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**MMA 707 Analytical Finance I**

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# Exotic Options:

# Quanto Basket Min Lookback Asian

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# Abstract

The purpose of this report was to construct a software application that uses Monte Carlo method with an intention to price a specific exotic option. The option that had been evaluated is a Quanto Basket Min Lookback Asian. A MATLAB application has also been created in order to help efficiently price the option.

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# 1.Introduction

A basic option such as an European option, for instance, has some pre-defined characteristics. For example:

- an option can be either a call or a put option
- option strike price is specified
- expiry date is the time to maturity
- it is either an European, American or Bermudan option
- settlement is defined as a physical or cash delivery
- and, normally, stock, currency, index or another asset represent the underlying for the option.

The difference between a standard option with the parameters described above and an exotic one is that exotic options usually have the same parameters defined in a different manner. They might also have additional properties. There exist many types of exotic options, one of a particular interest to us is Quanto Basket Min Lookback Asian.

# 2.Description of a Quanto Basket Min Lookback Asian

Asian options are options that are based on an average value over a certain time period. They are mostly written on currencies and commodity products which are traded in low quantities. The main advantage of purchasing an asian option is its stability, due to the average price used as an underlying, and lower price (in comparison with a plain vanilla option) resulting from the reduced volatility (average price options are less volatile than the asset they are traded on). The name "Asian" comes from the first time they had been used by the Banker's Trust Tokyo office in 1987.

Lookback options can be described as securities that are giving their holder a right to buy the underlying asset for its lowest value recorded during the options lifetime. They also allow their holder to sell the underlying security for the highest value observed in the market up to the options maturity.

Quanto is another type of an exotic option and it is written on a foreign currency asset, however the payoff is defined in local currency at some fixed exchange rate. Generally, trader taking a short position in a quanto option is being exposed to the currency risk, which has to be reflected in the options' price.

Basket options are a type of Multi-asset options with the underlying security represented by not one, but several distinct assets with specified weights. This portfolio of assets is usually constructed within the same market sector.

Quanto Basket Min Lookback Asian option has some properties of all the exotic options briefly described above, however it also has its own uniquely defined properties. The strike price is determined by the minimum value of the underlying asset over an initial time period. Payoff is calculated for a normalized underlying security. Quanto options are categorised as the ones that free the buyer from currency risk. The first period lowest value is the strike price and the last is an average price.

The price and the payoff are given by the following formulas:

$$\Pi(0) = e^{-rT}E^{Q}[\,\Pi(T)] = \; e^{-rT}E^{Q}\left[max\left\{\frac{A(T)-K}{B(t_0)},0\right\}\right]$$

With:

$$A(T) = \frac{1}{N-M}\sum_{j=M+1}^{N} B(t_j)$$

$K = min_{j=1,\dots,M}\, B(t_j)$ j = 1,...,N

$B\left(t_j\right) = \sum_{i=1}^{d} \phi_i S_i\,(t_j)$

Where A(T) is the Asian price, K – minimum lookback price, $B\left(t_j\right)$ - price of the basket at time $t_j$, $\phi_i$ - corresponding weight of the underlying asset i in the basket, $S_i(t_j)$ – represents the price of the underlying asset i, $t_j$ reflects the reset times, N is the number of the reset dates, M is the amount of the lookback dates and, finally, d is the number of the underlying assets.

# 3. Financial Background

Now we will enter the Black-Scholes world. We already know that the Black-Scholes model is constructed through a risk-free and a risky asset. Prices of these two assets are being governed by the following equations:

$$dB(t) = rB(t)dt,$$

$$dS(t) = \mu S(t)dt + \; \sigma S(t)dW(t)$$

The first equation describes risk-free asset price (B(t)) dynamic. The second equation represents the risky asset price (S(t)) movement and is a stochastic differential equation. Risky asset price is characterised through a constant drift µ and a stochastic term $\sigma SdW$. This term is a geometrical Brownian motion or a Wiener process, with $\sigma$ – as volatility.

We also know that according to the Girsanov theorem there exists such a probability measure Q that under this measure:

$$dS(t) = rS(t)dt + \; \sigma S(t)dW^{Q}(t)$$

With $W^Q(t) = W^P(t) + \left(\frac{\mu - r}{\sigma}\right)t$, which is defined as a standard Brownian motion under the probability measure Q. It can be proven that in this case the measure Q is a risk neutral probability measure. However, for the evaluation of our Quanto Basket Min Loockback Asian option it will not be necessary, as long as the risk neutrality is assumed in the definition.

When solving the differential equation mentioned above, we end up with the following result:

$$S(T) = S(t)\exp\left((r - \sigma^2/2)(T - t) + \sigma(W^Q(T) - W^Q(t))\right)$$

# 4. Monte-Carlo Simulations

Here we will give a short description of how Monte-Carlo simulations work and how they can be used to price complex instruments. The easiest approach would be to start with a plain European call option in the Black-Scholes world. Risk-free interest rate is continuously compounded and price of the underlying is governed by the following stochastic process:

$$dS(t) = rS(t)dt + \sigma S(t)dz(t)$$

If we consider natural logarithm of the stock price: x(t)=ln(S(t)). We will find that it can be described by dynamics below:

$$dx(t) = vdt + \sigma dz(t)$$

$$v = r - \frac{1}{2}\sigma^2$$

In other words: $x(t + \Delta t) = x(t) + v\Delta t + \sigma\big(z(t + \Delta t) - z(t)\big)$

Z increment in the equation above is distributed with zero mean and Δt variance. Considering this, we are able to simulate the random process with $\sqrt{\Delta t} \cdot \varepsilon$ and a normally distributed sigma. We obtain:

$$S(t_i) = \exp\big(x(t_i)\big)$$

$$x(t_i) = x(t_{i-1}) + v\Delta t + \sigma\sqrt{\Delta t} \cdot \varepsilon$$

With an appropriate software application, such as Matlab or Excel, investors are capable of simulating multiple stock price paths. The more simulations are carried out – the better the accuracy of the estimated option price. This is the main downside of Monte-Carlo simulations: they have proved to be time-consuming and requiring fast and powerful computers.

After we have simulated various paths of the stock price, we are in a position to calculate our options profits. As long as in our example we are using a call option, its payoff will be: max(S(T)-X,0). The payoffs are estimated for all cases. Then, we can find option price by summing all payoffs, discounting them to the present date and taking a mean value of that. For a call option, the formula will be:

$$C(0) = \exp\,(-rT)\frac{1}{N}\sum_{i=1}^{N}\max(S(T,i) - X, 0)$$

The standard deviation of the simulations can be written as :

$$SD = \frac{1}{N-1}\sqrt{\sum_{i=1}^{N}\big(C(T,i)\big)^2 - \frac{\left(\sum_{i=1}^{N} C(T,i)\right)^2}{N}} \cdot \exp(-2rT)$$

Then, it is very easy to calculate the standard error:

$$SE = \frac{SD}{\sqrt{N}}$$

If we have more than one option, a portfolio of derivatives, then the general algorithm for the option price calculation is as follows: we sum up all the expected payoffs from the generated simulations and discount them to the present value with an appropriate interest rate. With n number of simulations discounted cash flow for the ith path will be:

$$S(i) = \sum_{k=1}^{n}\big(-\mathrm{r}(k)t(k)\big)CF(k)$$

If we sum all the discounted cash flows and take an average of this sum:

$$A = \frac{1}{n}\sum_{k=1}^{n} S(k)$$

At this point we can use central limit theorem, stating that A will converge to the true expected value if n is approaching infinity. The same theorem gives us the size of the standard error of the average value A: $\frac{\sigma}{\sqrt{n}}$

Looking at the value of our standard error, we are able to draw conclusions on how certain the estimated instruments value is. There are several methods that can help improving the accuracy, one of them: is a larger amount of simulations. However there are others as well: variance reduction techniques, for example. They focus not only on increasing n, but on narrowing the size of σ to shorten the confidence interval.

Overall, Monte-Carlo simulations are a highly useful tool in pricing complicated instruments, especially when it comes to derivatives for which there exists no closed-form solution or if volatility or interest rate, for example, represent stochastic variables. It is important to always keep in mind that the value achieved through Monte-Carlo simulation is just an approximation of the true value.

# Conclusion

In this assignment we have briefly described some types of the exotic options, such as lookback, basket, etc. We have also listed theoretical models and concepts that underline pricing algorithm for the Quanto Basket Min Lookback Asian option. Our main task was to evaluate this option using Monte-Carlo simulation techniques, which had been successfully accomplished through a Matlab application.

# References

1. “Lecture notes in Analytical Finance I”, 2011 Jan Röman
2. “Pricing Asian Options using Monte Carlo Methods”, 2009 Hongbin Zhang