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| MÄLARDALENS HÖGSKOLA |
| Barrier Options |
| Valuation, Greeks and Plotting |
|  |
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# Introduction

Barrier options are extensions of standard stock options. Payoffs at maturity of standard puts and calls are only depending on the strike price. But barrier options have payoffs that depend on both the strike and the barrier levels. Investors can use them to enhance returns from future market scenarios more complex than the simple bullish or bearish expectations embodied in standard options. On the other hand, the premiums are lower than those of standard options with the same strike and expiration, this because the expected payoff is lower.[[1]](#footnote-2)

We will define the barrier options, introduce the Reiner- Rubinstein formulas and the option Greeks in the first section. In the second section we focused on valuating barrier options and plotting by using VBA program. The last section is about the results we got from our program.

# **Definition of Barrier Option**

*Barrier options* are a modified form of standard options that include both puts and calls. It is characterized by a barrier level and a strike level. It is also characterized by a cash rebate which associated with crossing the barrier. Just like the standard options, the payoff at expiration is determined by the strike level. The barrier option contract specifies that the payoff depends on whether the stock price (S) ever crosses the barrier level (H) during the life of the option, which depends on the type of barrier discussed below.

If the barrier is crossed, some barrier option contracts specify a rebate to be paid to the option holder. In our case, we assume that the rebate is zero.

Types of Barriers

An ***up*** *barrier* is a barrier above the current stock level. If it is crossed, it will be from below. A ***down*** *barrier* is a barrier below the current stock level. If it is crossed, it will be from above.

***In*** *barrier options* will only pay off if the stock finishes in the money and if the barrier is crossed sometime before the expiration. When the stock crosses the barrier, the in barrier option is *knocked in* and becomes a standard option of the same type with the same expiration time and the same strike. But once the stock never crosses the barrier, the option will expire worthless.

***Out*** *barrier option* will only pay off if the stock finishes in the money and the barrier is never crossed before the expiration time. As long as the stock has not crossed the barrier, the out barrier option will still be a standard option the same type with the same expiration and strike. But once the stock crosses the barrier, the option will be *knocked out* and expires worthless.

Therefore, the barrier options can be divided as up-and-out, up-and-in , down-and-out, down-and-in.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Barrier |  |
| Option | Type |  | Location |
| Call | Down-and-Out |  | Below Spot |
|  | Down-and-In |  | Below Spot |
|  | Up-and-Out |  | Above Spot |
|  | Up-and-In |  | Above Spot |
| Put | Down-and-Out |  | Below Spot |
|  | Down-and-In |  | Below Spot |
|  | Up-and-Out |  | Above Spot |
|  | Up-and-In |  | Above Spot |

Table 1 *Types of Barrier*

How to calculate barrier options with zero debates

Back to 1991, Rubinstein and Reimer summarized the formulas to calculate the barrier option. This is a European-style barrier option based on the Black-Scholes model.

Notations:

Formulas:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Type |  | X < H |  | X > H |
| Down-and-In | S > H | A-B+D |  | C |
| Up-and-In | S < H | B-C+D |  | A |
| Down-and-Out | S > H | B-D |  | A-C |
| Up-and-Out | S < H | A-B+C-D |  | 0 |

Tabel 2 *Prices of Call barriers*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Type |  | X < H |  | X > H |
| Down-and-In | S > H | A |  | B-C+D |
| Up-and-In | S < H | C |  | A-B+D |
| Down-and-Out | S > H | 0 |  | A-B+C-D |
| Up-and-Out | S < H | A-C |  | B-D |

Tabel 3 *Prices of Put barriers*

The Greeks:

The mathematical characteristics of Black-Scholes model are named after the Greek letters in the equations. Those are also known as the options Greeks. The common five Greeks measure the sensitivity of the stock options prices in relation to these factors: change in underlying stock price, volatility, time value and interest rate.[[2]](#footnote-3)

Each option contract has a limited life which is known as the expiration date or time to maturity. The expected value of an option contract will be more and more certain as the option more and more approaches its expiration date. The uncertainty of an option is the *Time Value.*

* Delta is the number of shares of stock that has the same instantaneous exposure as the option has to infinite small changes in the stock price. It tells option traders how fast the price of the option will change as the underlying stock/future moves.

Delta is always positive for calls and negative for puts

* Gamma () is the sensitivity of delta to changes in stock price. We define it as the ratio of the change in delta for an infinitesimal change in stock price to the corresponding percentage change in stock price. It shows us how fast our position delta will change as the market price of the underlying asset changes.

Gamma is always positive, for both puts and calls.

* Vega () indicates how much, theoretically at least, the price of the option will change as the volatility of the underlying asset changes. It shows the theoretical price change for every one percentage point change in volatility.

Vega is always positive, for both puts and calls.

* Theta is the calculation that shows how much of this time value is eroding as each trading day passes

Theta is always negative, for both puts and calls. If time passes the value decreases.

* Rho () is the change in option value that results from movements in interest rates. The value is represented as the change in theoretical price of the option for a one percentage point movement in the underlying interest rate.

Options Greek Rho is always positive for call options and negative for put options.

# VBA Program

For the valuation of the Barrier options, we wrote a program on excel using Microsoft Visual Basics. We declare all input values as in the table below:

|  |  |
| --- | --- |
| Strike | X |
| Barrier Level | H |
| Spot price | s |
| Volatility | sd |
| Time to Maturity | T |
| Cost of carry | B |
| Risk free rate | r |

Tabel 4 *Input values*

To make our calculation more easy and readable, we declare different type of input functions in the table below. These functions will help us in the calculation of the prices of call or put barrier sub functions:

|  |  |  |
| --- | --- | --- |
| Function name | Formula | Data type |
| XX | Log(s / x) / (sd \* Sqr(T)) + (1 + mu(b, sd)) \* sd \* Sqr(T) | Double |
| XY | Log(s / h) / (sd \* Sqr(T)) + (1 + mu(b, sd)) \* sd \* Sqr(T) | Double |
| YX | Log(h \* h / (s \* x)) / (sd \* Sqr(T)) + (1 + mu(b, sd)) \* sd \* Sqr(T) | Double |
| YY | Log(h / s) / (sd \* Sqr(T)) + (1 + mu(b, sd)) \* sd \* Sqr(T) | Double |
| mu | (b - (sd ^ 2 / 2)) / (sd ^ 2) | Double |

Tabel 5 *Barrier sub functions*

The prices of the call or the put barriers sub function are declared as in the table below:

|  |  |  |
| --- | --- | --- |
| Function name | Formula | Data type |
| A | fi \* s \* Exp(b - r) \* Application.WorksheetFunction.NormSDist(fi \* XX(x, s, T, sd, b)) - fi \* x \* Exp(-r \* T) \* Application.WorksheetFunction.NormSDist(fi \* XX(x, s, T, sd, b) - fi \* sd \* Sqr(T)) | Double |
| BB | fi \* s \* Exp(b - r) \* Application.NormSDist(fi \* XY(h, s, T, sd, b)) - fi \* x \* Exp(-r \* T) \* Application.NormSDist(fi \* XY(h, s, T, sd, b) - fi \* sd \* Sqr(T)) | Double |
| CC | fi \* s \* Exp(b - r) \* ((h / s) ^ (2 \* (mu(b, sd) + 1))) \* Application.NormSDist(eta \* YX(x, s, T, sd, b, h)) - fi \* x \* Exp(-r \* T) \* ((h / s) ^ (2 \* mu(b, sd))) \* Application.NormSDist(eta \* YX(x, s, T, sd, b, h) - eta \* sd \* Sqr(T)) | Double |
| D | fi \* s \* Exp(b - r) \* ((h / s) ^ (2 \* (mu(b, sd) + 1))) \* Application.WorksheetFunction.NormSDist(eta \* YY(h, s, T, sd, b)) - (fi \* x \* Exp(-r \* T) \* ((h / s) ^ (2 \* mu(b, sd))) \* Application.WorksheetFunction.NormSDist(eta \* YY(h, s, T, sd, b) - eta \* sd \* Sqr(T))) | Double |

Tabel 6 *Price of the barrier sub function*

* We declare all function data type as double because it provides the greatest and the smallest possible magnitudes for a number and the default value is 0.
* **Application.WorksheetFunction.NormSDist** or **Application.NormSDist** is a function used in excel to call the cumulative normal distribution in excel.

Calculations of the price of call or put barriers:

We decided to join the call and put for different barrier type using the nested *if* statement. If we calculate a down option the eta has to be 1, if we calculate an up option the eta should be -1. We also used the *if* statement to make a distinction between the strike being greater or smaller than the barrier level. The same we did for the spot price.

An example:

For **call\_put\_down\_in,** we know that we have the **down** part so we set eta equal to 1. If **fi** is equal 1 we have a **call price**, with the spot price greater than the barrier level. Then we use the conditions above: If our strike price is less than the barrier level we call the function **call\_put\_down\_in =** A-BB +D otherwise we have **call\_put\_down\_in =** CC. The second case is where **fi** is equal -1 we have a **put price** , still with the spot price greater than the barrier level, then if our strike price is less than the barrier level we call the function **call\_put\_down\_in = A** otherwise we have **call\_put\_down\_in =** BB - CC+D.

# Results

To discuss the results of our program we will first verify the results. The results of a valuation and the Greeks of barrier options can be verified by the following fact:

*Down-and-out call (put) + down-and-in call (put) = plain vanilla call (put*)

The same for up-and-in and up-and-out.

We used to following numbers as input for our variables:

|  |  |
| --- | --- |
| Strike (X) | 100.00 |
| Barrier Level(H) | 95.00 |
| Volatility(sd) | 20% |
| Time to Maturity(T) | 1 |
| Cost of Carry rate(b) | 0.05 |
| Risk free rate(r) | 5% |

Tabel 7: *Input for variables*

For the options where the spot has to be above the barrier we used 100 as spot price. For the options where the spot as to be below the barrier we used 90 as spot price.

Valuation

The results for the valuation are as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| CALL (s=100) | PRICE |  | CALL (S=90) | PRICE |
| Down-and-In | 4.814 |  | **Up-and-In** | 5.091 |
| Down-and-Out | 5.636 |  | **Up-and-Out** | 0.000 |
| PUT (s=100) | **PRICE** |  | **PUT (S=90)** | **PRICE** |
| Down-and-In | 5.563 |  | **Up-and-In** | 5.924 |
| Down-and-Out | 0.010 |  | **Up-and-Out** | 4.290 |

Tabel 8 *Results for the valuation*

We can check these results by calculation of the price of a plain vanilla put and call using the Black and Sholes formula. This gives the following results:

|  |  |  |  |
| --- | --- | --- | --- |
| Option | | Price | Check |
| Call(S=100) | 10.45058 | | 4.814+5.636=10.451 |
| Put (S=100) | 5.573526 | | 5.563+0.010=5.573 |
| Call(S=90) | 5.091222 | | 5.091+0.000=5.091 |
| Put (S=90) | 10.21416 | | 5.924+4.290=10.214 |

Tabel 9 *Calculation of price of plain vanilla put and call*

As can be seen from the table the results are correct.

Greeks

**The change we used to calculate the Greeks is 0.001. This resulted in the following table:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| GREEKS | DELTA | GAMMA | THETA | RHO | VEGA |
| CALL |  |  |  |  |  |
| Down-and-In (S=100) | -0.436 | 0.036 | -4.867 | 19.215 | 39.077 |
| Down-and-Out(S=100) | 1.073 | -0.01767816 | -1.545537427 | 34.084 | -1.548 |
| Up-and-In(S=90) | 0.430 | 0.021819711 | -5.213991255 | 33.665 | 35.354 |
| Up-and-Out(S=90) | 0.000 | 0 | 0 | 0.000 | 0.000 |
| PUT |  |  |  |  |  |
| Down-and-In (S=100) | -0.365 | 0.018879161 | -1.672 | -41.752 | 37.668 |
| Down-and-Out(S=100) | 0.002 | -0.00011765 | 0.015239195 | -0.024 | -0.139 |
| Up-and-In(S=90) | 0.336 | 0.003821766 | -1.833547502 | -28.797 | 32.772 |
| Up-and-Out(S=90) | -0.90605464 | 0.017997962 | 1.375584468 | -32.613 | 2.582 |

Tabel 10 *Results in change of Greeks*

**These results can be checked with the following table, which gives the Greeks for the plain vanilla options:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Option** | **Delta** | **Gamma** | **Theta** | **Rho** | **Vega** |
| Call(S=100) | 0.636831 | 0.018762 | -6.41403 | 53.23248 | 37.52403 |
| Put (S=100) | -0.36317 | 0.018762 | -1.65788 | -41.8905 | 37.52403 |
| Call(S=90) | 0.4298317 | 0.0218197 | -5.2144808 | 33.59363 | 35.34799 |
| Put (S=90) | -0.5701683 | 0.0218197 | -0.4583337 | -61.5293 | 35.34799 |

Tabel 11 *Checking results in Greeks for plain vanilla options*

**One remark has to be made for the calculation of the rho, because the cost of carry rate b is defined as r-q we also have to increase b with the same amount as r to get the correct values for Rho.**

Graphs

**We plotted three different kinds of graphs:**

* **Changing the strike**
* **Changing the Spot price**
* **Changing the volatility**

***Changing the strike***

**If the strike is increased the price of the put options will increase. This because the payoff functions of put options is, which will increase if the strike goes up. For the call option it is the other way around the payoff is, therefore the graphs are decreasing. This can be seen in graph 1 and graph 2.**

***Changing the spot price***

**Increasing the spot price will make the call options more valuable, because of their payoff function. This is illustrated in graph 3. This graph shows that four of the function gets a value above the barrier of 95, and three become worthless. The up-and-out call is worthless the whole time because the strike is above the barrier.**

***Changing the volatility***

**Increasing the volatility will normally increase the value of the options. For barrier options this depends on the type of option. If the option is a down-and-in or up-and-in option, the value will increase because there is a higher chance to hit the barrier and the options will be valuable. (See graphs 4 and 5) For the down-and-out and up-and-out options, the value is more constant. To explain this we added the value of a plain vanilla put option in graph 5. From this graph we can see that the slope of the plain vanilla put option is the same as the put up-and-in option. Therefore the slope of the up-and-out options should be zero.**

Graph 1 *Changing strike when the spot rate is above barrier*

Graph 2 *Changing strike when spot rate is below barrier*

Graph 3 *Changing the spot price*

Graph 4 *Changing volatility with spot above barrier*

Graph 5 *Changing volatility with spot below barrier*

# References

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1. Reiner ,E. and Rubinstein, M. (1991) [↑](#footnote-ref-2)
2. Wystup, U. (2002) [↑](#footnote-ref-3)