



MÄLARDALEN UNIVERSITY
SWEDEN

Division of Applied Mathematics

School of Education, Culture and Communication

Box 833, SE-721 23 Västerås

Sweden.

MMA 707 - Analytical Finance I

Teacher: Jan Roman

Exotic Options

American Digitals, One -Touch Down and One-Touch Up

Ikechukwu Chinaka

Henry Obeng Tawiah

Isaac Nsiah Yeboah

18th October, 2010

Abstract

In this report, we studied Exotic Options in particular, American Digital Options (One-Touch-Up and One-Touch down) and made a comparison with Vanilla Options. Models were also built in Excel for the pricing formulas.

Table of Contents

1.0 Introduction.....	3
1.1 American Digital Options.....	3
1.2 Barrier Options.....	4
1.3 Call Barrier Options.....	5
1.4 Put Barrier Options.....	5
1.5 Practical Example.....	5
1.6 Price Curves.....	7
1.7 American Digital and Vanilla options Compared.....	11
1.8 Conclusion.....	12
References	13

1.0 Introduction

A standard option has some well-defined properties

Type - Call or Put

Strike - A given strike price K

Style - European, Bermudan or American

Expiry Date - The time to Maturity

Settlement - Physical or Cash

Underlying - Stock, Currency, Index etc.

Exotic options are important features of over-the-counter market. It differs from the common vanilla American or European Option in terms of the underlying assets, the calculation of how the investor receives a certain pay off or hedging.

Exotic options can be constructed in many different ways and examples include

- Cash or Nothing Options
- Knock – Out and Knock – In Options
- Barrier Options
- Asian Options
- Binary/Digital Options

1.1 American Digital Options

Like most options, American Digitals (Binary) come in both European and the American style (early exercise types). Like all American options, its market price may not be lower than the intrinsic value, so the American option price rises more steeply than the European equivalent as it gets closer to the strike. However, the option has no time value when it is ITM (In the Money) and should be exercised immediately. American style digitals are often referred to as “one-touch Binary/digitals” or “Binary-at-Hit” and can easily be priced using set of formulas given by Reiner and Rubinstein (1991).

The Pay-Off of American Digital option is similar to a Vanilla Option but with the American Digitals, the holder receives a Unit amount if and when the underlying Asset trades above the Strike Price K for the first time. This introduces additional time option to the problem. The fair value of an American Digital is, according to the general principles:

$$V(S, T) = E\{e^{-r\tau}; \tau \leq T\}$$

Where expectation is taken with respect to a risk-neutral probability and τ represents the first hitting time of the strike price $S = K$.

One-Touch Option: This is a type of exotic option that gives an investor a payout once the price of the underlying asset reaches or surpasses a pre-determined barrier. This type of option allows the investor to set the position of the barrier, the time to expiration and the payout to be received once the barrier is broken.

Only two outcomes are possible with this type of option:- the barrier is breached and the trader collects full payment agreed upon at the outset of the contract, or the barrier is not breached and the trader loses the full premium paid to the broker.

This type of option is useful for traders who believe that the price of an underlying asset will exceed a certain level in the future but who are not sure that the higher price level is sustainable. Because a one-touch option has only one barrier level, it is generally slightly less expensive than a double one-touch option. These types of options are more popular with traders in the commodity and forex markets.

One-Touch-Down digital Option

This option is priced under the condition when $S > K$ and the formula is given by

$$C_{\text{One-Touch-Down}} = k \left[\left(\frac{H}{S}\right)^{\mu+\lambda} N(Z) + \left(\frac{H}{S}\right)^{\mu-\lambda} N(Z - 2\lambda\sigma\sqrt{T}) \right]$$

One-Touch-Up Digital Option

This option is also priced under the condition when $S \leq K$ and the formula is given by

$$C_{\text{One-Touch-up}} = k \left[\left(\frac{H}{S}\right)^{\mu+\lambda} N(-Z) + \left(\frac{H}{S}\right)^{\mu-\lambda} N(2\lambda\sigma\sqrt{T} - Z) \right]$$

$$Z = \frac{\ln\left(\frac{H}{S}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad \mu = \frac{r-q-0.5\sigma^2}{\sigma^2} \quad \lambda = \sqrt{\mu^2 + \frac{2(r-q)}{\sigma^2}}$$

r – Risk Free Interest Rate

K – Strike Price

S – Price of Asset

T – Time to Maturity

σ - Volatility

H – Barrier

q – Dividend rate

1.2 Barrier Options

Barrier options are a generic name given to derivative securities with payoffs which are contingent on the spot price reaching a certain level, or barrier over the life time of the option. A number of different types of barrier option regularly trade over the counter (OTC) market. They attractive to some market participants because they are less expensive than the corresponding regular vanilla options.

These Barrier Options can either be classified as either a **Knock-out-Option** or a **Knock-in-option**

A knock-out-option ceases to exist when the underlying asset price reaches a certain barrier. A knock-in-option comes only in existence only when the underlying asset price reaches a certain barrier.

In single barrier options, it is easy to show that barrier options with a knock-in feature can be priced by buying an option without any knock-out feature and selling a knock-out option. For example, an American Binary option with a knock-in barrier H , the option premium is equal buying an American binary knock-out option with a barrier at H . All the options have the same payment barrier.

1.3 Call Barrier Options

A **down-and-out call** is a type of knock-out option. It is a regular call option that ceases to exist if the asset price reaches a certain barrier level, H . The barrier level is below the initial price of the asset.

A corresponding Knock-in option is a **down-and-in call**. This is a regular call that comes into existence only if the asset price reaches the barrier level.

An **up-and-out call** is a regular call option that ceases to exist if the asset price reaches a barrier level, H that is higher than the current asset price.

An **up-and-in call** is a regular call option that comes into existence **only** if the barrier is reached. When H is less than or equal to K , the value of the up-and-out call is zero.

1.4 Put Barrier Options

Put barrier options are defined similarly to call barrier options. An **up-and-out put**, is a put option that ceases to exist when a barrier, H , that is greater than the current asset price is reached.

On the contrary, an **up-and-in put** that comes to existence only if the barrier is reached. When the barrier is greater than or equal to the strike price K .

A **down-and-out put** is a put option that ceases to exist when a barrier less than the current asset is reached.

A **down-and-in put** is a put option that comes into existence when the barrier is reached.

It should be noted that all these valuations make the assumptions that the probability distribution for the asset price of an American digital option is lognormal.

1.5 Practical Example

Let us consider the pricing of an option with the following parameters. Volatility (σ) 0.2, risk free interest rate 0.05, a strike price (K) 110 and a barrier (H) pegged at 80.

Under the condition of the Digital option where it is only priced when ($S \leq K$), from the excel sheet, it is seen that the Investor will not immediately be in the money when the stock price is below the strike price. Before the option will expire in the money, the stock price has to be between at maximum of 72 and a minimum of 44. When the price drops below 44, the option expires out of the money, therefore having no price.

For $S > K$, with the same parameters above, the Stock will immediately be in the money when the price goes above the strike price, the stock will expire out of the money when the stock price is above a certain point which is 133. Above this point the option will have no value and will be worthless.

Fig 1.0 Digital Excel Work Sheet

Price of Security	S	110
Strike Price	K	110
Risk Free Interest Rate	R	0,08
Time to Maturity	T	1
Barrier	H	80
Volatility	Σ	0,2
Dividend Rate	Q	0
Normal Dist. Z	Z	-1,527
Mean Value	μ	1,5
Lambda Value	λ	2,5
Digital option Up ($S \leq K$)	Cu	184
Digital Option Down ($S > K$)	Cd	2

From the Excel we used the same parameters to price vanilla options. It can be seen that when the price goes above the strike price for vanilla option for example 120, the price of the vanilla option is 20.9 while the price of the barrier option is 2. Relatively for a downward trend, for a stock price at 50 the barrier price is 4 while the price compared to the price of a put option is 51.5. From this simple application it can be seen that the price of a Barrier option is cheaper than vanilla.

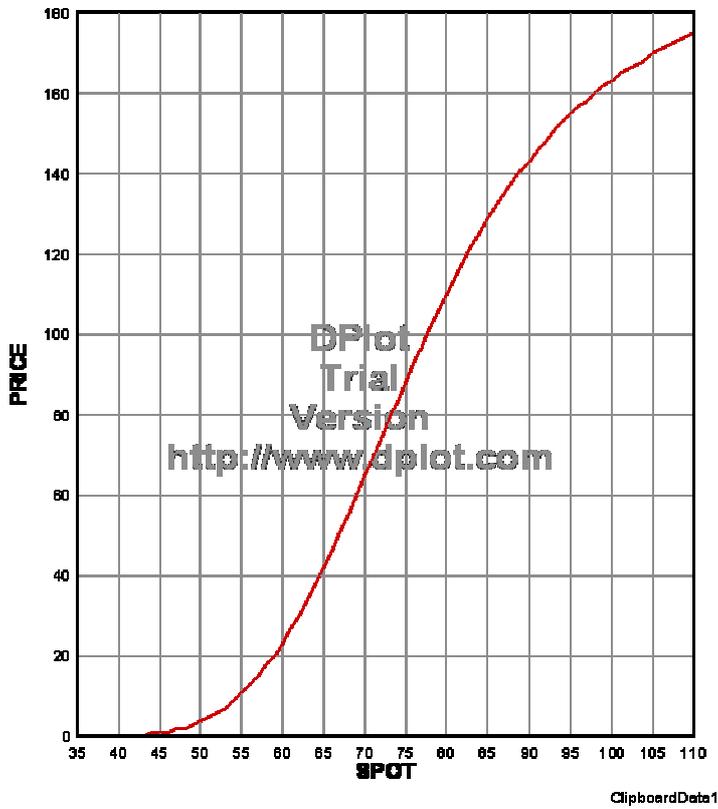
Fig 1.1 Vanilla Option Excel Sheet.

Current Price	S	50
STRIKE PRICE	K	110
TIME TO MATURITY	T	1
Volatility	σ	0,2
Interest Rate r	r	0,08
Dividend Yield	q	0
Normal Dist. Z	d1	-3,4422868
Normal Dist. Z	d2	-3,6422868
Price of Call Option	Pcall	0,000700668
Price of Put Option	Pput	51,53583398

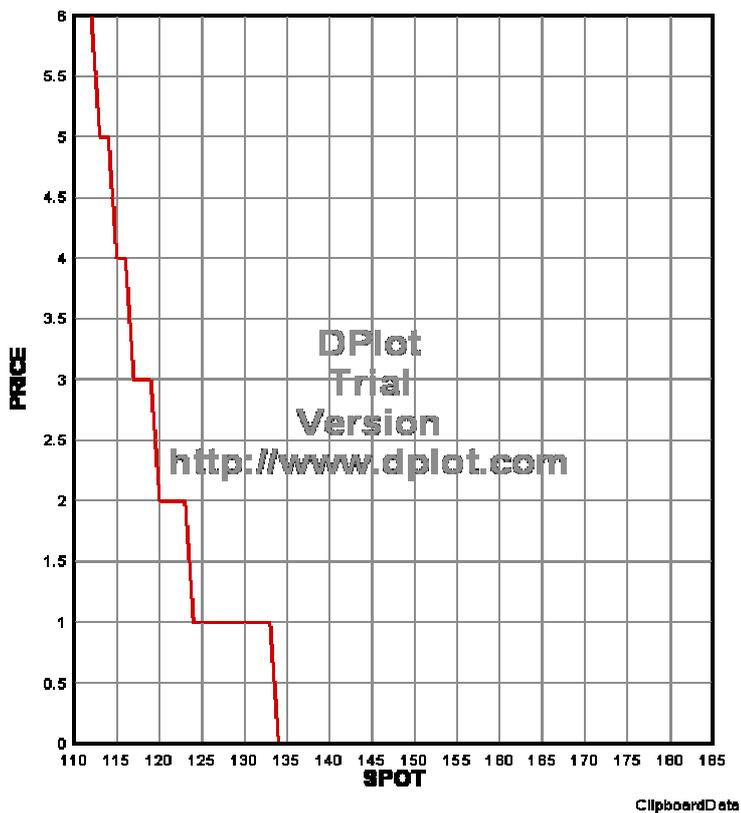
1.6 Price Curves

Price Curves for American Digital Options When the Strike is less than the Barrier ($K < H$).

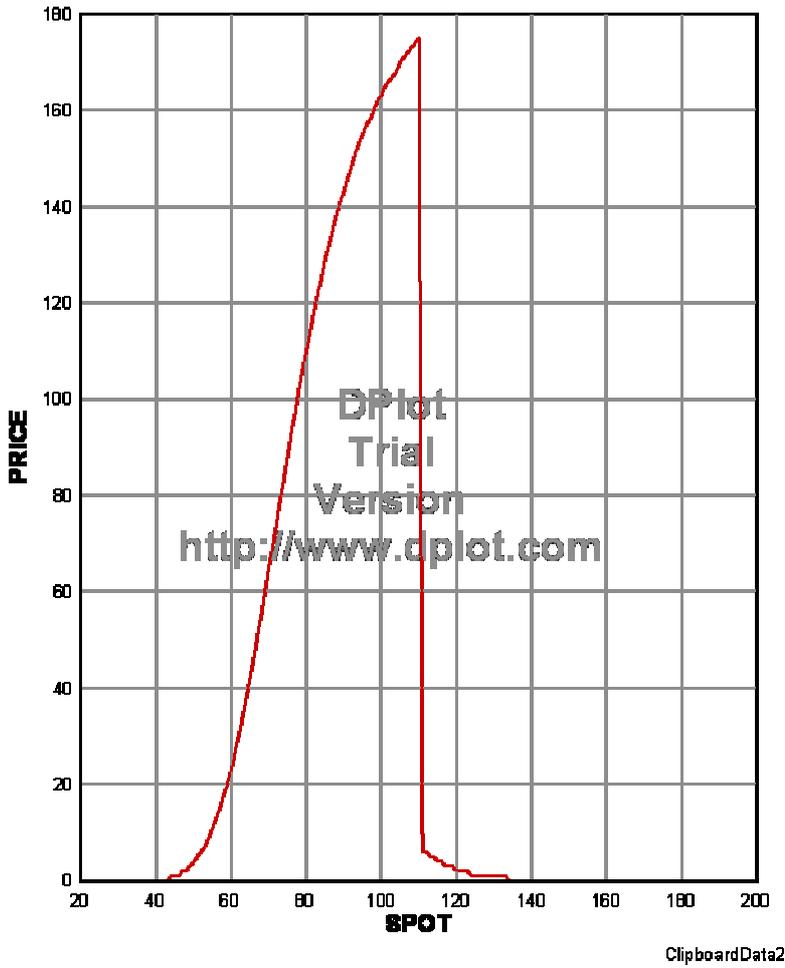
Digital Option up ($S \leq K$)



Digital Option Down ($S > K$)

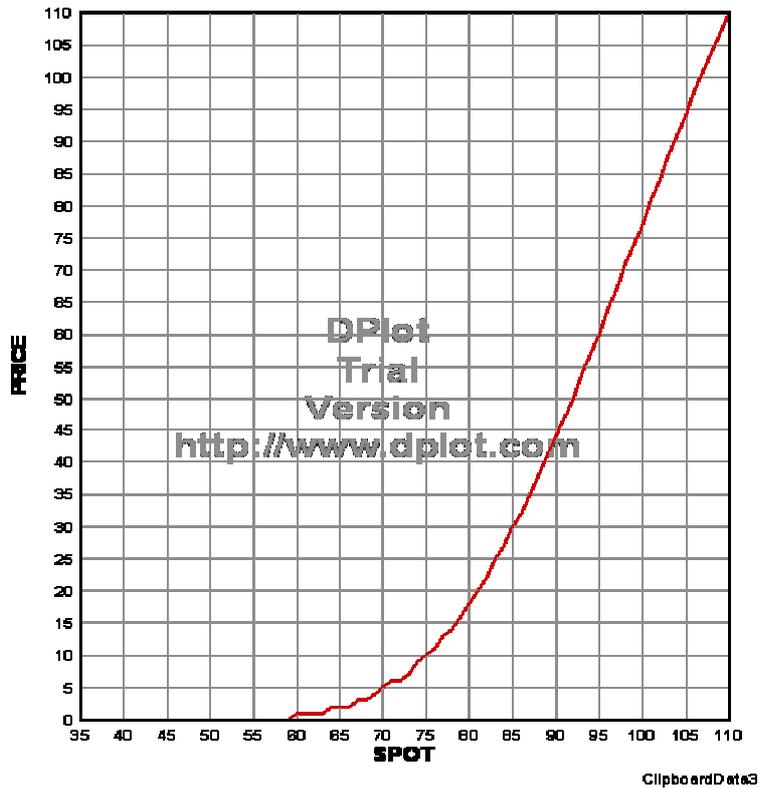


Digital Option UP and Down ($S \leq K$), ($S > K$)

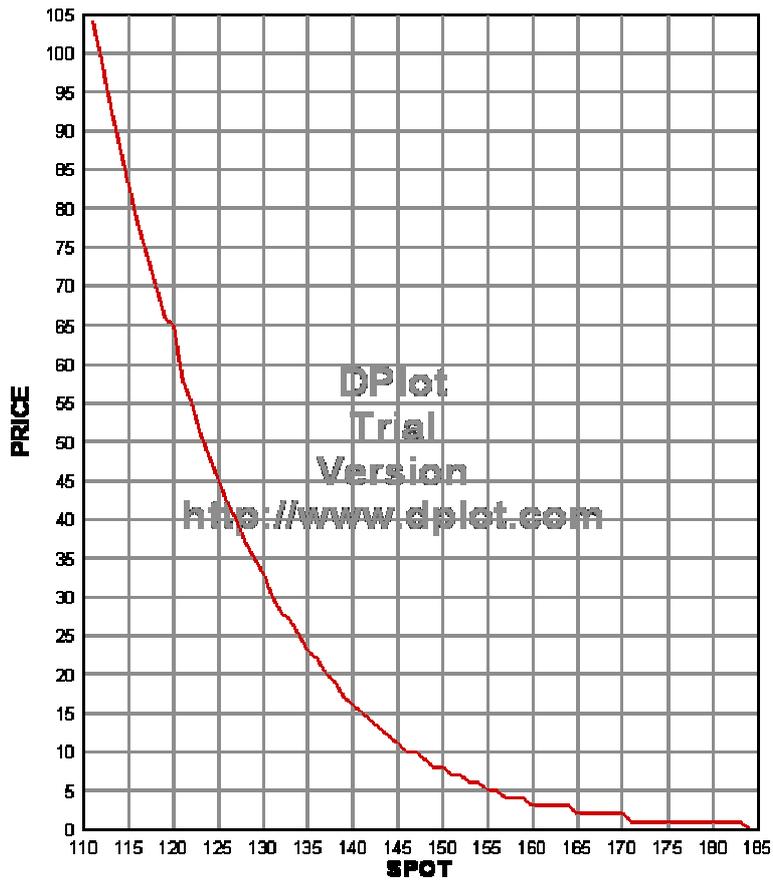


Price Curves For American Digital Options When the Strike Price Is Equal to the Barrier ($K = H$)

Digital Option up ($S \leq K$)

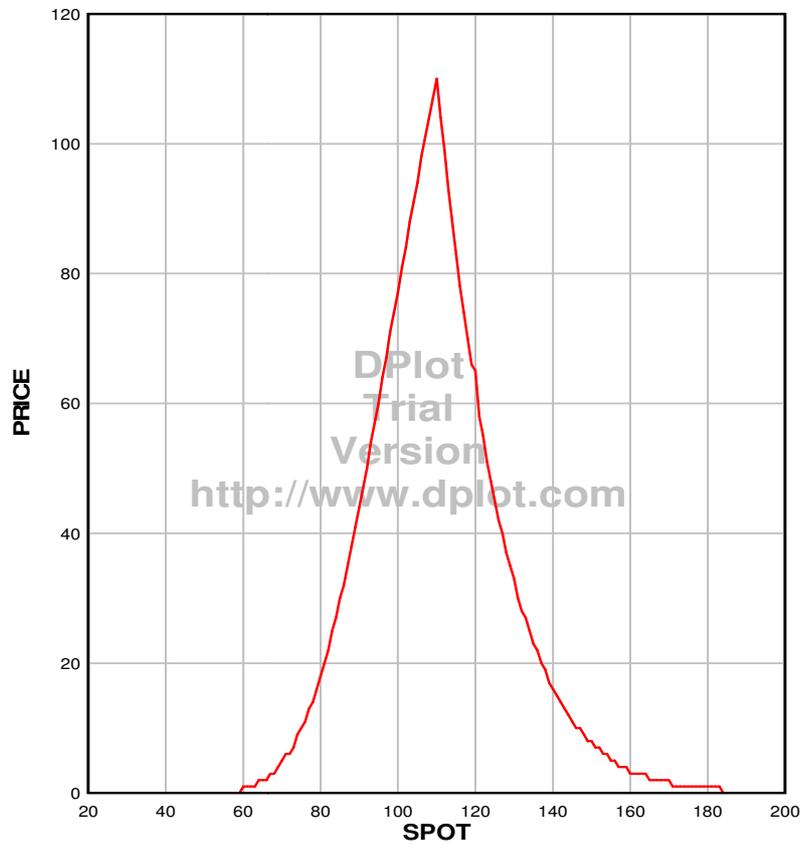


Digital Option Down ($S > K$)



ClipboardData4

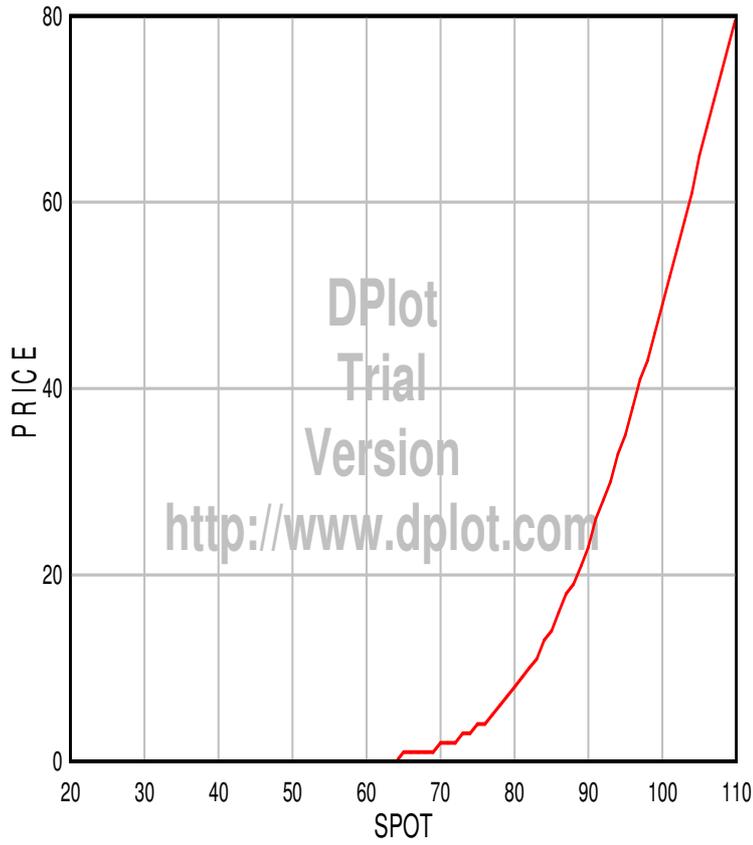
Digital Option UP and Down ($S \leq K$), ($S > K$)



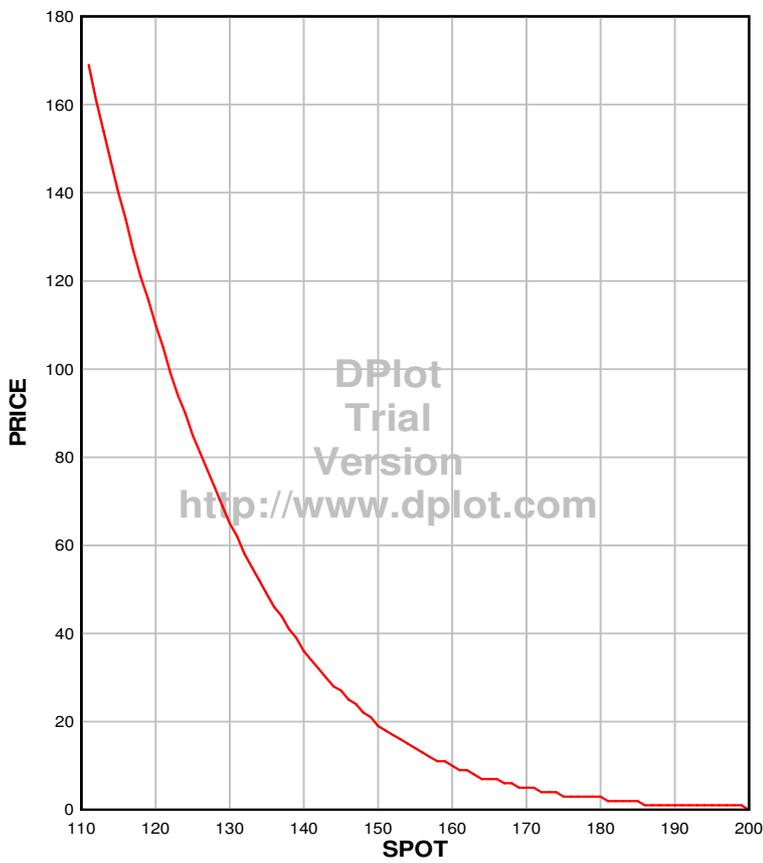
ClipboardData5

Price Curves For American Digital Options When the Strike Price Is Equal to the Barrier ($K > H$)

Digital Option up ($S \leq K$)

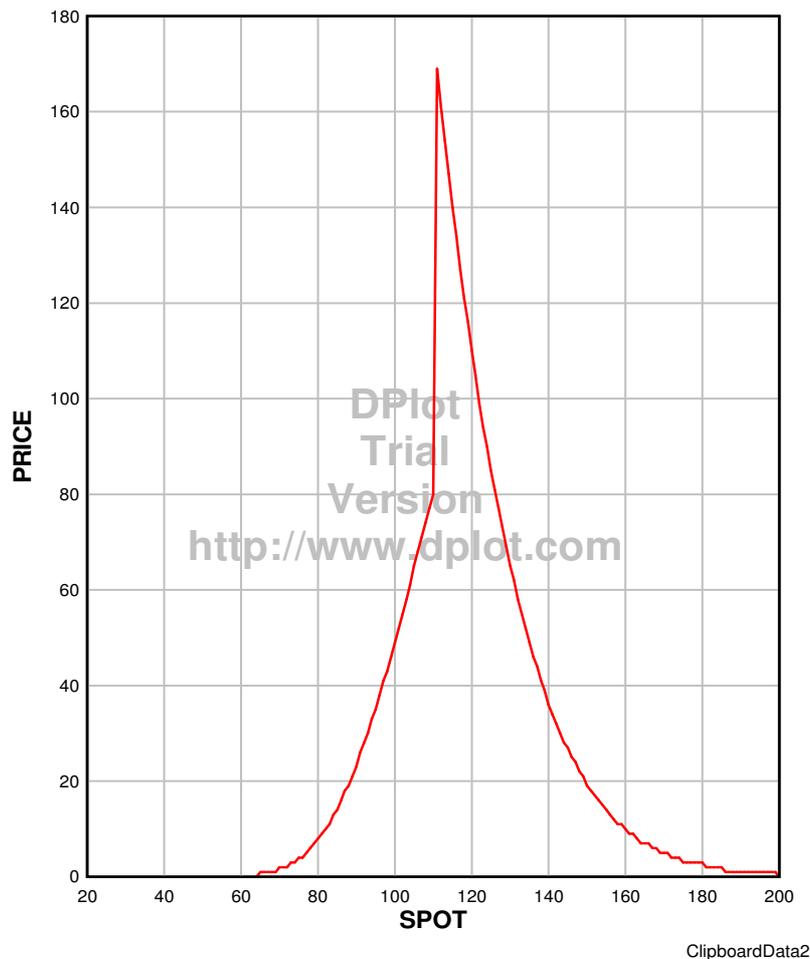


Digital Option Down ($S > K$)



ClipboardData1

Digital Option UP and Down ($S \leq K$), ($S > K$)



1.7 American Digital and Vanilla Options Compared

American digital options are generally harder to value than Vanilla Options because of the barriers included in the American digital.

Digital Barrier Options are cheaper than Vanilla Options.

American digital options have many varieties in trading compared with Vanilla which has only call and put options.

The pay-off of barrier options depends not just on the terminal level of the underlying asset but also its path but with Vanilla, the pay-off depends on the terminal level of the underlying asset

1.8 Conclusion

American digital options have no time value when it is in the money (ITM) and should be exercised immediately. In a barrier option, it is essential to define not only the type of barrier (knock-in or knock-out) but also the direction of the barrier (up or down). The definition is that the barrier is triggered if the spot trades "at or beyond" the barrier level at some time during the trade. So the definition of a knock-out barrier is a vanilla option that disappears if the spot trades at or beyond the barrier level during the trade in barrier period. The definition of a knock-in barrier is a vanilla option that comes into existence if the spot trades at or beyond the barrier level during the trade in barrier period.

References

- [1] Jan Roman, *Analytical Finance I lecture Notes*, MDH, 2010
- [2] John C. Hull, *Option, Futures and Other Derivatives*, 7th Edition, Prentice Hall New Jersey, 2009
- [3] <http://www.investopedia.com/terms/b/binary-option.asp>, last visited, 13th October 2010
- [4] <http://www.global-derivatives.com/index.php/pricing-models-topmenu-37/27?task=view>, last visited 13th October, 2010
- [5] <http://www.hoadley.net/options/devtoolsaddin.htm#exotic>, last visited 13th September, 2010