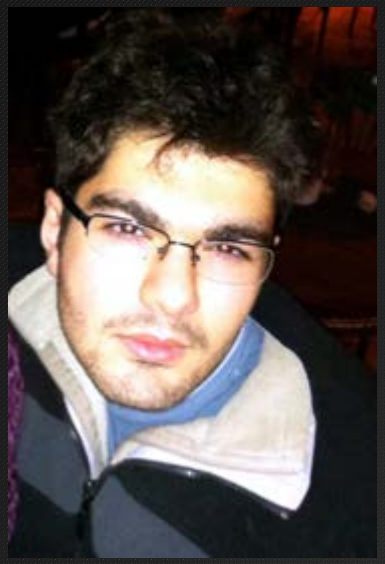
Bermudan Options with the Binomial Model

Who are we?



Amir Kazempour
Iran

Jessica Radeschnig
Sweden

Hossein Nohrouzian
Iran

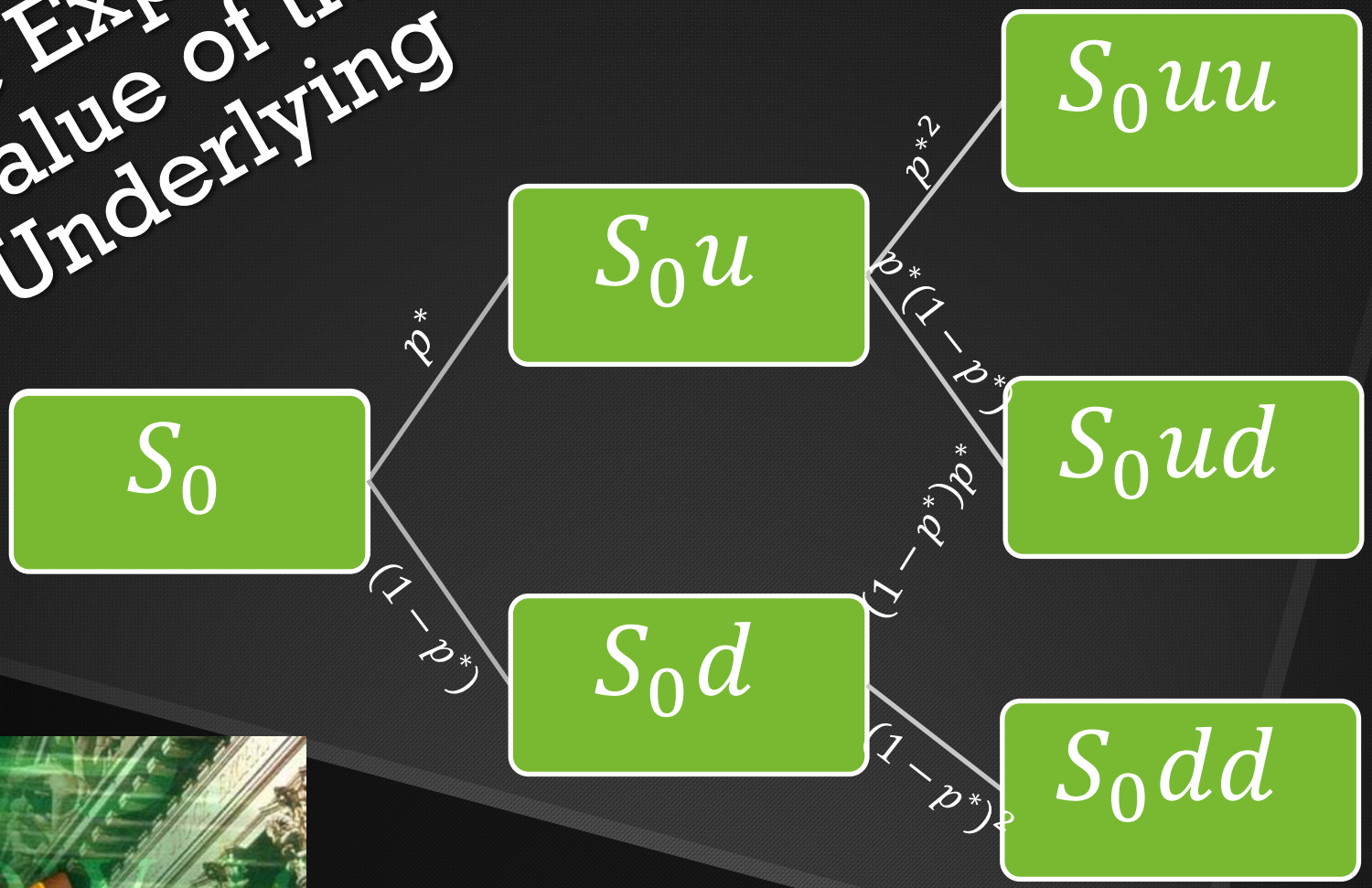
Analytical Finance

Agenda

- The Binomial Model
- Pricing Different Options
- Example
- MATLAB Implementation
- Questions

The Binomial Model

The Expected Value of the Underlying



The Binomial Model

The Up and Down Factors

$$u = e^{\sigma\sqrt{\Delta T}} \qquad d = e^{-\sigma\sqrt{\Delta T}}$$

The Binomial Model

The Risk-Neutral Probability Measure

$$p^* = \frac{e^{r\Delta T} - d}{u - d}$$

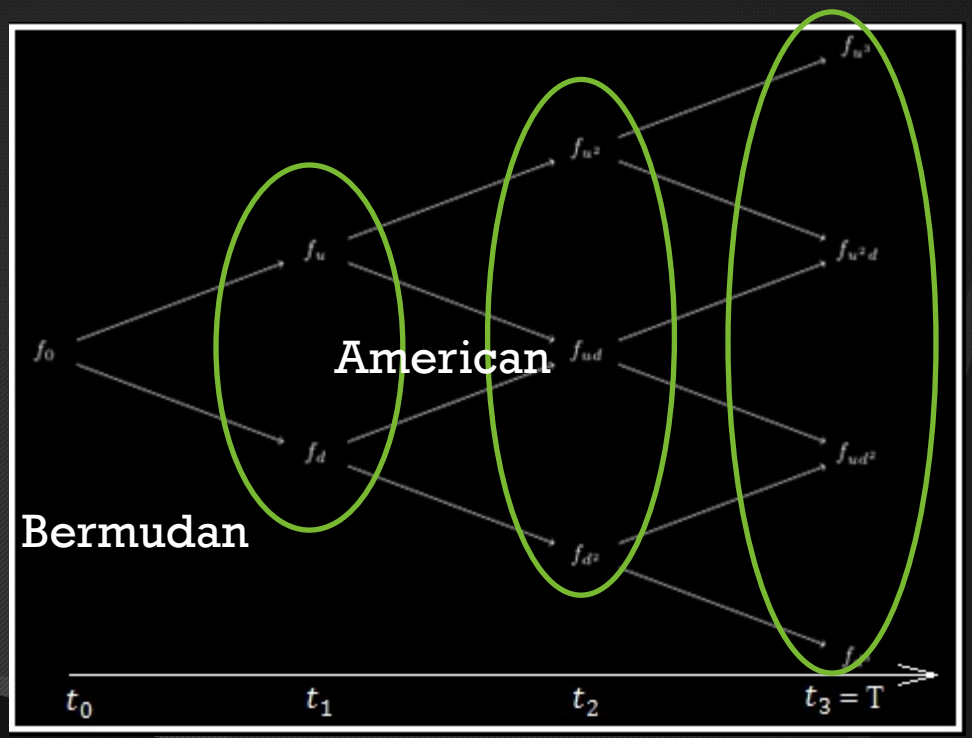
Pricing Different Options

Pay-off Functions		
	Position	
Option Type	Long	Short
Call	$\max(S_T - K, 0)$	$-\max(S_T - K, 0)$
Put	$\max(K - S_T, 0)$	$-\max(K - S_T, 0)$

Pricing Different Options

$$f_T = Long\ Pay - Off \qquad for \qquad t = T$$

$$f_t = e^{-r\Delta T} p^* f_u(t+1) + (1-p^*) f_d(t+1) \qquad for \qquad 0 \le t \le \mathrm{T}$$



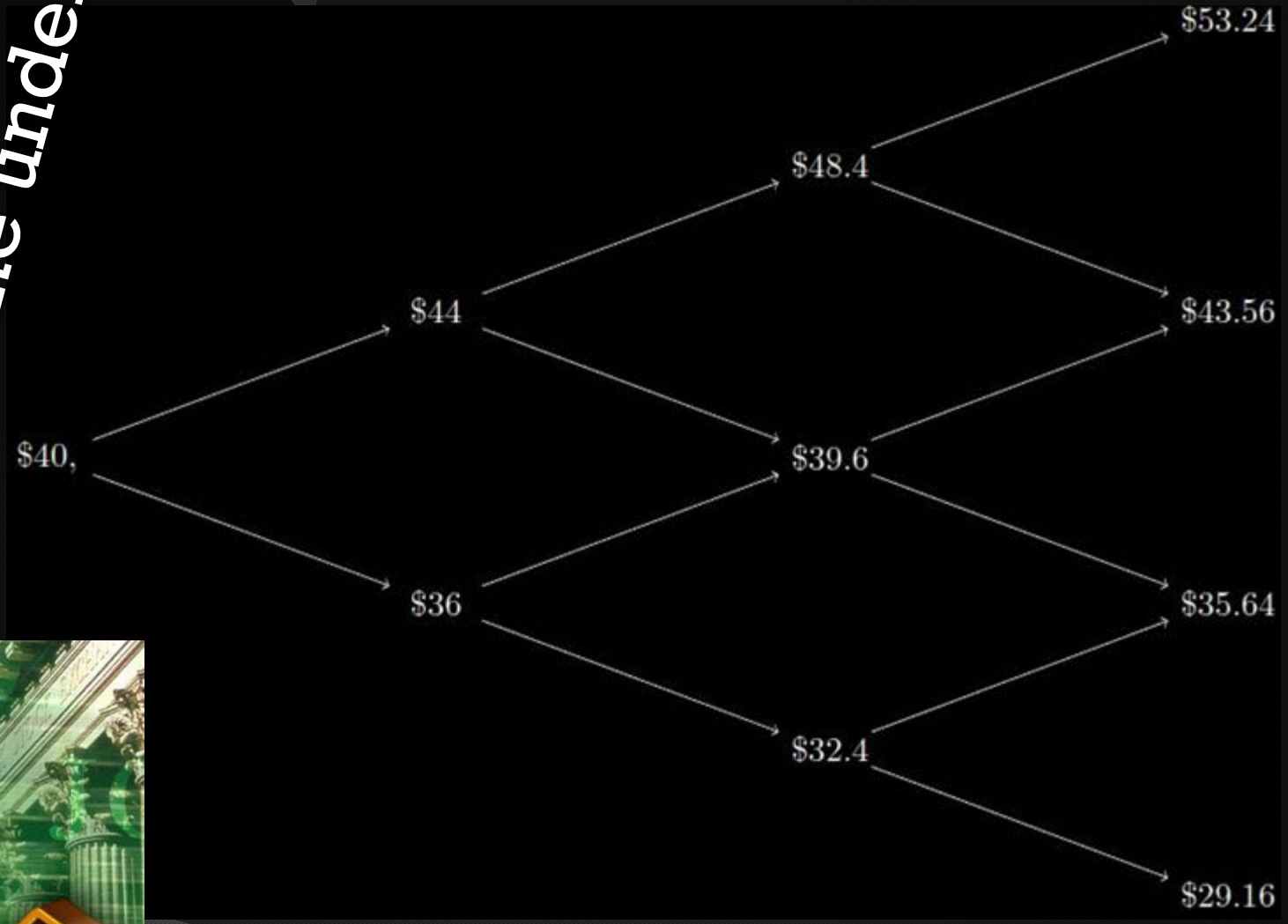
Example

European Put Option

- S_0 =$40
- K=$42
- r=12%
- ΔT=3-month
- T=9 months
- u=1.1
- d=0.9

Example

Value of the underlying



Example

Calculating the Risk-Neutral Probability:

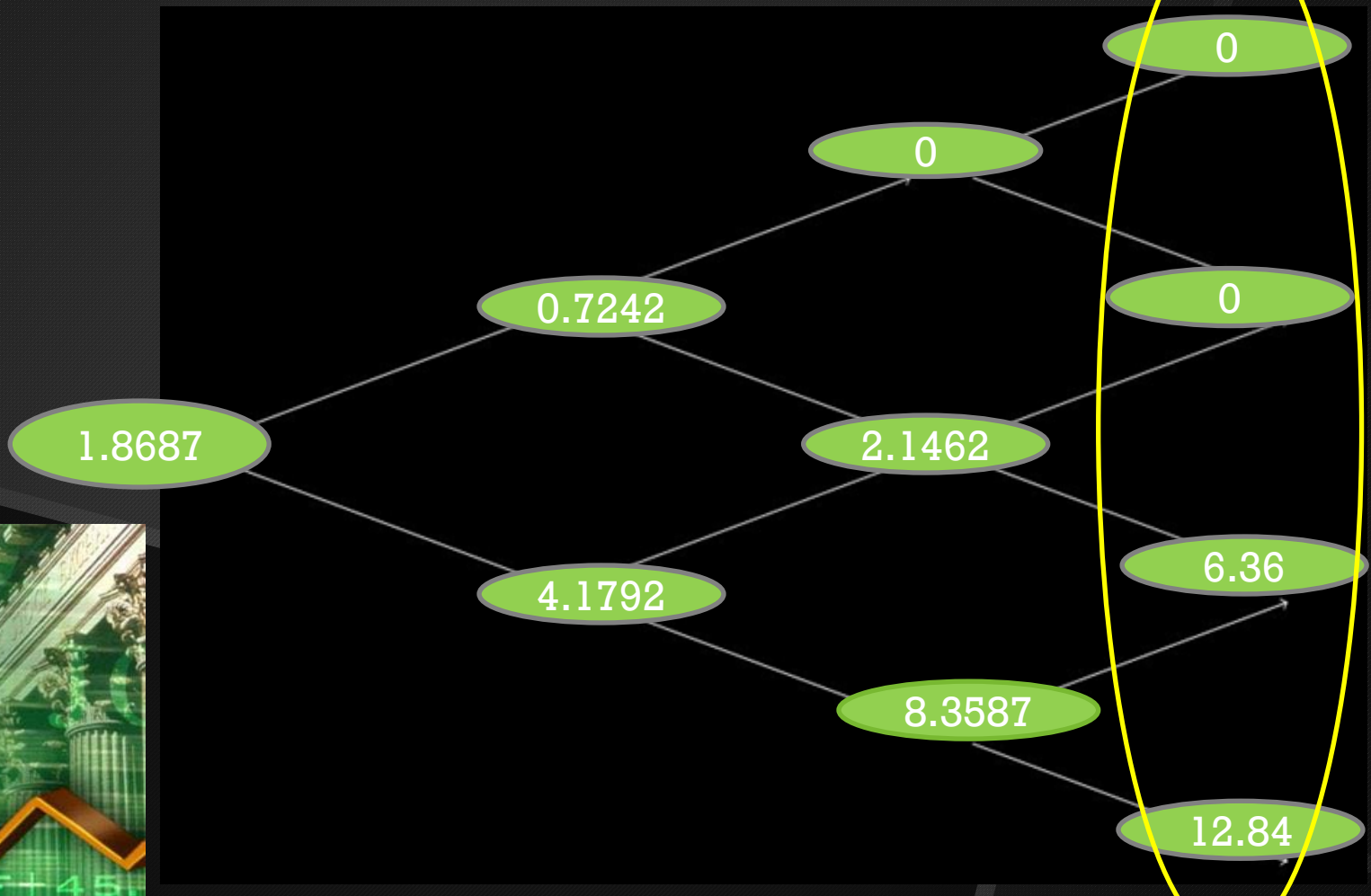
$$p^* = \frac{e^{r\Delta T} - d}{u - d}$$

$$p^* = \frac{e^{0.12\left(\frac{3}{12}\right)} - 0.9}{1.1 - 0.9} \approx 0.6523$$

Example

Price of the European Option

$$P_E = e^{r\Delta T}\left[p^{*3} f_{u^3} + 3p^{*2}(1-p^*) f_{du^2} + 3p^*(1-p^*)^2 f_{ud^2} + (1-p)^3 f_{d^3}\right] = \$1.8687$$



Price of the American Option

Example

max{42-53.24,0}

max{42-43.56,0}

max{42-35.64,0}

max{42-29.16,0}

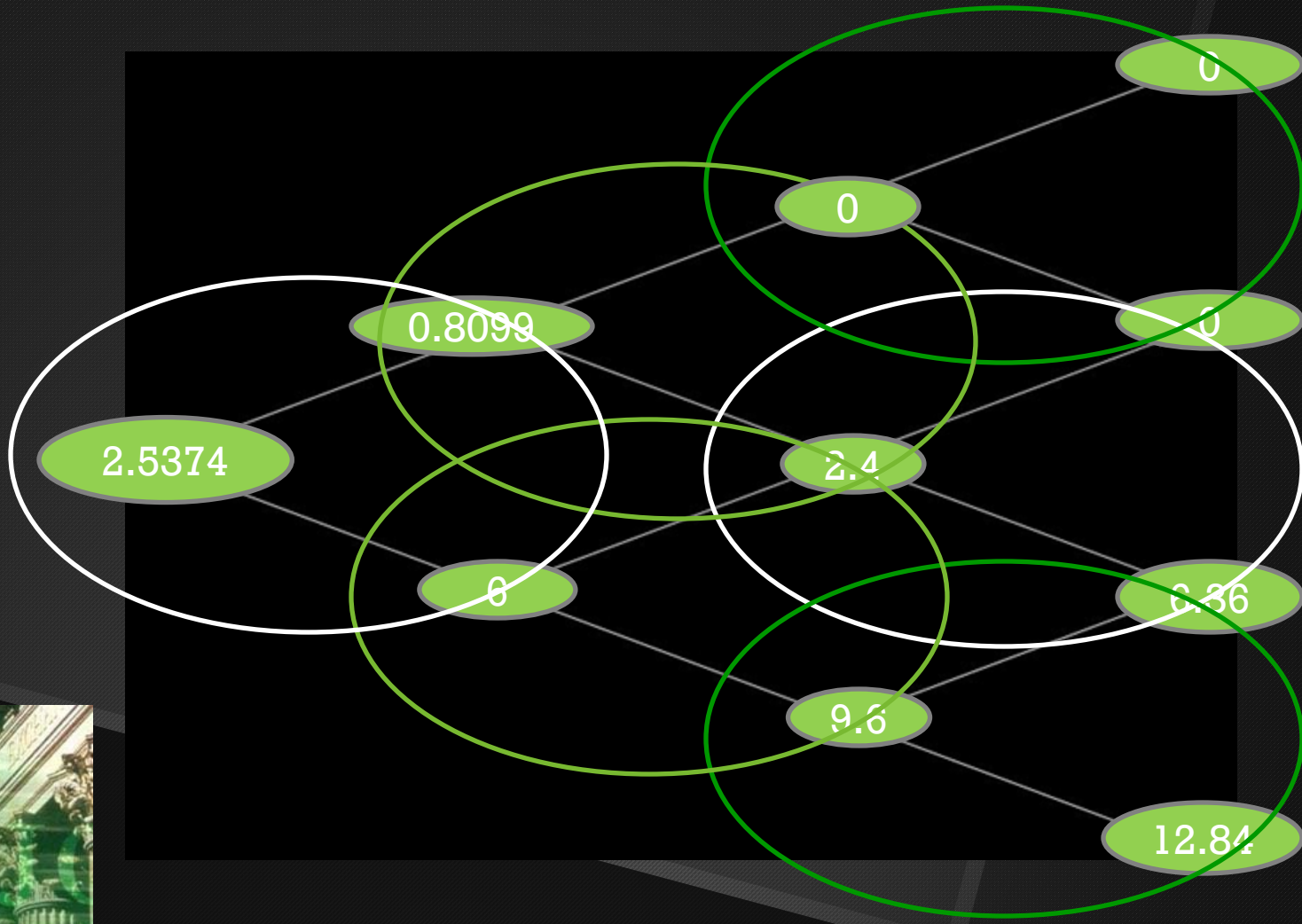
max{0,0}

max{2.4,2.1462}

max{9.6,8.3587}

max{0,0.8099}

max{6,4.7587}



$$P_A = e^{-r\Delta T}[p^*(0.8099) + 6(1 - p^*)] = 2.5374$$

Price of the Bermudan Option

Example

max{42-53.24,0}

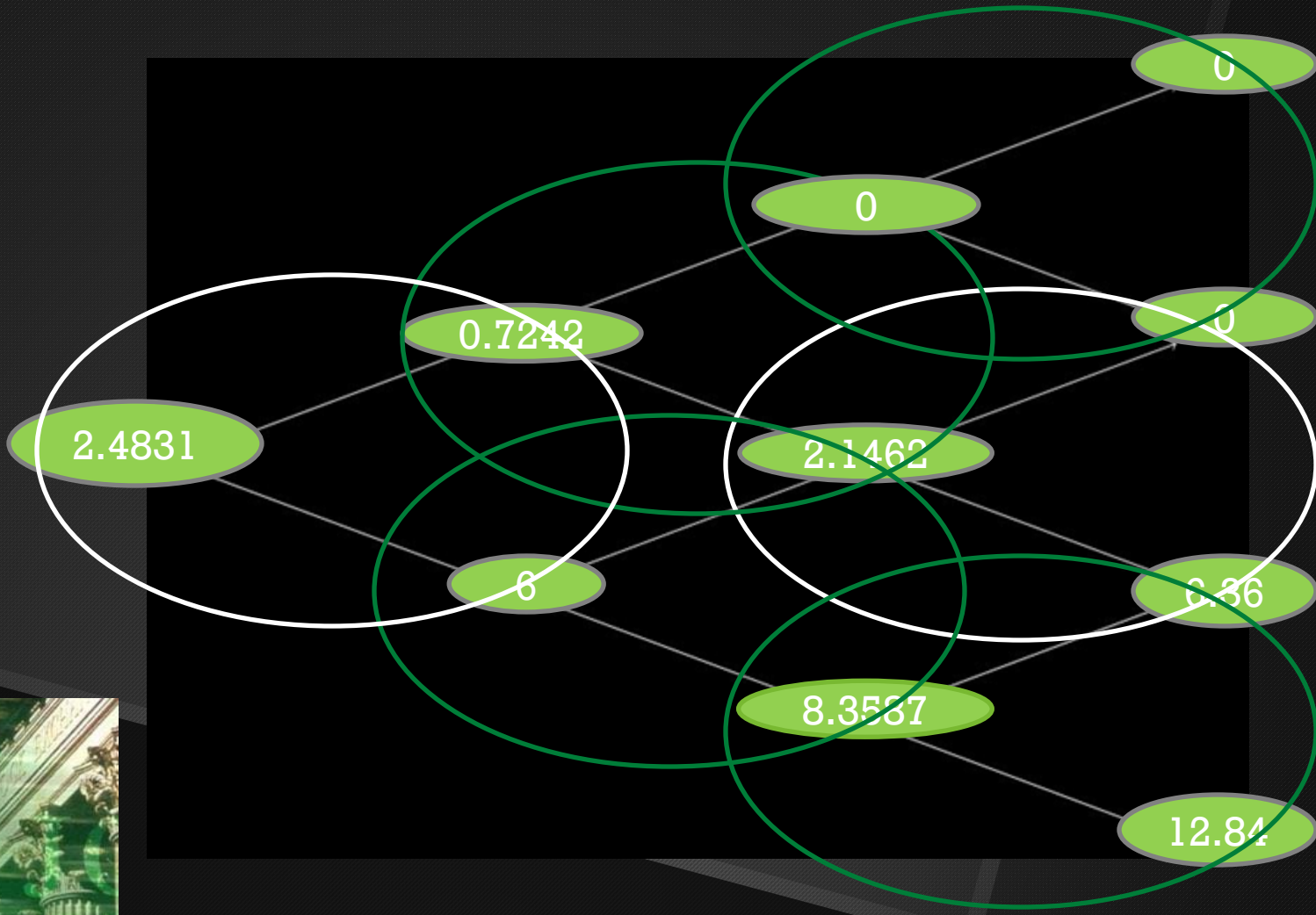
max{42-43.56,0}

max{42-35.64,0}

max{42-29.16,0}

max{0,0.7242}

max{6,4.1792}



$$P_B = e^{-r\Delta T}[p^*(0.7242) + 6(1 - p^*) = 2.4831]$$

Comparing the Results:

$$1.8687 \leq 2.4831 \leq 2.5374$$

$$European\ Price \leq Bermudan\ Price \leq American\ Price$$

MATLAB Implementation

- Calculate the fair price
  - European Option
  - Bermudan Option
  - American Option
- Binomial Option Pricing Model(BOPM)
  - Numerical Method
  - Discrete Time
  - Proposed first by:

    - JC Cox, SA Ross, M Rubinstein - Journal of financial Economics, 1979 (Option Pricing: A simplified approach)

MATLAB Implementation

binCalculator Function

- `function [euPrice, amPrice, brPrice] = binCalculator(S,K,r,sigma,T,N,exercise_Frequency,put_True)`

  S: Spot price, e.g., 50.

  K: Strike pricem, e.g., 50.

  r: Risk-free interest rate e.g., 0.1 for 10%.

  sigma: Volatility, e.g., 0.3 for 30%.

  T: Years to maturity, e.g., 1 for 1 year.

  N: Number of steps in the binomial tree.

  Exercise Frequency: Number of times that the Bermudan option can be exercised in a year, e.g., 12 for the monthly exercise.

  put_True: 1 for put option, 0 for a call.

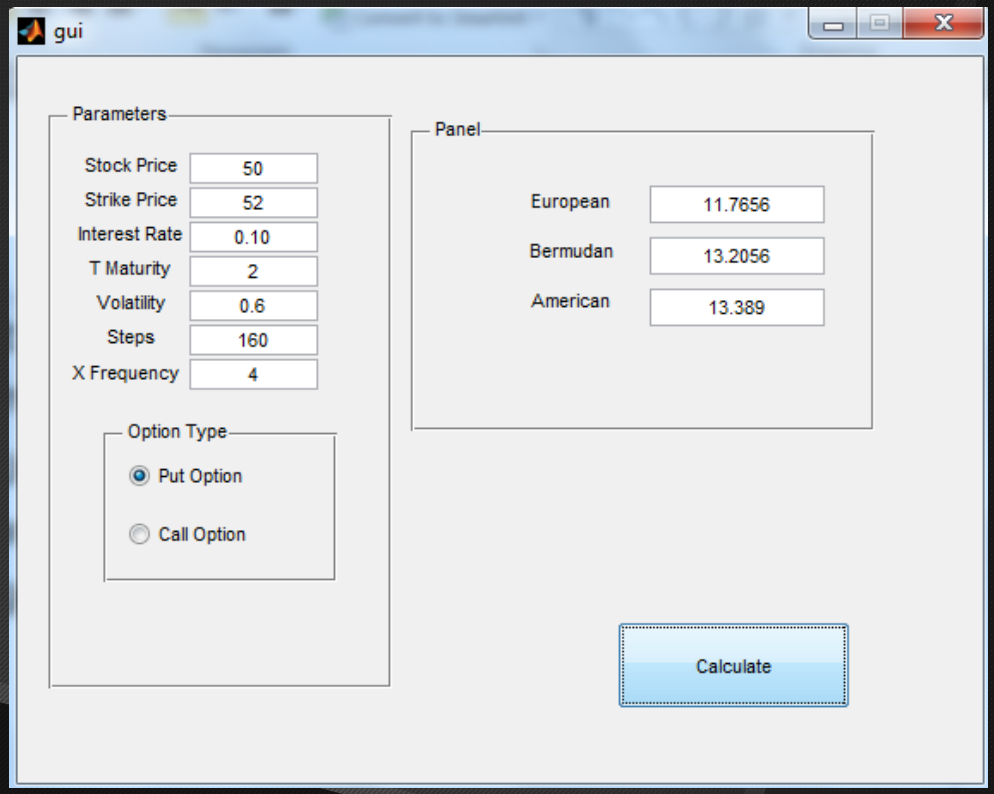
MATLAB Implementation

Graphical User Interface

- It reads the binCalculator input parameters from the textboxes.
- User will choose between Put Option and Call Option.
- Shows the prices calculated by the model.

MATLAB Implementation

Graphical User Interface



MATLAB Implementation

MATLAB Codes

- Create the binomial tree, using the up and down factor.
- Create the exercise_True vector: It indicates if the Bermudan can be exercised at a certain node.
- Depending on the value of the put_True, the pay off tree, binTreeEE, is created.

  - Pay-off at each node if the option is exercised at that node.

MATLAB Implementation

MATLAB Codes

- **binTreeNE**
  - European option's binomial tree.
  - Value of each node is equal to the expected pay-off of the two successive nodes, under the risk neutral probability.
- **binTreeAm**
  - American option's binomial tree.
  - Each node is the greater of the
    - binTreeEE value

    and
    - Expected pay-off of the two successive nodes.

MATLAB Implementation

MATLAB Codes

- binTreeBr
  - Bermudan option's binomial tree
  - If the exercise_True at the node is not zero, then, the pay-off is calculated as the one in American option.
  - If the exercise_True at the node is zero, the pay-off is calculated as the one in the European option.

MATLAB Implementation

MATLAB Codes

- Running the code in MATLAB
- Checking the binomial trees created by the application.
- Running the GUI
- Calculating the options' fair prices for different values of exercise frequency

Questions?

Thank You!