Monte Carlo Simulation European Option Price

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Abstract

In this project, Monte Carlo Simulation is examined by putting its use in European Options into practice by help of MS Excel and Visual Basic.

First of all, history of Monte Carlo method is discussed. And the main fields that Monte Carlo Simulation can be used and its applications in financial engineering are explained.

Secondly, a brief introduction to the model that we used for European options is made. After the introduction, transition of the model to MS Excel is provided. Algorithms are created in Visual Basic in order to create a connection between user inputs and excel, assign random variables to the model by help of the functions and provide a user friendly background in excel.

Finally, payoffs for both call and put options are obtained from MS Excel. The results are analyzed and compared. Necessary graphs that can visualize data and results are prepared. A conclusion for the application of Monte Carlo Simulation in European options is made.

1 Introduction

Monte Carlo was named and popularized by physicists Stanislaw Ulam, Enrico Fermi, John von Neumann, and Nicholas Metropolis. The most famous early use of monte carlo simulation was in random method that calculates the properties of neutron by Enrico Fermi. When the electronic computers were built Monte Carlo methods were studied carefully. It was used for an early work in development of hydrogen bomb and Monte Carlo Methods become popular in physics, physical chemistry operations research fields.

Monte Carlo methods are generally used in systems where significant number of coupled degrees of freedom exists. In other words, Monte Carlo methods are used in modeling with important amount of uncertain inputs such as calculating risk in business and evaluating multidimensional integrals under complex boundary conditions. Additionally, Monte Carlo method plays a significant role in computational physics. It has applications for quantum chromodynamics calculations and it is used in designing of heated shields and aerodynamic forms.

In financial engineering, Monte Carlo simulation is used to value and study complex portfolios and instruments by simulation of uncertain variables that affects their values by deciding functions outcome over a set of results. Monte Carlo can be used in valuing options and equity, valuing bonds and portfolio evaluating.

2 European Option Price Using Monte Carlo Simulation

2.1 European Option

European Option is an option that can only be exercised at the end of its life. In other words, you must ride the rollercoaster until the maturity date, and only then can you cash in.

2.2 Monte Carlo Simulation

We divide the maturity T into several intervals,and this is the formula to calculate the stock price

$$S_{t_i} = S_{t_{i-1}} e^{(r-\sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon}$$

The payoff function of the call option is given by

$$C_T = \max\{0, S_T - K\}$$

And the payoff function of the put option is given by

$$P_T = \max\{0, K - S_T\}$$

Where the r is the interest rate, σ is volatility, ε is the random number,K is the strike price,T is the maturity.

Then,we can extimate the premium of the european option by using the following function.

$$f = e^{-rt} E^Q[f_T]$$

2.3 Simulation by Excel/VBA

In European option pricing formula that is created by using monte carlo

simulation: underlying price of asset, strike price, volatility, risk free interest rate, time to option maturity and number of simulations are used as inputs for gaining the payoff values for both call on put option cases. Random value assigning functions are defined in Visual Basic in order to obtain different results for every simulation.

First of all, necessary functions are defined in a module that can be used while defining the main function that will give results of the simulation. "GetMean" and "GetVariance" functions are defined in order to obtain mean and variance of series of numbers. "GetMinimum" and "GetMaximum" functions are defined in order to calculate max and minimum of set of numbers.
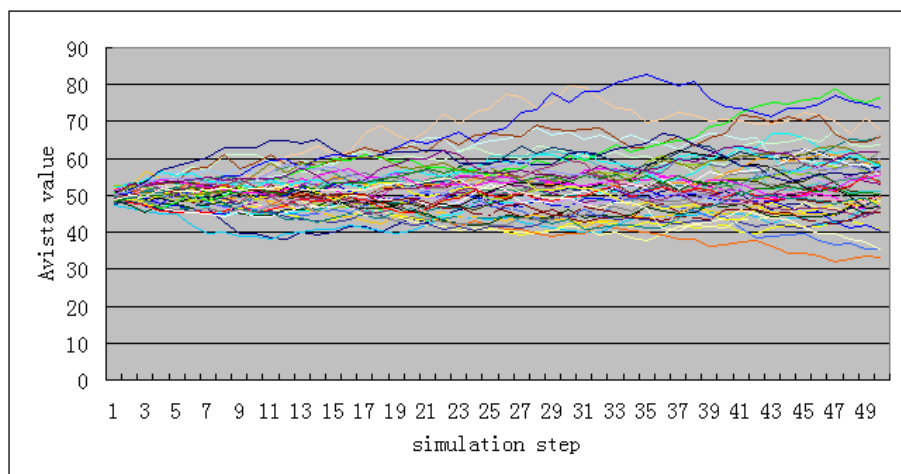
Secondly, another module for the main function is defined in order to provide randomness in European option pricing formula. A function that allocates random numbers called "NRandVars" which takes its input from the number of simulations that we will have, is defined.

Thirdly, the main program is defined in visual basic in order to connect the modules that are created with European option pricing function, and obtain necessary results and connect it with excel interface. First, we define call price, put price and probability to reach the strike as our unknown variables. Secondly, we enter our inputs which are underlying price of asset, strike price, volatility, risk free interest rate, time to option maturity and number of simulations by providing connection between the excel sheet and visual basic application. After defining inputs, the formulation of European option pricing model in Visual Basic is made by connecting necessary functions module and random variable function module into

the option pricing model. Finally, the results that we obtained from pricing function are assigned to the variables that we defined as unknown such as put price and they are connected to excel interface in order to illustrate the results.

Application Interface

In conclusion, the price of call and putoption and probability to reach the price can be obtained from excel by enter necessary inputs that are mentioned before and pressing calculate button in interface. Additionally, a histogram is prepared in order to visualize avista value vs. simulations step in excel.

3 Summary

Monte Carlo simulation is a popular method for pricing financial options., and it is a technique that involves using random numbers and probability to solve problems.

We use Monte Carlo Methods to construct "stochastic" or probabilistic financial models as opposed to the traditional static and deterministic models. In valuing an option on equity, the simulation generates several thousand possible (but random) price paths for the underlying share, with the associated exercise value (i.e. "payoff") of the option for each path. These payoffs are then averaged and discounted to today, and this result is the value of the option today.

Although Monte Carlo methods provide flexibility, and can handle multiple sources of uncertainty, the use of these techniques is nevertheless not always appropriate. In general, simulation methods are preferred to other valuation techniques only when there are several state variables (i.e. several sources of uncertainty). These techniques are also of limited use in valuing American style derivatives.

After several weeks hard working, our team successfully use Monte Carlo simulation to price the European Option. Although there is still a distance between our simulation and real financial world, I believe that we have tried our best. Thanks to our team members and thanks for your time.

4 References

http://en.wikipedia.org/wiki/Monte_Carlo_methods_in_finance

http://en.wikipedia.org/wiki/Monte_carlo_simulation#History