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The problem with Volatility

MMA 707 Analytical Finance I
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Abstract

This report is intended to analyze the problem with volatility. Specifically, it deals with the floating volatility, historical volatility and the implied volatility for stocks, options and index. Our report is constructed of two main parts: the first part focuses on the implied volatility, which introduces theories and formulas of the implied volatility and dig out the relationship between the historical volatility and the implied volatility; the second part concentrates on the floating volatility and compared it with the historical volatility.

Content

Introduction

Volatility is a crucial variable for calculating the financial derivatives, besides, it also plays a vital role for investors to make decisions. What is more, there is a tendency that investors judge the financial derivative by the volatility rather than the payoff. Owing to these reasons, we write this report to solve some problems relative to the volatility.

The report introduces the definitions and formulas about the floating volatility, historical volatility and the implied volatility. The stock ElectroluxB and the share index OMXS30 are observed to calculate the floating volatility and historical volatilities, the call & put option ElectroluxB and the index call & put option OMXS30 are also observed to calculate the implied volatilities. By employing the Matlab, it is much more efficiently to calculate these volatilities, making comparison and drawing pictures to make the results more intuitive.

Implied volatility

Definition

If the market price (premium) of the option is known, the volatility, specifically, the implied volatility can be calculated according to the Black-Scholes model. In details, it can be obtained by equating the Black-Scholes formula with a given strike price and the maturity to the observed option market price with the same strike price and the maturity. The value of the volatility in the Black-Scholes formula that yields the observed option price is the implied volatility (σ^{implied}).Let $C_{\text{market}}(K, T)$ denotes the observed market call price with strike price K and time to maturity T, and $C_{BS}(\sigma, K, T)$ denotes the Black-Scholes price of the call with the same strike price and maturity, then $\sigma^{\text{implied}}$is the value of the volatility in the Black-Scholes formula

$$C_{BS}\left(\sigma^{\text{implied}}, K, T\right) = C_{\text{market}}(K, T)$$

Since the implied volatility cannot be worked out by the Black-Scholes formula analytically, it must be done numerically, by using a root-finding algorithm. This requires to be expressed as the root of the objective function,

$$f(\sigma) = C_{BS}\left(\sigma^{\text{implied}}, K, T\right) - C_{\text{market}}(K, T)$$

Additionally, higher implied volatility reflects a great expected fluctuation (in either direction) of the underlying stock price. Generally, at-the-money options have a lower implied volatility than out-of-the-money option.

To sum up, the **implied volatility** is the value of the volatility that, when employed in the Black-Scholes formula, results in a model price equal to the market price.

Formula

From the Black-Scholes model:

$\Pi = SN(d_1) - Ke^{-rT}N(d_2)$

Where; $d_1 = \frac{\ln\{S/K\} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$;

$d_2 = d_1 - \sigma\sqrt{T}$

For call option, the implied volatility $= S_0N(d_1) - Ke^{-rT}N(d_2) - C$

For put option, the implied volatility $= Ke^{-rT}N(-d_2) - S_0N(d_1) - P$

Implied volatility *versus* Historical Volatility

Historical Volatility

Historical volatility reflects the price movement of the underlying asset during a past period, that is to say, it looks backward at price action and measures the degree of change in the price of a security. It also refers to the actual or realized volatility. Historical volatility is obtained by calculating the standard deviation of the historical daily stock price data.

N: period of the observation.

P_t: closing price of day t.

t: from 1 to N+1

Return $x_t = \ln\left(P_t/P_{t-1}\right)$

X (mean of return) $= \frac{1}{n}\sum_1^n x_t$

$$HV_{daily} = \sqrt{\frac{1}{(n-1)} * \sum_1^n (x_t - X)^2}$$

$$HV_{year} = HV_{current} * \sqrt{250}$$

Comparison between HV and IV

Implied volatility looks to the future; it is derived from the options price which suggests some information and clues for future investment. On the other hand, as the definition of the historical volatility illustrates, it indicates the historical movement. The difference of the emphasis of the two volatilities is the reason that why many investors emphasis slightly more on the implied volatility.

The relationship between the implied volatility and historical volatility is often used to determine whether the option is expensive or not. For instance, the option is relative expensive if the implied volatility is comparative higher than the historical volatility and vice versa.

Application

Numerical solutions to the implied volatility by Matlab

In order to compute the implied volatility by Black-Scholes model, we need to write a few files in Matlab.

1) The cumulative normal distribution. We use the polynomial approximation method which gives us the six decimal place accuracy. Then we apply the Horner’s scheme to calculate the cumulative normal distribution. In the Matlab code, it is called function cndf(d1).
2) The Black-Scholes formula for the put option and the call option. We define every variable there, e.x initial price, and strike price.
3) Newton method for solving the Black-Scholes equations. In the code, this file is called bynewton.
4) In order to implement the Newton method, we need to find the first derivate with respect to σ(volatility), that is the greek Vega. And the file is called fprimevega.

So, if we input every variable which are needed, and implement the bynewton file, you will get the solution for the volatility which from the Black-Scholes formula, or the so called implied volatility.

Call option ElectroluxB

We choose the call option ElectroluxB that the maturity date is 21st, Jan, 2011.The followings are the variables for pricing the volatility.

The expiration time is 3 months, the rate is 0.75%(repo rate from 1st,Oct,2010), and the price for the underlying asset is SEK175.5 on the day 11th, Oct, 2010 .The call option prices and the strike price vary with different kinds of call options, and the latter one are shown in the figure.

The following graph illustrates how the implied volatility changes with the different strike prices for call option ElectroluxB[1]. Generally speaking, the implied volatility decreases first and then increases. To be more specific, the implied volatility maintains its trend and hover around an average of 0.305(30.5%) in the given 3 months time periods. The minimum implied volatility is 0.295(29.5%) with the given strike price 180, while the maximum implied volatility is almost 0.32(32%) with the given strike price 135.

[1] Data are collected from the Nasdaq OMX Nordic market
http://www.nasdaqomxnordic.com/optionsandfutures/microsite?Instrument=SSE81

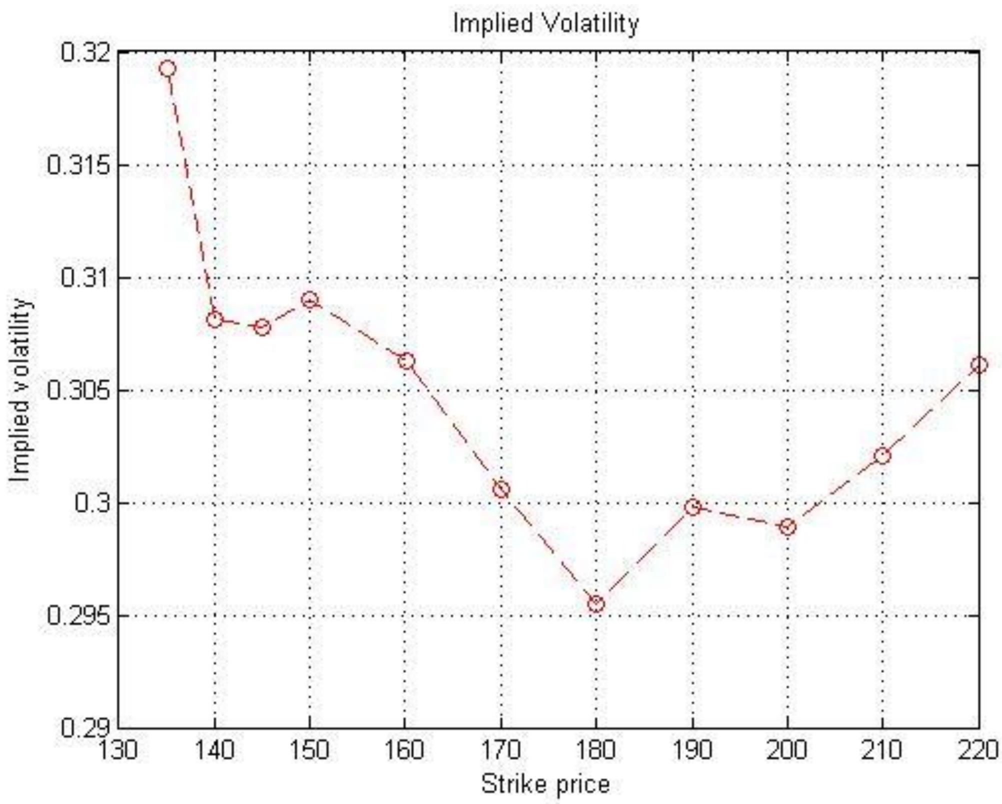

Figure1 *Implied volatility for call option ElectroluxB*

As we can see, the figure shows a volatility smile due to the reason that at-the-money options tend to have lower implied volatilities than in- or out-of-the-money options. In other words, the volatility is lower when the spot price of the underlying asset equals the strike price. In our case, the implied volatility reaches lowest when the strike price is 180 and the actual underlying stock price is 175.5.

Put option ElectroluxB

The following graph is for put option ElectroluxB[2], which has the same variables as the call option ElectroluxB. The implied volatility maintains its trend and hover around an average of 0.34(30.4%) in the given option. The minimum implied volatility is 0.3(30%) with the given strike price from 190 to 200, while the maximum implied volatility is

[2] Data are collected from the Nasdaq OMX Nordic market
http://www.nasdaqomxnordic.com/optionsandfutures/microsite?Instrument=SSE81

almost 0.415(41.5%) with the given strike price 135. It shows a volatility smile again due to the same reason as we mention before.

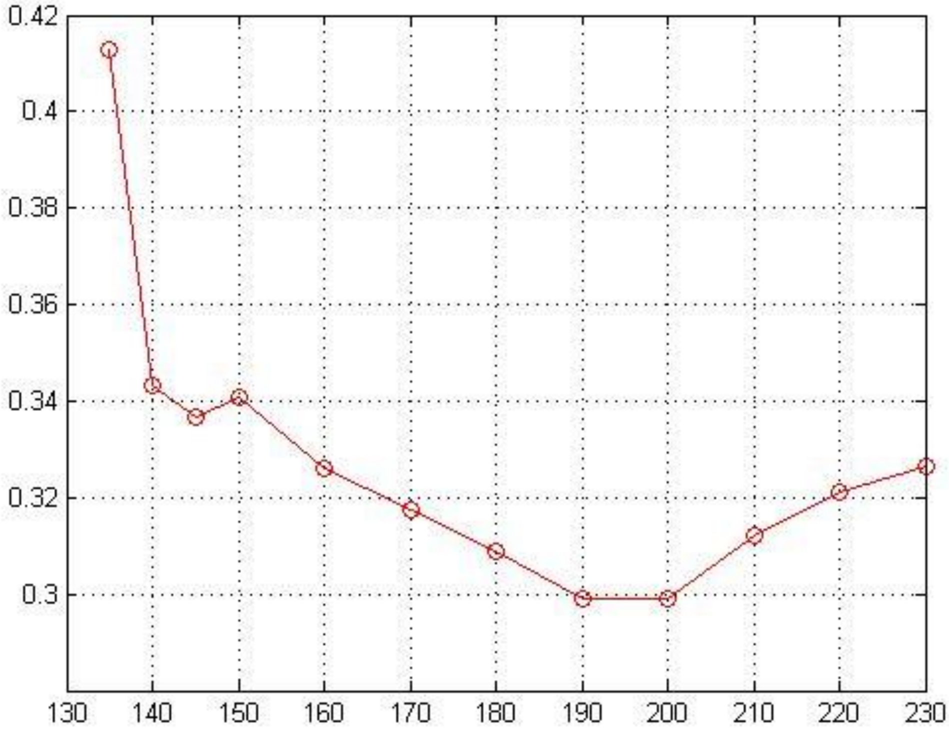

Figure2 *Implied volatility for put option ElectroluxB*

Index call and put options OMXS30

Besides, we also observe the index call &put options OMXS30[3] with the following variables: expiration time (3 months), the rate (0.75%), and the value for the underlying asset is SEK1092.34.

However, the figure below indicates how the index call & put options varies with distinct strike price. It is a more typical case for illustrating the theory that the implied volatility decreases while the strike price increases. The implied volatility decreases

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[3] Data are collected from the Nasdaq OMX Nordic market
http://www.nasdaqomxnordic.com/optionsandfutures/microsite?Instrument=SE0000337842
http://www.nasdaqomxnordic.com/optionsandfutures/microsite?Instrument=SE0000337842

from 0.31(31%) when then strike price is 900 to 0.2(20%) with the given strike price 1200.

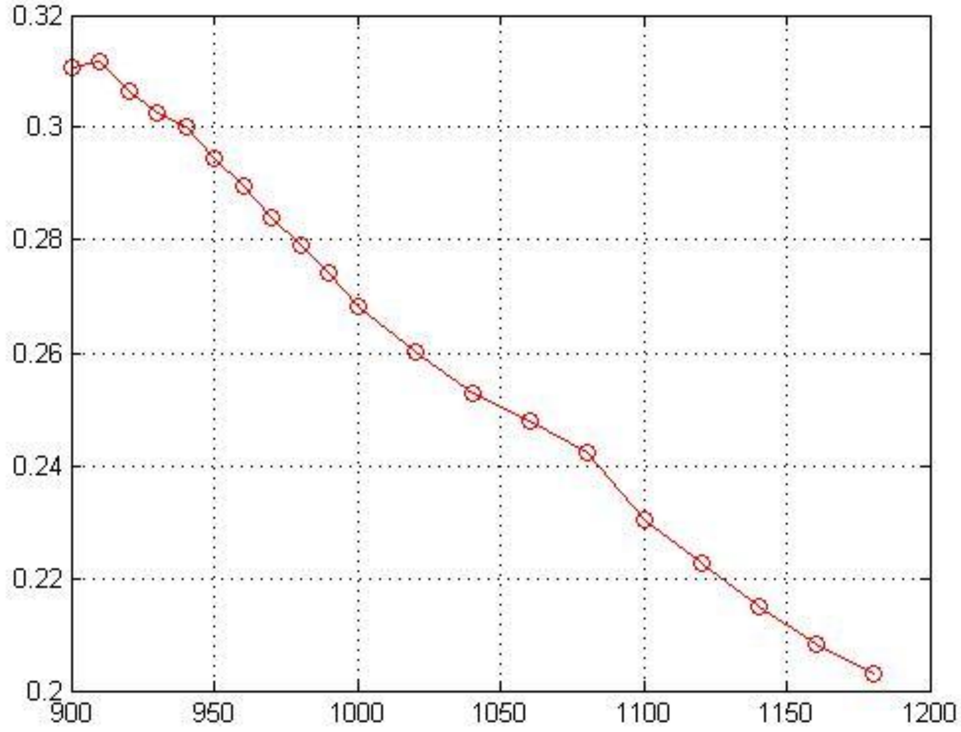

Figure3 *Implied volatilities for index call & put options OMXS30*

From figure1, 2 and 3, we can observe that the index options have the tendency that the implied volatility jumps down continuously rather than jump up. On the contrary, the stock options have comparative symmetric ups and down. All in all, we may conclude that the volatility smile for stock options looks like figure1 and 2, generally, more symmetric and smile-like, while the index option looks like the volatility skew.

Floating volatility

The floating volatility is analogous to the historical volatility. The only difference lies in their calculations. In order to calculate the underlying asset's floating volatility, we need to apply a moving window procedure. While in the calculation of historical volatility, we only add the time together as a whole. For example, we are supposed to calculate a 3-month floating volatility for a stock. Then we have to compute the volatility for the first 3 month, and moving to the next 3month window starting from actually the 2nd day till the 64th day (assuming that there are 21 trading days in a month). However, when it comes to historical volatility, we only add one more day each time.

Comparing the Historical Volatility with the 3-month Floating Volatility

We are interested in calculating the 3-month floating volatility of the OMXS30 index and the stock Electrolux B. In the following graphs, we have compared their historical volatility with their floating volatility for a given period. We used the data from 2009.10.9 to 2010.10.09 and calculate the daily volatility for 189 trading days.

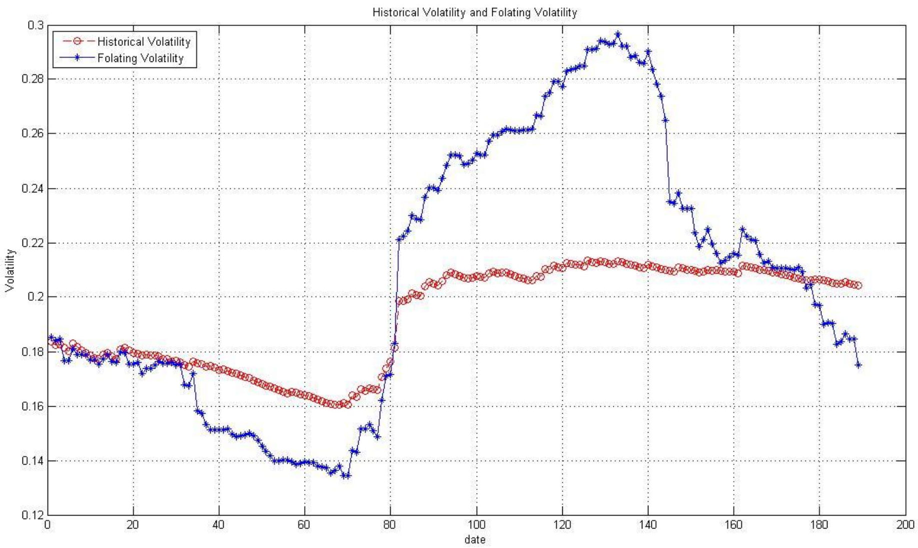

Figure 4 *Historical Volatility and 3-month Floating Volatility of OMXS30 Index*[4]

4 Data are collected from the Nasdaq OMX Nordic market.
http://www.nasdaqomxnordic.com/indexes/historical_prices/?Instrument=SE0000337842

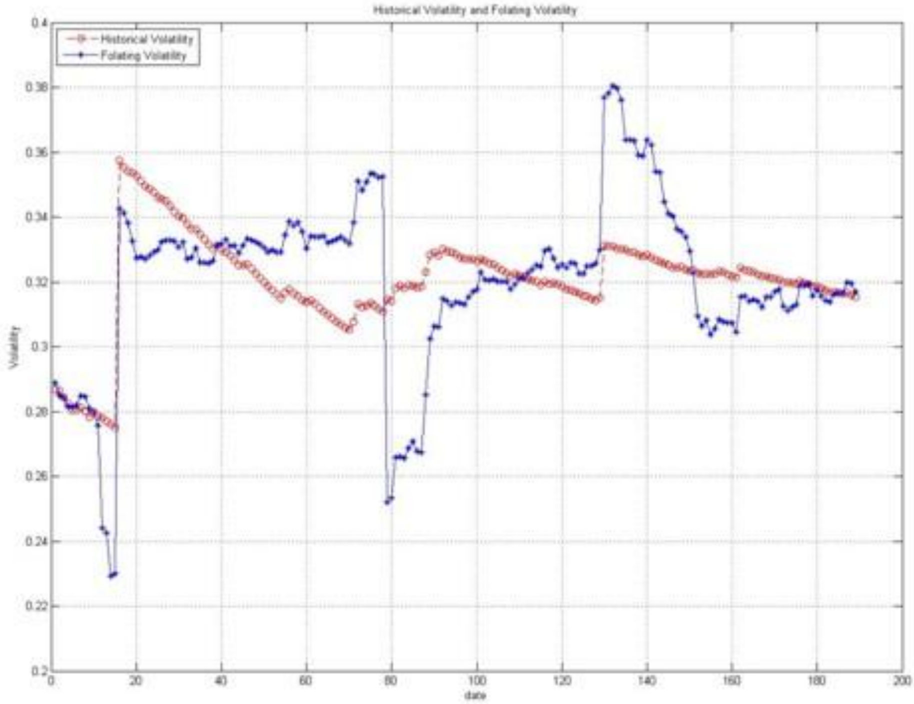

Figure 5 *Historical Volatility and 3-month Floating Volatility of the stock Electrolux B*[5]

We can see that the floating volatility tends to vibrate much greater than the historical volatility in the two pictures.

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[5] Data are collected from Nasdaq OMX Nordic market.
http://www.nasdaqomxnordic.com/shares/Historical_prices/?Instrument=SSE81

Conclusion

There exists high risk in investment due to the changeable volatility. According to what we have discussed in the previous sections, we could get a comprehensive understanding about the volatility. For the floating volatility, it is a good method for investors to judge the performance of the security by the historical data, which represents more fluctuated than historical volatility. In that way, investors can make more rational and reasonable decisions to earn the expected payoff. For the implied volatility, it is a “looking forward” method, which can supply some crucial clues for investors to decide whether invest it or not form the option price. Hence, more and more investors rely on it to make decisions.

The stocks, options and the index options are observed in this report to make a complete guideline to show how the theories bond with the practice. Several figures are drawn by the Matlab to enable the results are more visual and easier for readers to grasp the information.

There is no doubt that the volatility is a valuable topic, and since we have touched the area of the topic we would like to suggest further research on volatility and its applications.

References

Steven Li, Queensland University of Technology **A New Formula for Computing Implied Volatility**

http://www.nasdaqomxnordic.com/optionsandfutures/microsite?Instrument=SSE81

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http://www.nasdaqomxnordic.com/indexes/historical_prices/?Instrument=SE0000337842

http://www.nasdaqomxnordic.com/shares/Historical_prices/?Instrument=SSE81

Appendix

Previous studies

Brenner and Subrahmanyam (1988) provide an elegant formula to compute an implied volatility, based on a condition that it is accurate when a stock price is exactly equal to a discounted strike price. Their formula is as follows:

$$\sigma \approx \sqrt{\frac{2\pi}{T}}\frac{C}{S}$$

Where, time to maturity, call option price and the stock price are denoted by T, S, C, respectively.

Corrado and Miller (1996) provide an improved quadratic formula which is valid when stock prices deviate from discounted strike prices. Their formula is given as:

$$\sigma \approx \sqrt{\frac{2\pi}{T}}\frac{1}{S+K}\left[C-\frac{S-K}{2}+\sqrt{\left(C-\frac{S-K}{2}\right)^2-\frac{(S-K)^2}{\pi}}\right]$$

This formula yields quite accurate estimates for some cases.

Where, T, S and C are the same as mentioned above. K is the strike price.

For real cases

At-the-money calls

$$\sigma = \frac{2\sqrt{2}}{\sqrt{T}}Z-\frac{1}{\sqrt{T}}\sqrt{8Z^2-\frac{6\alpha}{\sqrt{2}Z}}$$

$$\alpha = \frac{\sqrt{2\pi}C}{S}$$

$$Z = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{3\alpha}{\sqrt{32}}\right)\right]$$

Remark: ① $8Z^2 - \frac{6\alpha}{\sqrt{2}Z} > 0$ holds.

② $0 < \frac{3\alpha}{\sqrt{32}} < 1$ holds as long as $0 < \frac{C}{S} < \frac{\sqrt{32}}{3\sqrt{2\pi}} = 0.7522$, which is valid for almost options in real life. Hence, the formula is available for all at-the money calls.

In- or out-of the-money calls

Deep in- or out-of-the-money calls

$$\sigma \approx \sqrt{\frac{2\pi}{T}}\frac{1}{S+K}\left[C - \frac{S-K}{2} + \sqrt{\left(C - \frac{S-K}{2}\right)^2 - \frac{(S-K)^2}{\pi}\frac{1+{}^{K}\!/_{S}}{2}}\right]$$

Nearly at-the-money calls

$$\sigma \approx \frac{2\sqrt{2}}{\sqrt{T}}\tilde{z} - \frac{1}{\sqrt{T}}\sqrt{8\tilde{z}^2 - \frac{6\tilde{\alpha}}{\sqrt{2}\tilde{z}}}$$

Where, $\tilde{z} = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{3\tilde{\alpha}}{\sqrt{32}}\right)\right]$

$$\tilde{\alpha} = \frac{\sqrt{2}}{1+\eta}\left[\frac{2C}{S} + \eta - 1\right]$$

$$\eta = \frac{K}{S}$$

η measures the moneyness of an option: $\eta = 1$ represents at the money, $\eta > 1$ represents out-of the money, $\eta < 1$ represents in-the-money. It is an important variable to determine whether we should use out-of-the money formula or in-the-money formula.

General formula for implied volatility

Implied volatility is a calculation that uses an option's Vega (its sensitivity to change in volatility) to derive an estimate of volatility.

Formula: $x_{i+1} = x_i - \frac{y_i - p}{v_i}$

Until $\left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \leq \varepsilon$, at which point x_i = implied volatility.

Where:

p = Option price

x_i = Volatility

y_i =Option's theoretical value at volatility xi

v_i =Option's Vega at theoretical value *yi* =Degree of accuracy