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EXOTIC OPTIONS WITH THE BINOMIAL MODEL

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Abstract

The aim of this paper is to present a binomial model which will value different exotic options i.e. cash-or-nothing, asset-or-nothing and power options. In the first chapter we present our theoretical research. The second part of this paper presents a program we wrote on MS Excel which valuates the European exotic options.

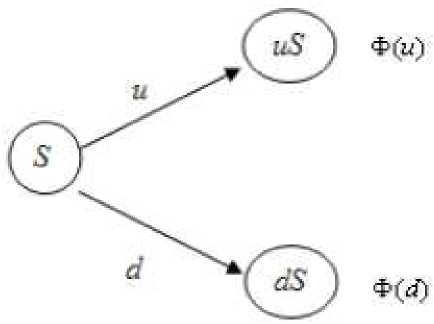
Table of contents

Introduction

The Binomial Model

The binomial model introduced by Cox, Ross and Rubinstein in 1979 is a very useful and popular technique for pricing an option[1] for two reasons – its simplicity and flexibility. First, this approach uses simple calculus, without the need of the complicated mathematical theory of stochastic Brownian motions and second, it gives a fast and numerically accurate method to price American options.[2]

The binomial option pricing formula is based on an assumption that the stock price follows a multiplicative binomial process over discrete periods. The rate of return of the stock over each period can have two possible values: u-1 with the probability q, or d-1 (1-d as shown in J. C. Hull, 2005) with the probability 1-q [3]. This can be represented with a following diagram:



Graph 1: One period binomial model [4]

As shown in Graph 1 the binomial model is characterized by *u* and *d* which are constants describing the potential price increase or decrease during a time period. Following we present the Cox-Ross-Rubinstein formula for *u* and *d*:

$$u = e^{\sigma\sqrt{dt}}, \qquad d = e^{-\sigma\sqrt{dt}}$$

where *dt* is the time interval between observations of the price and σ the volatility.

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[1] J. C. Hull, *Options, Futures, and Other Derivatives*, 6th Edition, Pearson Prentice Hall, New Jersey, 2005
[2] T. H F Cheuk, T. C F Vorst, *Currency lookback options and observation frequency: a binomial approach*, Journal of International Money and Finance, Vol. 16, No. 2 (1997), pp. 173 – 187
[3] J. C. Cox, S. A. Ross, M. Rubinstein, *Option Pricing: A Simplified Approach*, Journal of Financial Economics 7 (1979), pp. 229 - 263
[4] J. R. M. Röman, *Lecture notes in Analytical Finance I*, Department of Mathematics and Physics Mälardalen University, Sweden, 2012

Binary options

Binary options are options with discontinuous payoff[5]. It is a type of option where the payout is either fixed after the underlying stock exceeds the predetermined threshold (or strike price) or is nothing at all.[6]

The most common binary options are:

- Cash-or-nothing call
- Cash-or-nothing put
- Asset-or-nothing call
- Asset-or-nothing put

Binary Call Options pay either:

- a fixed cash settlement amount, if the underlying index settles at or above the strike price at expiration or
- nothing at all, if the underlying index settles below the strike price at expiration.

Binary Put Options pay either:

- a fixed cash settlement amount, if the underlying index settles below the strike price at expiration or
- nothing at all, if the underlying index settles at or above the strike price at expiration.[7]

Valuing cash-or-nothing options

In a risk-neutral world, the probability of the asset price being above the strike price at the maturity of an option is $N(d_2)$. Therefore the value of cash-or-nothing call and put in the Black-Scholes is:

$$Call: Qe^{-rT}N(d_2), \qquad Put: Qe^{-rT}N(-d_2)$$

where Q is the fixed amount which is paid at the strike time T and r is the risk-free interest rate.[8]

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[5] J. C. Hull, op. cit., p.535

[6] A. Thavaneswaran, S.S. Appadoo, J. Frank, *Binary option pricing using fuzzy numbers*, Applied Mathematic Letters 26 (2013), pp. 65 – 72

[7] http://www.cboe.com/Products/indexopts/bsz_spec.aspx

[8] J. C. Hull, op. cit., p.535

Valuing asset-or-nothing-options

Analogously to the cash-or-nothing options we can value the asset-or-nothing options. They value is given by:

$$Call\colon S_0 e^{-qT} N(d_1), \qquad Put\colon S_0 e^{-qT} N(-d_1)$$

where S is the initial stock price, q is the dividend rate.[9]

Power options

Power options are a class of exotic options in which the payoff at expiry is related to the x^{th} power of the stock price, where x is some power $x > 0$. For a power option on a stock with price S_t having strike price K and time to expiry T, the payoff is $\max(S_T^x - K, 0)$ for a call, and $\max(K - S_T^x, 0)$ for a put. Within the Black–Scholes model, closed-form solutions exist for the price of power options.[10] [11]

Valuing power options

Notations:

- c – value of power call option
- p – value of a power put option
- x – power of stock price
- S – a stock with given price
- q – fixed dividend yield
- r – assumed fixed interest rate
- K – strike price
- T – time to expiry

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[9] J. C. Hull, op. cit., p. 536

[10] E. G. Haug, *The Complete Guide To Option Pricing Formulas*, 2nd Edition, McGraw-Hill, 2006

[11] http://demonstrations.wolfram.com/PricingPowerOptionsInTheBlackScholesModel/

The value of a power call option is given by:

$$c = S^x \mathrm{e}^{\left[(\mathrm{x}-1)\left(\mathrm{r}+\frac{\mathrm{x}\sigma^2}{2}\right)-\mathrm{xq}\right]\mathrm{T}}\mathrm{N}(\mathrm{d}_1) - \mathrm{K}\mathrm{e}^{-\mathrm{rT}}\mathrm{N}(\mathrm{d}_2),$$

while the value of a put is:

$$p = \mathrm{K}\mathrm{e}^{-\mathrm{rT}}\mathrm{N}(-\mathrm{d}_2) - S^x \mathrm{e}^{\left[(\mathrm{x}-1)\left(\mathrm{r}+\frac{\mathrm{x}\sigma^2}{2}\right)-\mathrm{xq}\right]\mathrm{T}}\mathrm{N}(-\mathrm{d}_1),$$

where

$$d_1 = \frac{\ln\left(\frac{\mathrm{S}}{\mathrm{K}^{1/\mathrm{x}}}\right)+\left(\mathrm{r}-\mathrm{q}+\left(\mathrm{x}-\frac{1}{2}\right)\sigma^2\right)\mathrm{T}}{\sigma\sqrt{\mathrm{T}}} \quad \text{and} \quad d_2 = d_1 - x\sigma\sqrt{T}$$ [12]

[12] E. G. Haug, op. cit., p. 116

Solution

To write our program we used VBA. It valuates the european cash-or-nothing and asset-or-nothing options and presents the result as a binomial tree.

The presented example shows the outcome for the figures shown in Table 1.

Binormial Tree Price for a European Cash-or-Nothing Option	
Spot Price(S)	50
Strike Price(K)	40
Payoff(Q)	10
Year to Maturity(T)	0,5
Risk-Free Rate(r)	0,04
Dividend Yield(q)	0,02
Volatility(sigma)	0,4
Steps(n)	250
Type(Call or Put)	Call
Option Price	7,377507506

Table 1 – Properties of the European cash-or-nothing option

Binormial Tree Price for a European Asset-or-Nothing Option	
Spot Price(S)	30
Strike Price(K)	40
Year to Maturity(T)	0,5
Risk-Free Rate(r)	0,05
Dividend Yield(q)	0,01
Volatility(sigma)	0,3
Steps(n)	250
Type(Call or Put)	Call
Option Price	3,697904048

Table 2 – Properties of the European asset-or-nothing option

Call at each notes:

Upper value = Underlying Asset Price

Lower value = Option Price

Values in red are a result of early exercise.

Strike price = 40

Discount factor per step = 0,9917

Time step, dt = 0,1667 years, 60,83 days

Growth factor per step, a = 1,0084

Probability of up move, p = 0,4848

Up step size, u = 1,1774

Down step size, d = 0,8493

			81,6075
			41,6075
		69,3122	
		29,6442	
	58,8695		58,8695
	19,5306		18,8695
50		50	
12,2369		10,3319	
	42,4668		42,4668
	5,57305		2,46685
		36,0687	
		1,18594	
			30,6344
			0
Node Time:			
0,0000	0,1667	0,3333	0,5000

Table 3 – Binomial tree for the European cash-or-nothing option

Conclusion

In this paper we presented information about a binomial model which valuates cash-or-nothing and asset-or-nothing options. In the first part we have shown the theoretical research and presented information about the binomial model and binary options. In the second part of this paper we introduced a program written on MS Excel which valuates the European cash-or-nothing and asset-or-nothing options.

References

1. A. Thavaneswaran, S.S. Appadoo, J. Frank, *Binary option pricing using fuzzy numbers*, Applied Mathematic Letters 26 (2013), pp. 65 – 72
2. E. G. Haug, *The Complete Guide To Option Pricing Formulas*, 2nd Edition, McGraw-Hill, 2006
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4. J. C. Hull, *Options, Futures, and Other Derivatives*, 6th Edition, Pearson Prentice Hall, New Jersey, 2005
5. J. R. M. Röman, *Lecture notes in Analytical Finance I*, Department of Mathematics and Physics Mälardalen University, Sweden, 2012
6. T. H F Cheuk, T. C F Vorst, *Currency lookback options and observation frequency: a binomial approach*, Journal of International Money and Finance, Vol. 16, No. 2 (1997), pp. 173 – 187
7. http://demonstrations.wolfram.com/PricingPowerOptionsInTheBlackScholesModel/
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