

Problem with Volatility

MMA 707 Analytical Finance I

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Abstract

Our report is intended to analyze the problem with volatility.

In this report, we first introduce the definition of the historical and implied volatility, and the formulas that we used to derive them. Then, basic on the data, we calculate the implied volatility by using the Black-scholes model. In the last, we study about the how good historical volatility matches the ATM options and where on the volatility smile we can find that historical volatility.

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Introduction

In finance, volatility is a measure for variation of price of a financial instrument over time. It is a general measure for traders analyze how risky he has been exposed by holding his financial instruments. Unfortunately, the volatility does not concern the direction of the price changes. That is because the way we calculate the standard deviation. Bare in mind, two instruments with different volatilities may have the same expected return, but for sure, the instrument with higher volatility will have larger swings in values over a given period of time. There are two volatility are commonly used. Historic volatility is derived from time series of past market prices. An implied volatility is derived from the market price of a market traded derivative (in particular an option).

The stock ABB and the share index OMXS30 are observed to calculate historical volatilities, the call & put option ABB and the index call & put option OMXS30 are also observed to calculate the implied volatilities.

With help of the Excel, we can easily calculate these historical volatilities.

Implied volatility

The implied volatility of an asset is an estimate of volatility, or rate of price change, for the asset. It is the volatility of the option implied by current market prices. In mathematical finance, volatility is defined as the annualized standard deviation of daily price changes. Implied volatility can be interpreted as the market expected future volatility. The Black-Scholes model is used to calculate a theoretical call price (ignoring dividends paid during the life of the option) using the five key determinants of an option's price: stock price, strike price, volatility, time to expiration, and short-term (risk free) interest rate. Theoretically, by reverse the Black-Scholes model, the implied volatility can be calculated, which means given the market call price, and other parameters (except volatility) in Black-Scholes model remain the unchanged. Unfortunately, since the model itself is a complex function, we actually need some root-finding method to estimate the root.

$$C_{BC}(\sigma^{implied}, K, T, S, r) = C_{market} \quad (1)$$

where C_{BC} denotes the price of the call generated by Black-Scholes model, and C_{market} denotes the observed option price from the market.

Additionally, higher implied volatility reflects a great expected fluctuation (in either direction) of the underlying stock price. Generally, at-the-money options have a lower implied volatility than out-of-the-money option.

Calculating the implied volatility under Binomial model is just as the same as under Black Scholes model as we showed in equation (1). In order to find the root in the model, we can apply the Newton-Raphson Method as follow:

$$\sigma_{n+1} = \sigma_n - f(\sigma_n) / f'(\sigma_n)$$

So start with σ_0 by guessing a initial value:

$$f'(\sigma_n) = [f(\sigma_n + \Delta\sigma_n) - f(\sigma_n)] / \Delta\sigma_n$$

eventually as soon as the following equation holds, our implied volatility is estimated.

$$f = P(S, K, r, T, \sigma) - p^* = 0$$

where p^* is the observed price of put, S is stock price, K is strike price, r is interest rate, T is time to maturity.

The same method apply to call option:

$$f = C^* - C(s, k, u, d, \sigma, T, r) = 0$$

Historical Volatility

The historical volatility is basically the volatility of a financial instrument over a given time period. Generally, this measure is calculated by determining the average deviation from the average price of a financial instrument in the given time period. Standard deviation is the most frequently used method. The following is the formula for calculating the historical volatility:

n : period of the observation.

P_t : the price in day t

t : from 1 to $T+1$

Return $X_t = \ln(P_t / P_{t-1})$.

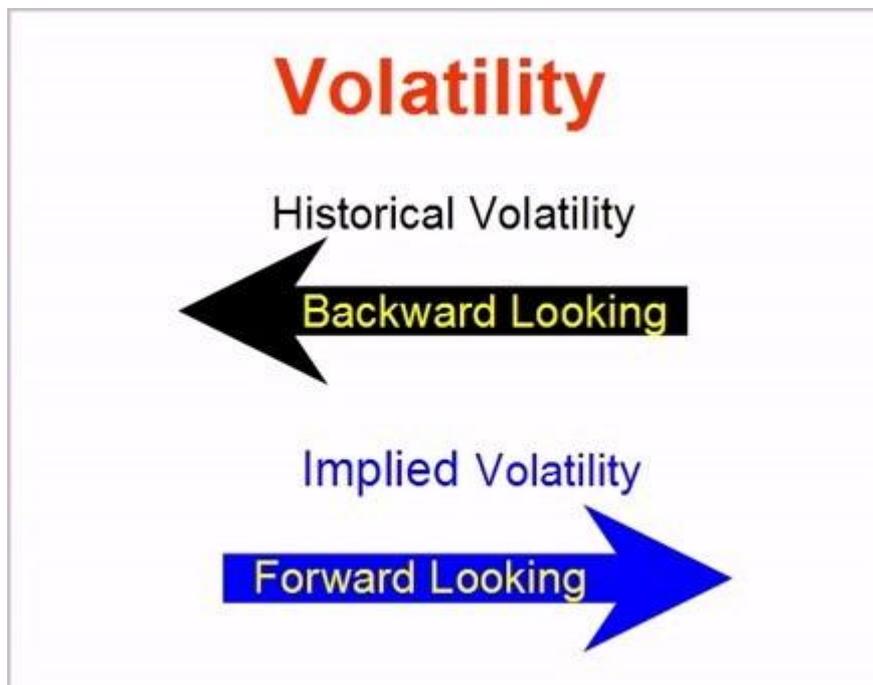
$$\bar{X}(\text{meanreturn}) = \frac{1}{T} \sum_{t=1}^T X_t$$

$$HV_{daily} = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (X_t - \bar{X})^2}$$

$$HV_{yearly} = HV_{daily} * \sqrt{252}$$

Comparison between Implied Volatility and Historical Volatility

Historical volatility reflects the price movement of the underlying asset during a past period, that is to say, it looks backward at price actions and measures the degree of change in the price of a security. While the implied volatility it looks forward.



The way implied volatility is estimated, has been widely accepted that gives better information regard the future movements that the historical one. It can also help to gauge whether options are cheap or expensive. Rising implied volatility causes option prices to rise or become more expensive; falling implied volatility results in lower option premiums. Therefore, with everything else being equal, when implied volatility on an option is high, it is better to sell that option; if the implied volatility is low, the option more suitable for buying.

Application

Getting implied volatility by Matlab

To get the Implied volatility by matlab we need some matlab code to get the implied volatility with call and put:

The main file:

```
BLSIMPV Black-Scholes implied volatility.  
    Compute the implied volatility of an underlying asset from the market  
    value of European call and put options using a Black-Scholes model.  
  
    Volatility = blsimpv(Price, Strike, Rate, Time, Value)  
    Volatility = blsimpv(Price, Strike, Rate, Time, Value, Limit, ...  
        Yield, Tolerance, Class)  
  
    Optional Inputs: Limit, Yield, Tolerance, Class.  
  
    Inputs:
```

and for call option we need input like this:

```
>> Volatility = blsimpv(100, 100, 0.05, 1, 14.2313)  
  
Volatility =  
  
    0.3000
```

and for the put option:

```
>> Volatility = blsimpv(100, 100, 0.05, 1, 9.3542,0.5,0,[],false)  
  
Volatility =  
  
    0.3000
```

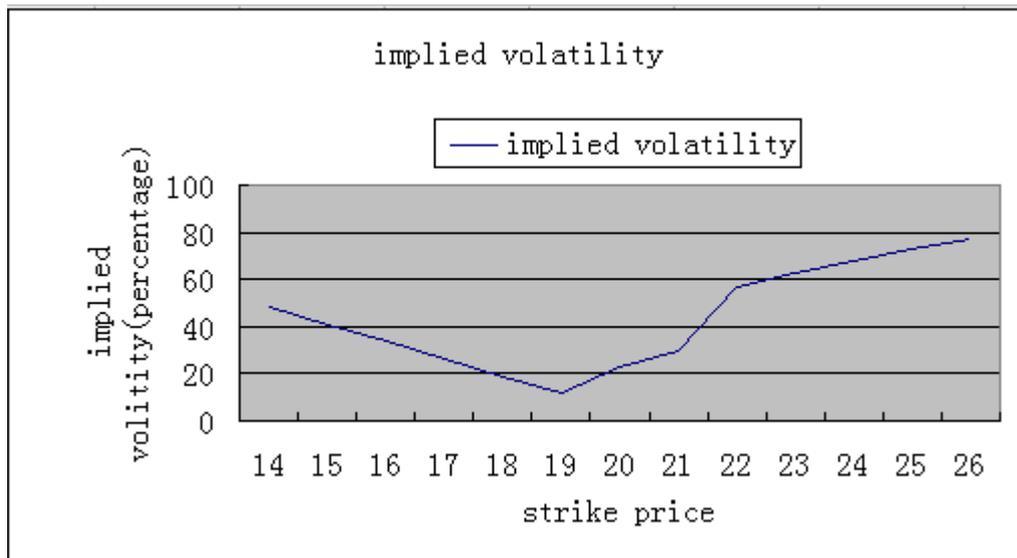
So we get the implied volatility with different strike price.

Then we use plot() to get the line which is the volatility smile indeed.

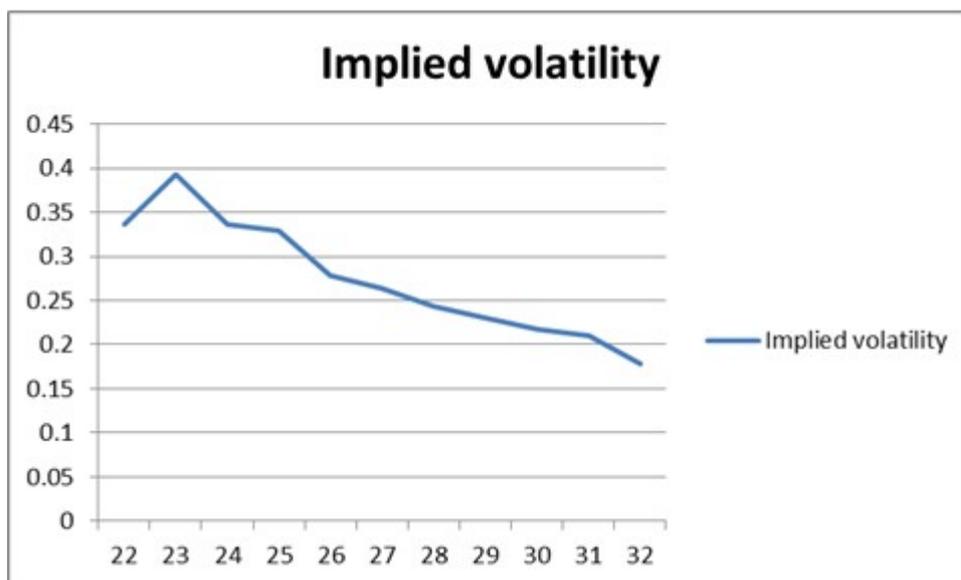
Implied Volatility Graphs

The following two graphs showing the implied volatility on call options from ABB and Microsoft varies with the strike prices on the same maturity date.

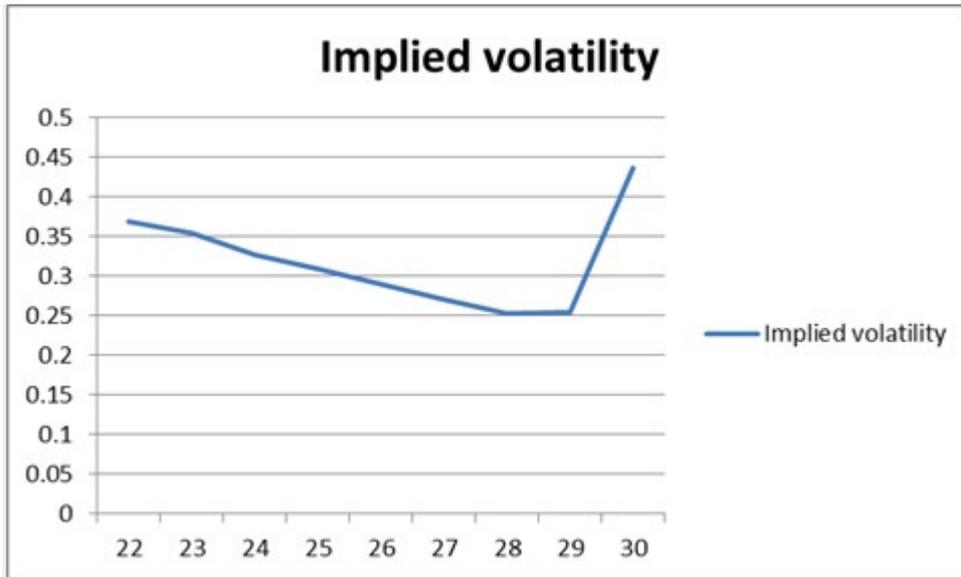
ABB



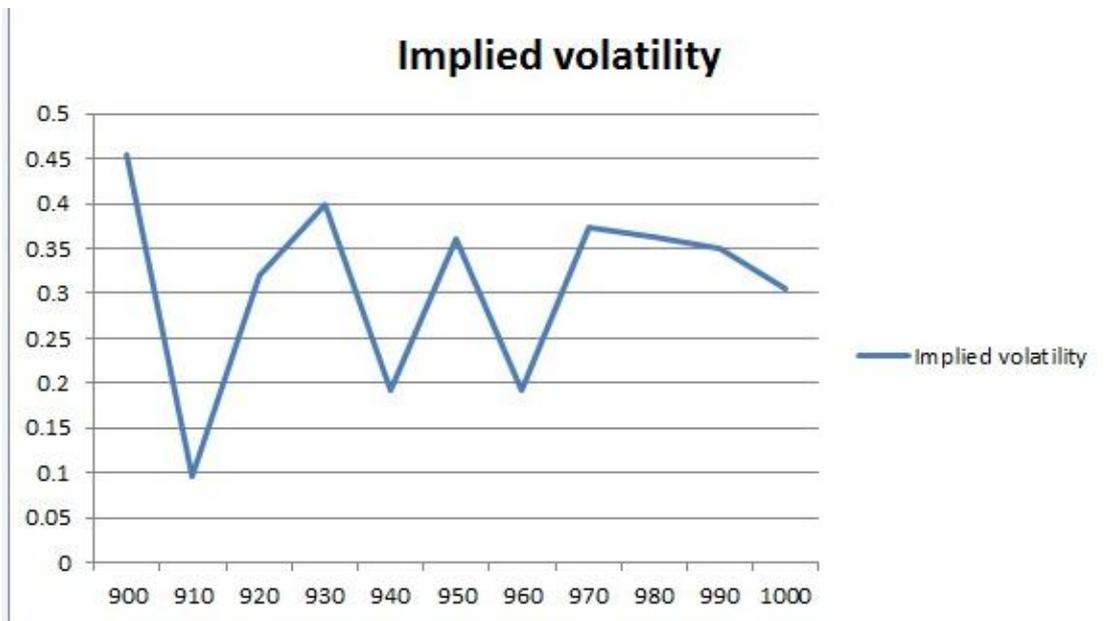
Microsoft



Put option with Microsoft.



Index call and put options OMXS30



Conclusion

According to what we have discussed and analyzed above about the volatility, volatilities play important role in trading decisions, especially implied volatility. However, since we lack of programming skills and relevant knowledge, the results were not as obvious as the theory states.

We highly suggest further research on volatility issues.

Reference

http://en.wikipedia.org/wiki/Implied_volatility

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