MMA707- Analytical Finance I

Curran Model to Calculate the Value of Asian Options

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ABSTRACT

Asian options are options in which the underlying variable is the average price over a particular period of time. Because of this reason, Asian options have a lower volatility and hence making them cheaper relative to their European counterparts. They are commonly traded on currencies and commodity products which have low trading volumes. Industrial consumers are highly exposed to volatility in the cost of electricity. This case study focuses on how a business hedges the electricity price volatility by using the average-price options. Using Monte Carlo Simulation, we calculated the option price at the end.

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INTRODUCTION

Asian Options were originally used in 1987 when Banker's Trust Tokyo office used them for calculating average options on crude oil contracts, and give the name Asian option. There are no known solutions for arithmetic options. It is not possible to analytically evaluate the sum of the correlated lognormal random variables. Asian Options usually referred to as average options, whose payoff is a function of the average of asset (or underlying) prices over a particular time period, and, thus, are dependent upon the particular path of asset prices.
In this article a presentation of some of the applications of Asian Options will be shown, then a brief description of methods to calculate Asian Options and their hedge parameters will be shown, followed by some information of the difficulties encountered when attempting to calculate Asian Options and it will be explained is mainly due to the numerous and varied different existing methods. We will study below how in competitive electricity markets, the price behavior of power is extremely volatile.

1.1 Definition

Asian options are financial contracts giving the holder the right to buy a certain asset for a pay-off price related to its average price during a particular time period (expiry date). The averaging can be arithmetic or geometric, the average price can be either discretely sampled or continuous sampled. Because of this fact; Asian options have a lower volatility and hence making them cheaper relative to their European counterparts. They are commonly traded on currencies and commodity products which have low trading volumes.
There are two types of Asian options available in the market, average rate options and average strike options. The average rate (call) option or ARO payoff at maturity is defined to be the difference (if positive) between the average of (asset) prices recorded over a particular time interval and a particular strike price. However, the average strike option or ASO, as Levy stated, pays the difference (if positive) between the asset price at expiry and the average of asset prices recorded over a particular time interval. Moreover, Geman and Yor state that, the ASO is less used in practice.

1.2 Difficulties

The difficulties in calculating Asian options are within a Black-Scholes framework, where the price of the underlying asset at any future time is modeled by a lognormal density function, the payoff at expiration, using an arithmetic average, is necessarily a sum of lognormally distributed random variables. Moreover, since the density of the sum of these components has no explicit representation, it’s not possible to produce a closed form solution for calculating arithmetic Asian options. However, since the product of lognormally distributed random variables is itself lognormal, various methods can produce a closed form solutions to this problem. Yet, in as much as the geometric average is less than or equal the arithmetic average on a set of numbers, these techniques tend to underpriced the value of an arithmetic Asian option. Furthermore, using put-call parity to determine the value of an arithmetic average rate put option, the geometric approach tends to overprice the value of an Asian put as well.

APPLICATION AND METHODS

There are many reasons why Asian options have become popular in the market. The main reasons, as Levy states, first, a company's exposure to future price movements are sometimes naturally expressed as exposure to the average of prices in the future. Secondly, average options are less sensitive to movements in the underlying asset price when the option's life is close to maturity. And thirdly, some accounting standards may require the translation of foreign currency assets or liabilities at an average of exchange rates over the accounting period. In addition, Asian options are commonplace in the currency and energy markets, where a firm that is susceptible to asset price fluctuations could use Asian options to speculate on the average of asset prices over a particular time interval. For example, a Brazilian coffee exporter to the United States who is concerned with appreciation of the Brazilian cruzado verses the US dollar might want to hedge the value of the cruzado against future dollar receipts. Moreover, an electric company, whose energy demands increase during the winter months, might want to hedge the costs of coal consumption in January over a particular time interval, say October through December, trying to reduce the impact of dramatic price fluctuations during the time of greatest consumption.
In following years, because of the popularity in the market, there has been a lot of interest and effort spent in evaluating average rate options. The methods of valuation of Asian options can be put into the following distinct classes of analytical approximation:

- Geometric method (Kemna & Vorst)
- Arithmetic Rate Approximation (Turnbull & Wakeman)
- Arithmetic Rate Approximation (Levy)
- Binomial Method & Trinomial Trees
- Finite Differences Method
- Monte Carlo simulations method with variance reduction techniques.
- The partial-differential equation (PDE) approach.

2.1-Geometric method (Kemna & Vorst)

In 1990, Kemna & Vorst put forward a closed form of evaluating geometric averaging options by altering the volatility and cost of carry term. Geometric averaging options can be priced through a closed form analytic solution because of the reason that the geometric average of the underlying prices follows a lognormal distribution as well, whereas with arithmetic average rate options, this condition is not possible. The payoff of geometric Asian options is given as:

$$Payoff_{Asian-Call} - \max\left(C\left(\prod_{i=1}^{n} S_i \right)^{1/n} - X \right)$$

$$Payoff_{Asian-put} - \max\left(C, X - \left(\prod_{i=1}^{n} S_i \right)^{1/n} \right)$$

The payoff of arithmetic Asian Options is given as:

$$Payoff_{Asian-call} = \max\left(0, \frac{\sum_{r=1}^{n} S_{ij}}{r} X\right)$$

$$Payoff_{Asian-put} = \max\left(0, X \frac{\sum_{r=1}^{n} S_{ij}}{r} X\right)$$

To expand on arithmetic averaging, this is seen as being the sum of the sampled asset prices divided by the number of samples:

$$Avg_A \frac{S_1 S_{2,.....,} S_n}{n}$$

And for geometric averaging, the average is taken.

$$Avg_G = \sqrt[n]{S_1 S_{2,.......} S_n}$$

Where the nth root of the sample value multiple together is taken. The payoff functions of Asian Options are given as

An average price Asian

$$V = Max(0, n(S_A - X))$$

And average strike Asian

$$V = Max(0, n(S_T - X_A))$$

Where n is a binary variable which is set to 1 for a call, and -1 for a put .Asia Options can be both European style and American style exercise.
The solution to the Geometric Asian Option for call and put is given as:

$$C_G = Se^{(b-r)(T-t)} N(d_1) - Xe^{-r(T-t)} N(d_2)$$

And

$$P_G = Xe^{-r(T-t)} N(d_2) - Se^{(b-r)(T-t)} N(d_1)$$

Where N(x) is the cumulative normal distribution function of:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (b + 0.5\sigma_A{}^2)T}{\sigma_A \sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + (b - 0.5\sigma_A{}^2)T}{\sigma_A\sqrt{T}}$$

Which can be simplified as? $d_2 = d_1 - \sigma_A\sqrt{T}$

The adjusted volatility and dividend yield are given as:

$$\sigma_A = \frac{\sigma}{\sqrt{3}}$$

$$b = \frac{1}{2}\left(r - D - \frac{\sigma^2}{6}\right)$$

σ is the volatility, r is the risk free rate of interest and D is the dividend yield.

2.2 Arithmetic Rate Approximation (Turnbull & Wakeman)

In 1991, Turnbull and Wakeman (TW) suggested an approximation by making use of the fact that the distribution under arithmetic averaging is *approximately* lognormal, and they put forward the first and second moments of the average in order to evaluate the option.

As there are no closed model solutions to arithmetic averages because of inappropriate use of the lognormal assumption under this form of averaging and a number of approximations have been put forward in literature. The analytical approximations for a call and under TW are given as

$$P_{TW} \approx Xe^{-rT_2} N(d_2) - Se^{(b-r)(T_2} N(d_1)$$

Where

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + (b - 0.5\sigma_A{}^2)T_2}{\sigma_A \sqrt{T}_2}$$

$$d_2 = d_1 - \sigma_A \sqrt{T_2}$$

Where T_2 is the time remaining up to expiry date. For average options which have already begun their averaging time, T is t (that is the original time to maturity), if the averaging time has not yet begun, T_2 became $T - \tau$.

The adjusted volatility and dividend yield are as:

$$\sigma_A = \sqrt{\frac{\ln(M_2)}{T} - 2b}$$

$$b = \frac{\ln(M_1)}{T}$$

To generalize the equations, we assume that the averaging time has not yet begun and given the first and second term moment as:

$$M_1 = \frac{e^{(r-D)\Gamma} - e^{(r-D)\Gamma}}{(r-D)(T-t)}$$

$$M_2 = \frac{2e^{(2(r-L)+\sigma^2)\Gamma}}{(r-D+\sigma^2)(2r-2q+\sigma^2)(T-t)^2} + \frac{2e^{(2(r-D)+\sigma^2)\Gamma}}{(r-D)(T-t)^2}\left(\frac{1}{2(r-D)+\sigma^2} - \frac{e^{(r-D)\Gamma}}{r-D+\sigma^2}\right)$$

If the averaging time has already begun, we must adjust the strike price as:

$$X_A = \frac{T}{T_2} X - \frac{(T - T_2)}{T_2} S_{Avg}$$

Where T is the original time to maturity, T_2 the remaining time to maturity, X the original strike price and S_{Avg} is the averaging asset price. Notes that if $r = D$ the formula will not give a solution and this was state by Haug in 1998.

2.3 Arithmetic Rate Approximation (Levy)

Another analytical approximation was puts forward by Levy which suggested giving more accurate results than the TW approximation. We shall look at the differences below. The approximation to a call is given as:

$$C_{Levy} \approx S_Z N(d_1) - X_Z e^{-rT_2} N(d_2)$$

And through put-call parity, we get the price of a put as:

$$P_{Levy} \approx C_{Levy} - S_Z + X_Z e^{-rT_2}$$

Where

$$d_1 = \frac{1}{\sqrt{K}}\left[\frac{\ln(L)}{2} - \ln(X_Z)\right]$$

$$d_2 = d_1 - \sqrt{K}$$

And

$$S_Z = \frac{S}{(r-D)T}(e^{-DT_2} - e^{-rT_2})$$

$$X_Z = X - S_{Avg}\frac{T-T_2}{T}$$

$$K = \ln(L) - 2[rT_2 + \ln(S_Z)]$$

$$L = \frac{M}{T^2}$$

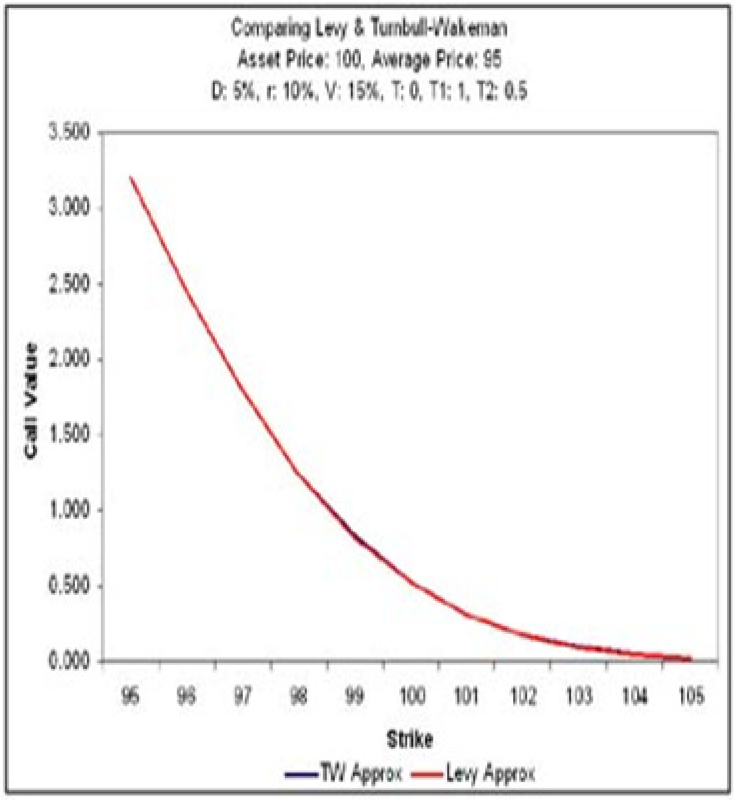
$$M = \frac{2S^2}{r-D+\sigma^2}\left\{\frac{e^{(2(r-D)+\sigma^2)T_2-1}}{2(r-D)+\sigma^2}\right\} - \frac{e^{(r-D)T_2}-1}{r-D}$$

Where the variables are the same as defined under the TW approximation.
For the various input, we compared the price an Asian call under the TW approximation to that of Levy approximation

Asset Price: 100, Average Price: 95
D: 5%, r: 10%, V: 15%, T: 0, T1: 1, T2: 0.5

X	TW Call Value	Levy Call Value	Absolute Error
95	3.202859	3.199390	0.0034690
96	2.444752	2.440545	0.0042066
97	1.787605	1.782873	0.0047318
98	1.246971	1.242086	0.0048849
99	0.827130	0.822518	0.0046122
100	0.520494	0.516509	0.0039841
101	0.310270	0.307114	0.0031558
102	0.175088	0.172788	0.0022995
103	0.093529	0.091982	0.0015470
104	0.047316	0.046352	0.0009645
105	0.022689	0.022130	0.0005593

We can observe that the absolute differences between the 2 approximations are very small, and that the two values can be said to be similar.



In addition, transposing the 2 Call Values as a function of the strike price give the similarity between the two methods.

2.4The partial-differential equation (PDE) approach

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The partial-differential equation (PDE) controlling the Asian option price *V(S, A, t) is*:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}+\frac{S-A}{t}\frac{\partial V}{\partial A}-rV=0$$

The discretely sampled arithmetic average value A at the *N*-th sampling time is:

$$A_N=\frac{1}{N}\sum_{i=1}^{i=N}S(t_i)$$

For the discretely sampled Asian option, the equation is the familiar Black-Scholes equation:

$$\frac{\partial V}{\partial t}+\frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0$$

Addition jump conditions across the sampling dates have to be satisfied:

1. The average value A satisfies the jump condition at the *N*-th sampling date:

$$A_{N-1}=A_N+\frac{1}{N-1}(A_N-S)$$

2. The option value *V(S, A, t)* satisfies the continuous condition:

$$V(S,A_{N-1},t^-)=V(S,A_N,t^+)$$

2.5 Binomial Method & Trinomial Trees

Asian options can be value using lattice/tree methods. At any point in time on the tree, the value of the option depends on the average of the price that the path has taken. Given the averaging nature of Asian options, a minimum and maximum range at each node must be determined, depending on the path which the underlying asset has taken. The problem is that as the number of nodes on a tree grows, so does the number of averages which must be taken, particularly in the central nodes this is because the number of averages to be taken is exponentially related to the number of possible asset paths.
In 1993, Hull & White attempts to solve this problem by adding a state variable to the tree nodes and approximation is undertaken with interpolation techniques in backward induction. The binomial tree can therefore be set up as:

1. The minimum and maximum averages at each time node can be determined as:

$$Average_{Min(i)} = \frac{\sum_{j-1}^{i} Sd^{j-1}}{i}$$

$$Average_{Max(i)} = \frac{\sum_{j-1}^{i} Su^{j-1}}{i}$$

Where u denotes the size of the up move and d denotes the size of a down move. i is the number of nodes.

2. The approximate average is calculated at each time node.
3. Payoff for each approximate average is determined through means of linear interpolation
4. Discount backwards towards the first time node:

$$V_{i-1k} = e^{-rT}(P_u V^u{}_{ik} + P_d V^d{}_{ik})$$

Where k = 1... A. and A is the first time node. P_u and P_d denote up and down probabilities of the binomial tree.
However, there is the problem that convergence is not guaranteed.
In 1997, Chalasani, Jha & Varikooty, came out with a method for the computation of the lower bounds of the Asian option with reasonable accuracy. Their work was base on Rogers & Shi 1995, the authors use a modified choice of random variable z used to estimate the conditional expectation of the option payoff. An improved lower bounds is given, which can be programmed using a binomial lattice.

2.6 Finite Differences Method

In the early mid 90s, several of papers documenting the use of finite differences to solve Asian options where published. Rogers & Shi in 1995, present a method using a one dimensional PDE which can be solved using finite differences. However, their method is prone to problems associated with the diffusion term, especially with lower volatilities and short time to maturities. Andreasen in1998 expand on Rogers & Shi's model by using a change of numeraire to solve the price of Asian options numerically.
Example of finite differences methods and the application towards Asian options can also be found in Tavella & Randall in 2000.

2.7 Arithmetic Rate Approximation (Monte Carlo Simulation)

Several methods using Monte Carlo simulation (MCS) have been developed to price arithmetic Asian options. The analytical approximations by TW, Levy and Curran can all be computed using a simulation method. Monte Carlo simulation can give relatively accurate prices for pricing option, and in the case of Asian options, which are highly path dependent, this method is particularly useful.
The control variant technique can be used to find more accurate analytical solutions to a derivative price if there is a similar derivative with a known analytic solution. With this in mind, MCS is then undertaken on the two derivatives in parallel.
Given the price of the geometric Asian, we can price the arithmetic Asian by considering the equation:

$$V_A = V_A^* - V_B^* + V_B$$

The estimated value of the arithmetic Asian through simulation is V_A and V_B is the simulated value of the geometric Asian, and V_B is the exact value of the geometric Asian given above.

A Case Study in Electricity Markets

The price behavior is extremely volatile in electricity markets. In economics and finance, volatility is **established** in the risks associated with holding assets when there is an uncertain risk associated with the future value of the assets. In competitive electricity markets, pricing are not gradually deregulated, being determined by market participants for each specific interval of the day (e.g. every day ahead), while taking into account various economic and operational factors. In addition, electricity market responds to underlying price drivers that differ dramatically from interest rates and other well developed money markets. Most financial instruments in money markets are traded electronically today. However, electricity responds to the active interchange between producing and using, transferring and storing, buying and selling, and finally consuming actual physical products. In the electricity markets, the supply side concerns not only the storage and transfer of the actual commodity, but also how to get the actual commodity out of the ground. On contrast, the final users consume the commodity with different perspectives. Residential users need power for heating in the winter and cooling in the summer, and industrial users consume energy in order to keep their own production running to avoid the high costs of shutting down and restarting. All market participants have different initiatives with regards to the pricing and determined by different fundamental engine drivers, which in turn significantly affect the behavior of energy markets, causing extremely complex price behavior. These problems lead directly to the need of derivatives contracts. In this study, we are concern on the application of Asian style options in electricity market to discuss how a business hedged or speculate the electricity volatility by using the average price options.

A company is highly exposed to volatility in the cost of electricity. An Asian option is applied based on the average price of a kilowatt hour (or other underlying commodity) over a particular period of time.

A target price of $0.059 was set up based on the past 3-year average price of electricity (Appendix 1). Specifically, the contract terms are as following

- If the average price per kilowatt hour during the next twelve months is greater than this target price, then the counterparty will pay the company the difference.
- If the average price per kilowatt hour during the next twelve months is less than this target price, then the company loses the price it paid for the option.

The question of interests is what the fair price for 1-million kWh per options is.

In this study, we applied Monte Carlo Simulation using Crystal Ball version 7.2 to obtain the proposed option price in one year scenario.

The historical behavior of electricity prices determines the starting point of the modeling process. The model will be based not on the actual prices, but on monthly percent changes in price (or return). To examine the theoretical probability distribution approximating the empirical distribution in these data, the histogram was making as it is shown in Figure 1.

Figure 1

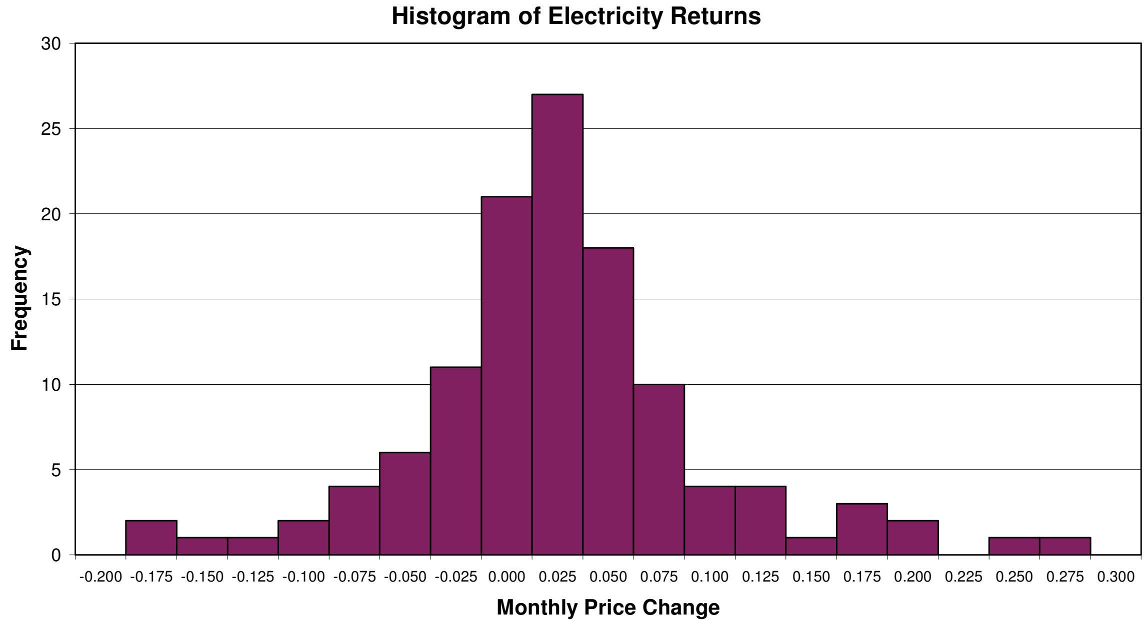
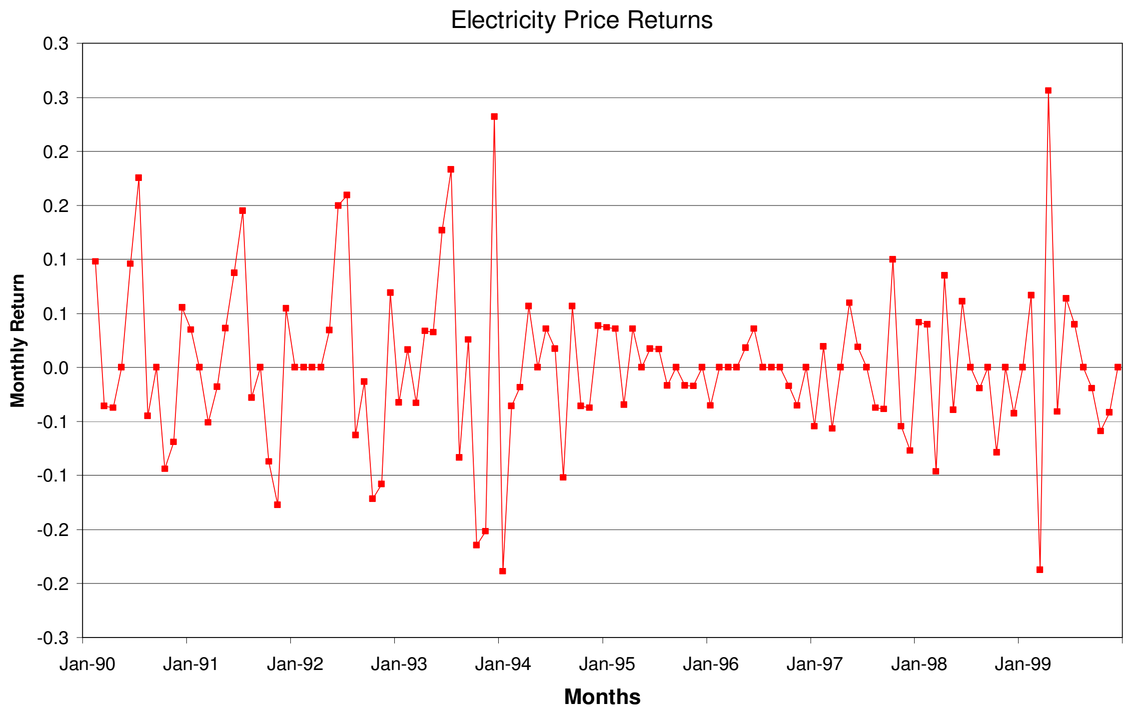


Figure 2

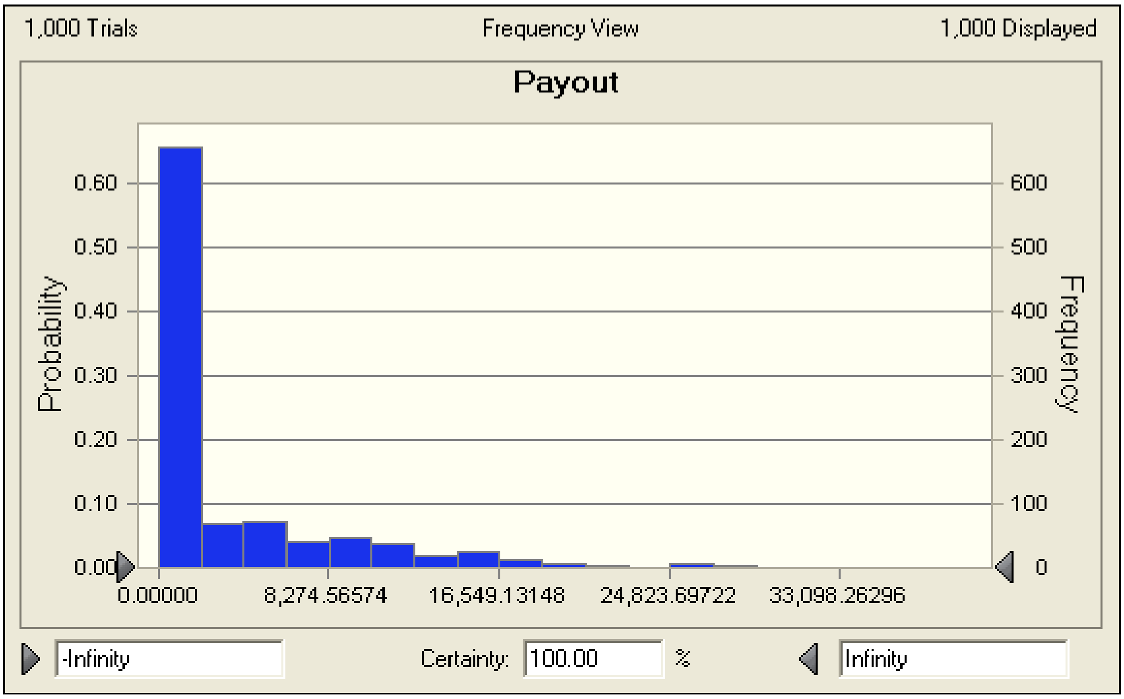


It would appear that the percent price changes are approximately normally distributed, and the sample mean and sample standard deviation from these data (0.001768 and 0.073462, respectively) were applied as the mean and standard deviation of the input random variable for the model.

3.1 Empirical Results

The chart frequency indicates that the option is frequently insignificant or **meaningless** (as evidenced by the tall bar at zero), but that the payout is occasionally $20,000 or more. Remember that the units here are millions of kilowatt hours

Figure 3



To evaluate a fair price, the piece of the simulated sample mean of approximately $3,183 per million kWh (Table 1) is used to calculate the Asian option price as it is shown in Table 2. We are 95% confident that the true fair price is somewhere between $2,843.71 and $3,523.51, which seems like a very wide interval. We could narrow the interval around our estimate by running a longer simulation.

Table 1: Statistics Output from Crystal Ball Simulation

Statistic	Fit: Beta distribution	Forecast values
Trials		1,000
Mean	3,183,61	3,183,61
Median	826,61126	0
Mode		0
Standard Deviation	5,481,86	5,484,60
Variance	30 050 773,59	30 080 854,45
Skewness	2.19	2.19
Kurtosis	8.51	8.49
Coeff. of Variability	1.72	1.72
Minimum	– 697,40098	0
Maximum	47,488.53	39,398.75
Mean Std. Error		173,43833

Statistically, a 95% confidence interval is given by:

Table 2 Asian Option Price Calculation

$\overline{X}$	$\pm 1.96 \frac{s}{\sqrt{n}}$
\$3,183,61	$\pm 1.96 \frac{\$5,484.60}{\sqrt{1,000}}$
	$\pm 1.96(\$171.67)$
	$\pm \$339.9$

CONCLUSION

Asian option may be more expensive than the standard option (e.g., options on currencies or oil spreads), and a simple, closed form expression of the Asian option price when the option is in the money. The various methods of analyzing Asian Option have made the instrument popular and easy to use.

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APPENDIX

Month	$/kwh	Month	$/kwh	Month	$/kwh
Jan-90	0.0510	May-93	0.0630	Sep-96	0.0580
Feb-90	0.0560	Jun-93	0.0710	Oct-96	0.0570
Mar-90	0.0540	Jul-93	0.0840	Nov-96	0.0550
Apr-90	0.0520	Aug-93	0.0770	Dec-96	0.0550
May-90	0.0520	Sep-93	0.0790	Jan-97	0.0520
Jun-90	0.0570	Oct-93	0.0660	Feb-97	0.0530
Jul-90	0.0670	Nov-93	0.0560	Mar-97	0.0500
Aug-90	0.0640	Dec-93	0.0690	Apr-97	0.0500
Sep-90	0.0640	Jan-94	0.0560	May-97	0.0530
Oct-90	0.0580	Feb-94	0.0540	Jun-97	0.0540
Nov-90	0.0540	Mar-94	0.0530	Jul-97	0.0540
Dec-90	0.0570	Apr-94	0.0560	Aug-97	0.0520
Jan-91	0.0590	May-94	0.0560	Sep-97	0.0500
Feb-91	0.0590	Jun-94	0.0580	Oct-97	0.0550
Mar-91	0.0560	Jul-94	0.0590	Nov-97	0.0520
Apr-91	0.0550	Aug-94	0.0530	Dec-97	0.0480
May-91	0.0570	Sep-94	0.0560	Jan-98	0.0500
Jun-91	0.0620	Oct-94	0.0540	Feb-98	0.0520
Jul-91	0.0710	Nov-94	0.0520	Mar-98	0.0470
Aug-91	0.0690	Dec-94	0.0540	Apr-98	0.0510
Sep-91	0.0690	Jan-95	0.0560	May-98	0.0490
Oct-91	0.0630	Feb-95	0.0580	Jun-98	0.0520
Nov-91	0.0550	Mar-95	0.0560	Jul-98	0.0520
Dec-91	0.0580	Apr-95	0.0580	Aug-98	0.0510
Jan-92	0.0580	May-95	0.0580	Sep-98	0.0510
Feb-92	0.0580	Jun-95	0.0590	Oct-98	0.0470
Mar-92	0.0580	Jul-95	0.0600	Nov-98	0.0470
Apr-92	0.0580	Aug-95	0.0590	Dec-98	0.0450
May-92	0.0600	Sep-95	0.0590	Jan-99	0.0450
Jun-92	0.0690	Oct-95	0.0580	Feb-99	0.0480
Jul-92	0.0800	Nov-95	0.0570	Mar-99	0.0390
Aug-92	0.0750	Dec-95	0.0570	Apr-99	0.0490
Sep-92	0.0740	Jan-96	0.0550	May-99	0.0470
Oct-92	0.0650	Feb-96	0.0550	Jun-99	0.0500
Nov-92	0.0580	Mar-96	0.0550	Jul-99	0.0520
Dec-92	0.0620	Apr-96	0.0550	Aug-99	0.0520
Jan-93	0.0600	May-96	0.0560	Sep-99	0.0510
Feb-93	0.0610	Jun-96	0.0580	Oct-99	0.0480
Mar-93	0.0590	Jul-96	0.0580	Nov-99	0.0460
Apr-93	0.0610	Aug-96	0.0580	Dec-99	0.0460