MMA 707 Analytical Finance

Options on options

13/18 of October, 2010

Members:

GBADAGO EVELYN DELA

NYAMEKYE KOFI AFRIYIE

OSEI-FOSU AUGUSTINE

Lecturer: Jan R. M. Röman

Department of Mathematics and Physics
Mälardalen University, Vasteras, Sweden

Abstract

This report cast a glance at Options on option that is we wrote about Option on option, which is also known as Installment options. We spoke about all the types of installment options and used some useful tool that did help us in giving better in-depth explanation and analysis of the work. Some of the tools that we did use were MathType, Excel, just to mention but few. We finally found out that installment option, in the first place is an interesting contract to enter and also very complex to deal with. There are a lot to know about installment option as you read on.

Table of content

1 Introduction

1.1 Goal

The goal of this report is to give answers to four following questions:

- What is an Installment Option?
- What sorts of Installment Options are traded?
- Why are these options attractive?
- How to price several chosen Installment Options?

1.2 Definition of Installment Option

Installment options, is the option where the premium is divided into different parts and is paid during the option lifetime.
Every installment date presents the moment when the holder takes the decision either to continue to pay the premiums or allow the contract to lapse.

1.3 Types of installment Option

Installment option is also called compound options at the same time options on options - i.e. the 'underlying' is another option.
There are four types:

- Put on a put
- Call on a call
- Call on a put
- Put on a call

1.4 Why write installment options

The total premium of the installment option is always higher than the vanilla options premium. This can be explained by the opportunities to terminate the contract without paying the whole sum of the premium. It is interesting for the investors who are ready to overpay for the advantage to terminate the payments and reduce the losses if their investment position goes wrong.

1.5 Why buy Installment Option

Unlike other option contracts, instead of paying a lump sum for a derivative instrument, holder of the installment option will pay the installments as long as the need for being long in the option is present. This considerably reduces the cost of entering into a hedging strategy. In addition, the non-payment of an installment suffices to close the position at no transaction cost. reduces the liquidity risk typically associated with other over-the-counter derivatives. Also, investors may lodge their shares in return for installments, thereby extracting cash to diversify their portfolios without losing exposure to their shares.

1.6 Problems with installment option

Installment option has so many advantages, but it has few setback such as, the holder has the right to stop making payments, thereby terminating the option on the due date of the first missed payment. The significance of the latter feature is that if the option is not worth the present value of the remaining payments, the holder does not have to continue to make payments. In return for the right to terminate payments, so investors end up not honoring contracts unlike other options.

1.7 In-depth to options

In this report three exotic options will be under thorough investigation:

- Put on a put
- Call on a call
- Call on a put
- Put on a call

2 Installment options

2.1 Introductions

In this chapter we are going to focus seriously on installment option. Installment options are one of the most widely-traded exotics on some markets. When compared to vanilla options, installment option has two characteristics that differentiate it from a vanilla: The option premium is paid periodically-usually monthly or quarterly-over the life of the option and the holder has the right to stop making payments, thereby terminating the option on the due date of the first missed payment. If an installment option is selling below the value of making an installment to keep it alive, the investor need not be concerned about losing the value of a fully-paid- for standard option. Investor can simply walk away from the installment option on any installment payment date. Unless the option is an American-style contract, however, it usually makes sense to continue payments on installment options which have a net present value on a payment date.

2.2 Advantages of installment options

Installment options give buyers two main benefits:

- More geared exposure to changes in the underlying asset price (or to changes

in volatility) than conventional options

- The flexibility to pay for the rights to the underlying asset

2.3 Disadvantages of installment option

These are some of the bad side of installment options:

- The holder has the right to stop making payments, thereby terminating the option at any time leading to most contract not honed.
- Because of the nature of installment options, always the outcome of the bid is uncertain.

2.4 Details of some of the type of installment options

As we rightly said at the introductory chapter, installment Options are options on options. Of which it constructed in one of the following four ways:1. Call on Call 2. Call on Put 3. Put on Put 4. Put on Call

These options are highly sensitive to the volatility of the volatility as there is also an underlying option, making it more difficult to hedge, relative to vanilla options.

European-Style installment Options

Considering a Black & Scholes environment, the payoff for a installment option is given as: $\max\left\{0, \phi PV_t\left[\max\left(0, \eta S^* - \eta X_\mu | T\right)\right] - \phi X\right\}$ (1)

Where S* is the value of the stock underlying the underlying option, X_μ is the underlying strike price and X is the installment strike. t is the expiry date of the installment and T is the expiry date of the underlying option.

The variables ϕ and η are binary variables in that they take either values of 0 or 1. η is given as 1 when the underlying option is a call, and -1 when the underlying option is a put option. ϕ is given as 1 when the installment is a call and -1 when the installment is a put.

1. Price of the underlying asset of the underlying option (S)
2. Exercise prices of underlying option and the installment option (X1 & X2)
3. Dividend payments (if any) on the underlying asset (q)
4. Risk free rate (γ)
5. Expiry dates for the underlying option (T1) and the installment option (T2)

The 4 formulae for pricing the options are as follows:

For a call on call:

$$Call_{call} = Se^{-qT_2}M\left(a_1,b_1;\rho\right) - X_2e^{-\gamma T_2}M\left(a_2,b_2;\rho\right) - e^{-\gamma T_1}X_1N\left(a_2\right) \qquad (2)$$

Call on put:

$$Call_{Put} = Se^{-qT_2}M\left(a_1,-b_1;-\rho\right) - X_2e^{-\gamma T_2}M\left(a_2,-b_2;-\rho\right) + e^{-\gamma T_1}X_1N\left(a_2\right) \qquad (3)$$

Put on call:

$$Put_{call} = X_2e^{-\gamma T_2}M\left(-a_2,b_2;-\rho\right) - Se^{-qT_2}M\left(-a_1,b_1;-\rho\right) + e^{-\gamma T_1}X_1N\left(-a_2\right) \qquad (4)$$

Put on put:

$$Put_{Put} = X_2e^{-\gamma T_2}M\left(-a_2,-b_2;\rho\right) - Se^{-qT_2}M\left(-a_1,-b_1;\rho\right) - e^{-\gamma T_1}X_1N\left(-a_2\right) \qquad (5)$$

Where the variables are defined as:

$$a_1 = \frac{\ln\left(S/S^*\right) + \left(\gamma - q + 0.5\sigma^2\right)T_1}{\sigma\sqrt{T_1}} \qquad (6)$$

$$a_2 = a_1 - \sigma\sqrt{T_1} \tag{7}$$

$$b_1 = \frac{\ln\left(\frac{S - q_1 e^{-\gamma(t_1 - t)}}{S^*}\right) + \left(\gamma + 0.5\sigma^2\right)\left(t_1 - t\right)}{\sigma\sqrt{t_1 - t}} \tag{8}$$

$$b_2 = b_1 - \sigma\sqrt{T_1} \tag{9}$$

$$M\left(a,b;\rho\right) = \frac{\sqrt{1-\rho^2}}{\pi}\sum_{i,j=1}^{n} A_i A_j f\left(B_i, B_j\right) \tag{10}$$

$$f\left(x,y\right) = \exp\left[a^{|}\left(2x - a^{|}\right) + b^{|}\left(2y - b^{|}\right) + 2\rho\left(x - a^{|}\right)\left(y - b^{|}\right)\right] \tag{11}$$

$$a^{|} = \frac{a}{\sqrt{2\left(1-\rho^2\right)}}, \quad (12) \qquad b^{|} = \frac{b}{\sqrt{2\left(1-\rho^2\right)}} \quad (13)$$

Where S* is the critical stock price for which the following criteria holds:

$$Call_{European}\left(S^*, X_1, q, \gamma, V, T_2 - T_1\right) = X_2 \tag{14}$$

It can be solved iteratively using the Newton-Rhapson method.

For overlapping Brownian increments, we can denote the correlation of the installment and underlying options as:

$$\rho = \sqrt{T_1/T_2} \tag{15}$$

Also note that in the equations,

$$M\left(a,b;\rho\right)$$

Is the bivariate cumulative distribution function.

American-Style installment Options

In a Black & Scholes world *without dividends*, American style installment options would not be valuable to hold as it is never to exercise American style options which pay no dividends.

2.5 Summary

We now know the types of installment option and how it works with principles such as, If an installment option is selling below the value of making an installment to keep it alive, the investor need not be concerned about losing the value of a fully paid for standard option. The investor can simply walk away from the installment option on any installment payment date. Unless the option is an American-style contract, however, it usually makes sense to continue payments on installment options that have a net present value on a payment date

2.6 References

John C. Hull, *Options, futures, and other derivatives,* 6th ed., Pearson, New Jersey, 2008.
M. Kijima, *Stochastic processes with applications to finance,* Chapman and Hall, 2003
R. Jan Röman, *Lecture note for analytical finance 1,* 2010

2.7 Internet Link

http://en.wikipedia.org/wiki/Compound_option, 13:03, 14 May 2010
http://www.mathfinance.de/wystup/papers/instalment.pdf, February, 2007
http://www.amex.com/servlet/AmexFnDictionary?pageid=display&word=Installment%20Option

3 Comparison of installment option and vanilla option

3.1 Introduction

In this chapter we are going to do detailed comparison between ordinary options and installment options. Vanilla option as we all know is the normal option with no special or unusual features, while installment options are option for which the underlying is another option. Therefore, there are two strike prices and two exercise dates. Also installment type of option usually exists for currency or fixed-income markets, where an uncertainty exists regarding the option's risk protection capabilities. The advantages of compound options are that they allow for large leverage and they are cheaper than straight options. However, if both options are exercised, the total premium will be more than the premium on a single option.

3.2 Installment option versus vanilla options

In our comparison let's take it one after the other, i.e. from the vanilla option to installment option. Let's consider the simplest case of European call or put option. The decision to be made is whether the option should be exercised at its expiration date T. There are two states to be considered, during the time interval [0,T] the state is holding the option and the option is not exercised. At time T a distinction must be made between the case where the option is not exercised, which leads to switching over to the second state, and where the option is not exercised, which mean there is no change in the state of the system. At time T these two state accrue from the payoff function of the option. For example the payoff function of a European call is given by $C(T, S_T) = \max\{S_T - K, 0\}$ (16)

Where S_T is the value underlying asset the option at time T and the switching cost H are equal to the exercise price K. without going too much into the mathematical details, the graphical representation can be seen below
As can be seen in the figure 1, during the time interval [0,T] the state of the system remain unchanged because there is no flexibility to alter anything. However at time T there are two opportunity to take action. If the holder of the option decides to exercise, the system perform European switch, represented by the dotted circle, and immediate afterwards ends. Since there is only one state over which switching can be chosen, the dotted circle contains 1. If the option is not exercised, the system will not change its state and its natural end is represented by the right closure of the solid line.
The next example deals with the representation of American option in the graphical decomposition model. In much the same way as its European counterpart there are two states to consider, see figure 2. The distinctive feature of the American option is that the second state, option exercised can be attained everywhere in the time interval [0,T] so that its first possible entry time is 0. While the system starts in the not exercise state, it is drawn as a solid line up to the random stopping time τ where exercising is triggered. Afterward the state not exercised in inactive denoted by the dashed line. At time τ the switch to the state exercised is performed which is afterwards active up to time T which could have been the latest possible entry and exit time. Since there is only one state to switch to, the solid circle indicating American switch contains 1. The natural end of both state is time T.
The last example of a financial option is a comparison between American and European compound option. The American contract allows its holder to exchange the compound option up to time T1 with a simple option on the underling expiring at $T_2 > T_1$. For example the resulting intrinsic value of the American compound option for two call options with exercise price K_1 is given by

$$\max\left\{c\left(t,S_t\right)-K_1,0\right\} \tag{17}$$

And the corresponding value of the compound option by

$$cc\left(t,x\right)=\sup_{t\le\tau_1\le T_1}E^x\left[e^{-r(\tau_1-t)}\max\left\{c\left(\tau_1,S_{\tau_1}\right)-K_1,0\ \right\}\right] \tag{18}$$

Where $cc(t,x)$ is the value of the compound option (call on call), $cc(t,x)$ is the value of the underlying call option, and St is the call option's underlying source of uncertainty, eg, the stock price. $\tau_1 \in [0, T_1]$ is the stopping time at which the American compound option is exercised.

In contrast, exercise of European compound option and its underlying simple option may only take place at the fixed time T_1 and T_2, respectively. Consequently, we get for the European compound option for calls the value function

$$cc\left(t,x\right)=\sup_{t\le\tau_1\le T_1}E^x\left[e^{-r(\tau_1-t)}\max\left\{c\left(\tau_1,S_{\tau_1}\right)-K_1,0\ \right\}\right] \tag{19}$$

Reflecting the restriction that the exercise date may not be chosen. The differences in the graphical representation are demonstrated below

Since the American compound option and its underlying simple option may be exercised instantaneously, the first entry time to both state is zero. On the other hand the latest exist time for both state is T_2. In much the same way the time interval over which the states of the European compound option are active can be obtained. Due to the fixed switching times in the European case the considerations get the result of above are straightforward.

Although the previous examples have been devoted to financial options, the same logic of graphical representation applies to real options, namely the perpetual option to invest (or option to wait) which illustrates irreversible switches and the entry and exit decision of firms which highlights the use of costly reversible switches.

Illustration diagrams

Figure 1 graphical representation of European option

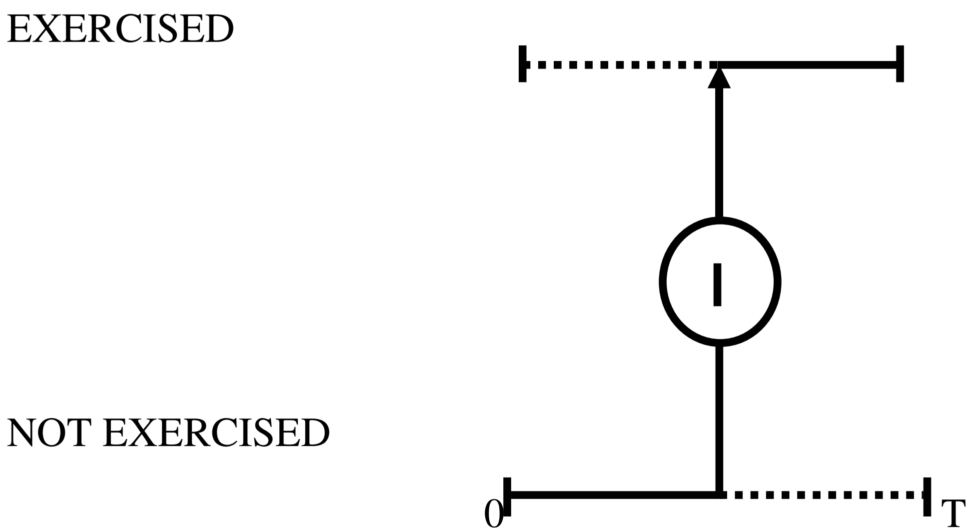

Figure 2 graphical representation of American option

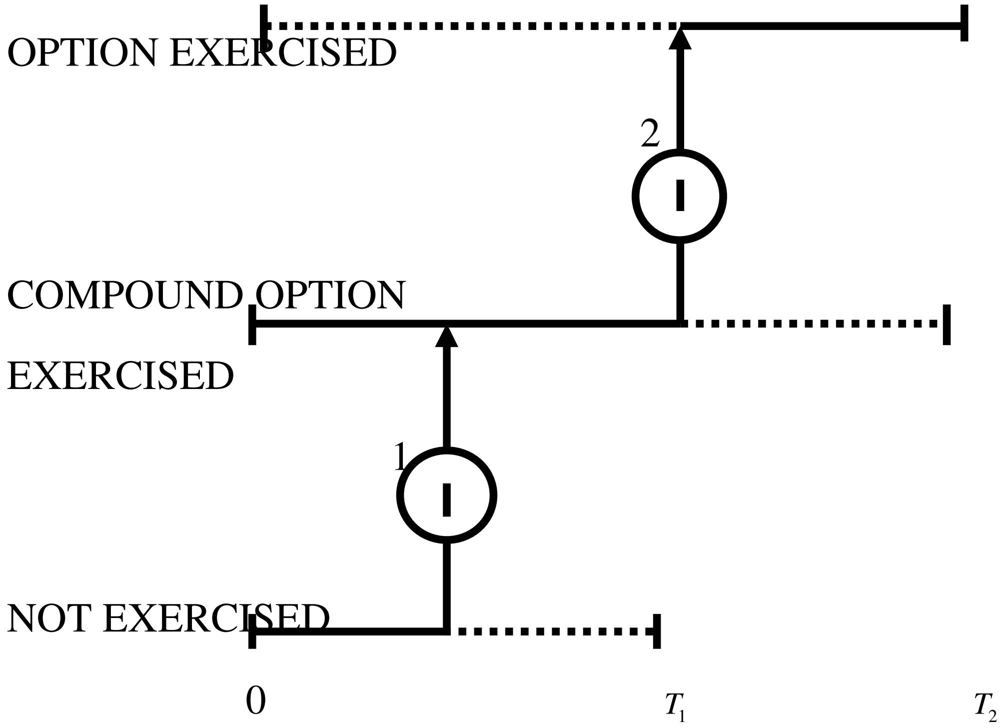

Figure 3 graphical representation of American compound option

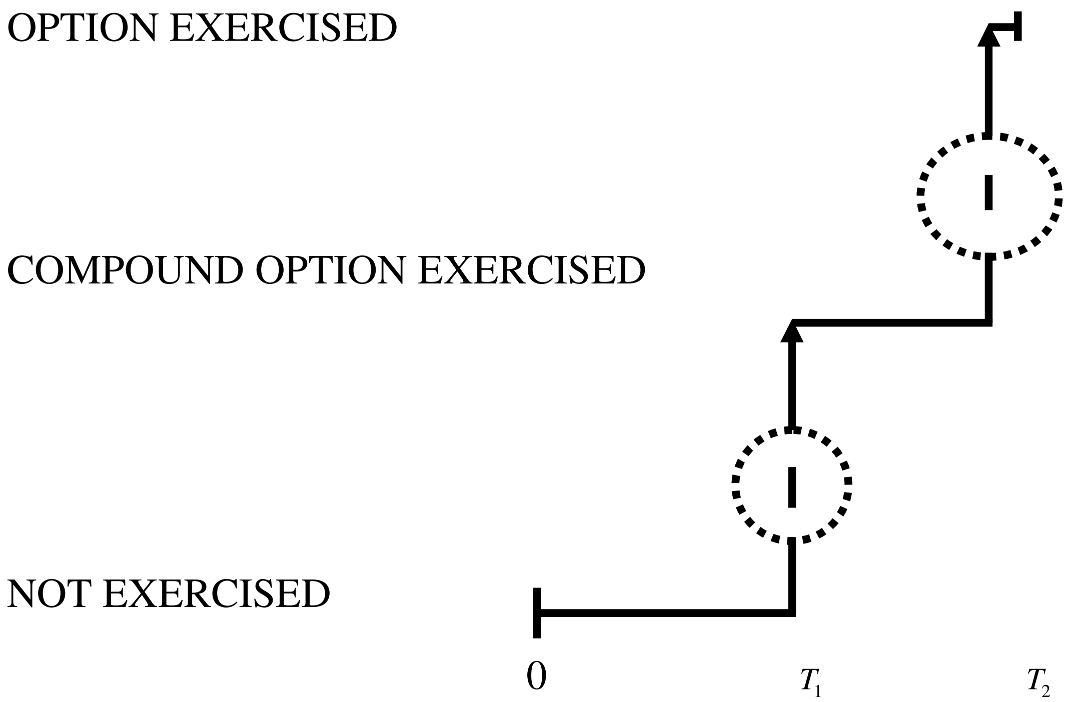

Figure 4 graphical representation of European compound option

3.3 Conclusion

Installment option is a complicated option to handle in practical sense, but interesting and involving when reading and writing about it. In the case of European vanilla call option, as can be seen in the graph, during the time interval [0,T] the state of the system remain unchanged because there is no flexibility to alter. In much the same way as its European counterpart there are two states to consider, see figure 2. The distinctive feature of the American option is that the second state, option exercised can be attained everywhere in the time interval [0,T] so that its first possible entry time is 0. A comparison between American and European compound option. The American contract allows its holder to exchange the compound option up to time T1 with a simple option on the underling expiring at $T_2 > T_1$ while in much the same way the time interval over which the states of the European compound option are active can be obtained.

3.4 References

John C. Hull, *Options, futures, and other derivatives,* 6th ed., Pearson, New Jersey, 2008.
M. Kijima, *Stochastic processes with applications to finance,* Chapman and Hall, 2003
R. Jan Röman, *Lecture note for analytical finance 1,* 2010

3.5 Internet link

http://books.google.com/books?id=8jHImiDlOGYC&pg=PA107&lpg=PA107&dq=graphical+presentation+of+compound+option&source=bl&ots=dukAlezajo&sig=5i0fKLW0kpUx91kel23dIRBSP4A&hl=en&ei=5XOvTKTfFoygOoCXqeUN&sa=X&oi=book_result&ct=result&resnum=7&sqi=2&ved=0CCkQ6AEwBg#v=onepage&q&f=true

Table of figures