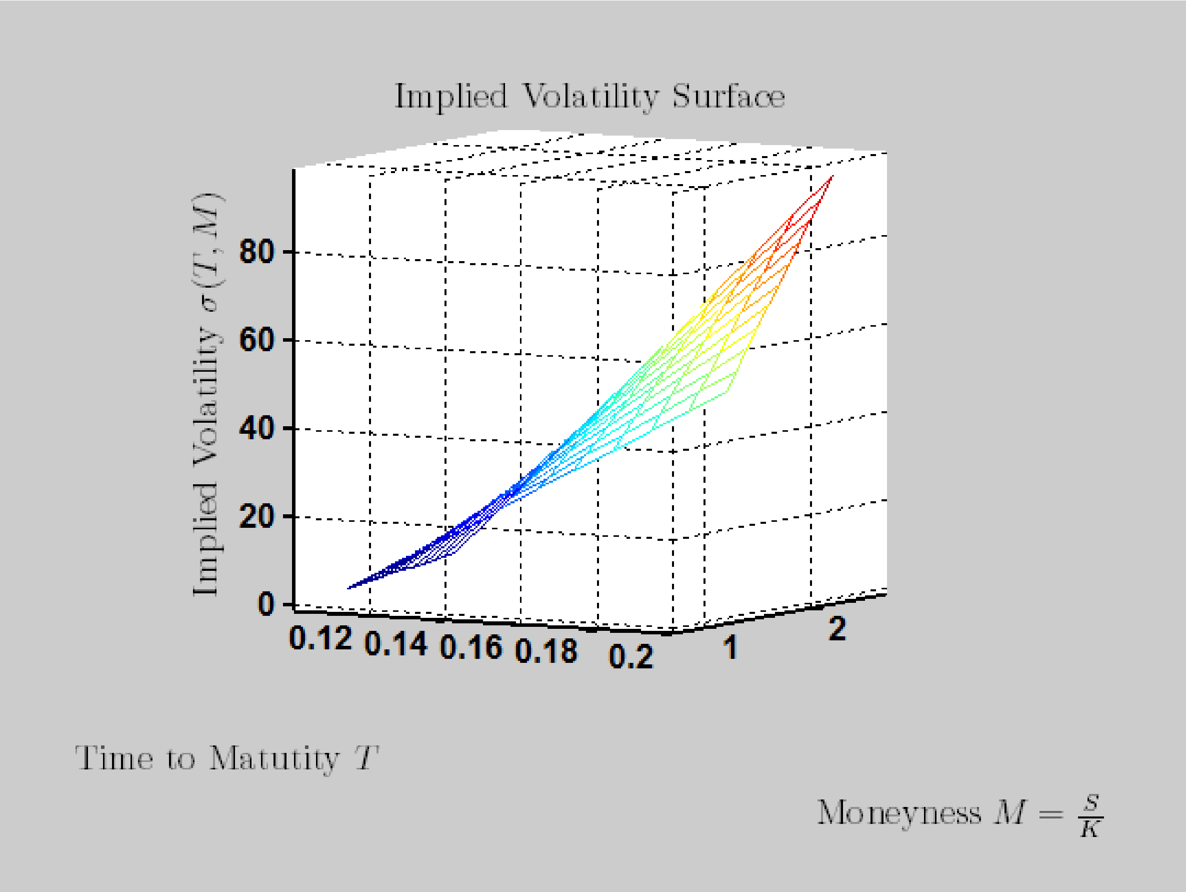
1,0.0849,0.0849^2,0.0849^3,1.090523,1.090523^2,1.090523^3,0.0849*1.090523,0.0849*0.0849*1.090523,0.0849*1.090523^2;];

Z=[0.1379;0.23095;0.2193;0.2013;0.1968;0.1948;0.1961;0.2057;0.2358;0.14445];

As a result,here comes to the surface equation for the call option of OMXS30.

$$\sigma(T,X) = -1.4651 + 0.9163*T + 0.4465*T^2 + 0.0628*T^3 + 0.6507*X - 2.0615*X^2 - 0.3845*T*X - 0.1882*X*T^2 + 2.0231*T*X^2;$$

In the end, the call option volatility surface for OMXS30 from October 2012, is plotted as following:



6. References

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