Mälardalens Högskola

Analytical Finance

Supervisor: Jan Röman

Subject:

Valuation of Asian basket digital option

Team members:

Ewa Lukasik
Jakub Lawik
Juan Mojica
Xiaodong Xu

Västerås 2008

1. Introduction

The area of consideration in this report is the valuation of Asian basket digital option. The pricing process was performed by means of the Monte Carlo simulation. It is one of the most widely accepted numerical methods, used in quantitative finance. With the development of computer hardware Monte Carlo simulation is predicted to raise in popularity, however, its biggest shortcoming is still long computational time.

Underneath we list detailed conditions of the analyzed contract:

Starting date: 13.10.2008

Expiration date: 13.11.2008

Currency: SEK

Notional amount: 1 SEK

Payoff of our contract is based on the change of the value of three stock indices:

- Nikkei 225 *(i=1)*
- FTSE 100 *(i=2)*
- DJIA *(i=3)*

$$\Pi(T) = \begin{Bmatrix} 1 \; if \; S^* \geq 1 \\ 0 \; if \; S^* < 1 \end{Bmatrix}$$

Where

$$S^* = \frac{1}{3}\sum_{i=1}^{3} R_i$$

$$R_i = \frac{\frac{1}{24}\sum_{t=1}^{24} S_{it}}{S_{i0}}$$

$S_i(t_j)$ - Value of the *i-th* index at the end of the *t-th* trading day (excluding S_{i0})

$S_i(0)$ - Value of the *i-th* index at the opening of the market on 13.10.2008

R_i – Ratio of the mean index to beginning value

S^* - indicator of the relative change of the values of indices during the contract period

i - Number of the index

t_j - Number of the trading day

Trading days are numbered from 1 to 24, where the number 1 is 13.10.2008 and the 24 is 13.11.2008

All the index values for our calculation in the contract payoff (except $S_i(0)$) are taken as the closing prices for specified date.

$S_i(0)$ - being accounted as values of *i-th* index the opening of the market on 13.10.2008

2. Theoretical background

1. Monte Carlo simulation

In finance, there is an amount of uncertainty and risk relative with estimating the future value of figures due to the wide variety of potential outcomes. Monte Carlo simulation is one technique that helps to simplify the model with uncertainty involved in estimating future outcomes. Monte Carlo simulation can be applied to complex, non-linear models or used to evaluate the accuracy and performance of other models.

To apply Monte Carlo simulation to estimate a financial value, there are typically three steps: generating sample paths, evaluating the payoff along each path and calculating an average to obtain estimation.

For example:

For a risk-neutral environment, the value of the derivative security is the discounted value of its future date cash flow:

$$\Pr ice = e^{-rT} E^{Q} \left[f\left(S_0, \cdots, S_T \right) \right].$$

Monte Carlo simulation approximates the expectation of the derivative's future cash flows with one simple arithmetic average of cash flows taken over a finite number of simulated price path:

$$\Pr ice = e^{-rT} \left[\frac{1}{N} \sum_{n=1}^{N} f\left(S_0, \cdots, S_T \right) \right].$$

The mean of the sample will be quite close to accurate price of derivate in a large sample of simulated price paths. And the rate of convergence is 1/√N.

Unfortunately, the use of random numbers yields an error bound which is probabilistic and the rate of convergence is slow. Therefore, high accuracy requirements may lead to long computation times.

2．Valuation issues

i. Choosing process for the underlying asset

Underlying assets (indices values) are assumed to follow geometrical Brownian motion. In the following sections, the model is going to be developed to incorporate additional features of the contract.

We begin with:

$$d(logS_i) = \mu_i dt + \sigma_i dW$$

Where:

$d(logS_i)$ - change in the natural logarithm of *i-th* asset's value

μ_i - drift rate for *i-th* asset

σ_i - volatility of *i-th* asset

dt - time increment

dW - Wiener process

Then to obtain process which is martingale after discounting, we set drift rate μ_i to $(r - \frac{\sigma^2}{2})$, as a result:

$$d(logS_i) = (r - \frac{{\sigma_i}^2}{2})dt + \sigma_i dW$$

r - risk free rate

Hence, we obtain the following form of geometric Brownian motion for index value process:

$$S_i(t) = S_i(0)e^{\left(r - \frac{{\sigma_i}^2}{2}\right)dt + \sigma_i dW}$$

The advantage of such a process for Monte Carlo simulation is that simulated values are accurate, so we can simplify our computation to a reasonable level. Some other processes require us to simulate path divided into large numbers of intervals for obtaining required level of accuracy. In considered case, it leads to 24 simulated time steps.

ii. Incorporation of dividends in the model

Characteristic feature of stocks or stock indices is dividends received during the holding period. Dividends reduce cost of holding asset physically. Hence, introducing them into no arbitrage argument leads to slight changes in the index/stock price process. Two basic methods used for incorporation of dividends are:

a) Reduction of asset price by value of forecasted dividends.

Dividend dates and dividend amounts have to be forecasted to use this method.

$$S_i(t_j) = S_i(0)e^{\left(r - \frac{{\sigma_i}^2}{2}\right)dt + \sigma_i dW} - \sum_{j=1}^{t} D_j e^{r(t-j)dt}$$

D_j – Value of dividend payment at time t_j

b) Reduction of asset price by dividend yield q

$$S_i(t_j) = S_i(0)e^{\left(r-q_i-\frac{{\sigma_i}^2}{2}\right)dt+\sigma_i dW}$$

q_i - dividend yield for *i-th* security

First method is applicable to single stocks as it is possible to assess timing and amount of future dividends. In mature financial markets, many companies follow stable dividend policy, which enables dividends forecasting with high level of accuracy.

Second method is much easier to implement from mathematical and analytical point of view. As we assume continuous dividend streams, proportionate to the value of the stock/index, there is no need to predict dividend dates. Here dividends are incorporated into the formula in a way that does not influence its mathematical properties. Significant discrepancies between assumptions underlying this method and dividend policies observable in the market make it rarely used for single stocks. However, this method copes well with describing behavior of the index. As index consists of many different stocks, dividends are paid frequently enough to justify continuous dividend payout assumption. Additionally, it was noticed that dividend yield matches pretty well with real dividend payouts in longer periods (months, quarters and years).

Basing on our previous arguments, we decided to employ the second method as it is better suited with dividends on indices.

iii. Currency interdependence (quanto)

Contracts which payoff is calculated in one currency, but underlying asset is quoted in the country using another currency are called quantos. In this way, they are somehow dependent on the exchange rate movement and the risk free rate in the foreign country. To implement this special feature which is present in the contract under analysis (we have our payoff in SEK but underlying assets are quoted in USD, JPY and GBP environments respectively), we need to change our price processes to:

$$S_i(t) = S_i(0)e^{\left(r - q_i - (r - r_f - cov(S_i, Ex_i)) - \frac{\sigma_i^2}{2}\right)dt + \sigma_i dW}$$

That reduces to:

$$S_i(t) = S_i(0)e^{\left(r_{fi} - q_i - cov(S_i, Ex_i) - \frac{\sigma_i^2}{2}\right)dt + \sigma_i dW}$$

$cov(S_i, Ex_i)$ - covariance between logarithmic return on *i-th* asset and logarithmic return on the currency in which *i-th* asset is denominated (in case of an index – in which currency are denominated securities included in an index)

r_{fi} - risk free rate in the country of the *i-th* asset

iv. Variance – Implied Vs historical volatilities

Input in our option valuation model include risk free rate, price of the underlying asset, strike price (or other limit implying when it is optimal to exercise), time to expiry and variance (there would be also covariances in our case). All the parameters excluding last one are observable on the market. But we never know the value of the variance (and covariances). This causes a need for estimation of volatilities and correlation parameters. There are several approaches:

a)Variance is constant in time.

Contra intuitively constant volatility is more efficient for longer time periods (quarters, years). It does not mean that volatility tends to be stable in the long run, but rather that other methods will not provide us with better estimate, while would significantly increase computational time and complication of the model.

We can distinguish two popular approaches under assumption of constant volatility in time.

- Volatility calculated from historical data

The simplest method, assumes that volatility in the future will be the same as it was in the past. Our estimation of volatility is just sample standard deviation from previous period. Estimating volatility for pricing purposes, we usually use data from recent period of equal length to the duration of the option, e.g. to value option with 3 months to expiry, data from recent 3 months would be used.

Formula:

$$S(R) = \frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2 = \frac{1}{n-1}\sum_{i=1}^{n} R_i^2 - (\frac{1}{n-1}\sum_{i=1}^{n} R_i)^2$$

$$R_i = \ln\left(\frac{X_i}{X_{i-1}}\right) = lnX_i - lnX_{i-1}$$

Where:

R_i- logarithmic rate of return for *i-th* period

X_i- price of security (value of the index) at time *i*

- Volatility implied from other derivative contracts traded on the market(recheck sentences below carefully please)

Under assumption of validity of the option pricing formulas, we can imply what level of the volatility is perceived by the market participants. Implied volatility is the value that calibrates the model (e.g. Black-Scholes), so that it yields current market price of the instrument. Rationale for using implied volatility is that price of volatility should be the same for all traded assets. Using other volatility than the market, means that you are buying (selling) uncertainty on other conditions than the market. This leads to arbitrage opportunities.

Remark: There is no exact analytical formula for implied volatility (or covariance). Values are obtained by means of numerical algorithms.

b) Variance is stochastic

It is true that volatility is unstable in time, but patterns which volatility follows are still under discussion. One of the findings, which is supported by the majority of market analysts and researchers is called "volatility clustering". It describes following property: period with high (low) volatility is usually followed by a period with high (low) volatility.

For that reason most popular models for stochastic volatility incorporate this autoregressive property. Examples of such models are ARCH and GARCH.

Those methods are especially successful in modeling volatility of short term contracts.

According to arguments above, GARCH model would be best suited for valuation of the analyzed option, but due to simplicity of the model we did not employ idea of stochastic volatility. Pricing of the option was performed with estimates based on historical data.

v. Multi-asset price process

In the case of the basket option, we need to model more than one price process. If those price processes are independent or even uncorrelated, we could simulate them just by generating random realizations of Wiener process and applying them to the previously obtained formula:

$$S_i(t) = S_i(0)e^{\left(r-q_i-(r-r_f-cov(S_i,Ex_i))-\frac{\sigma_i^2}{2}\right)dt+\sigma_i dW} = S_i(t) = S_i(0)e^{\left(r_f-q_i-cov(S_i,Ex_i)-\frac{\sigma_i^2}{2}\right)dt+\sigma_i dW}$$

In the financial world there are thousands of reciprocal relations between different markets. Hence, behaviors of indices are correlated. To implement this correlation into our model we will use method called Cholesky decomposition.

Instead of generating Wiener process in usual manner:

$$dW = \sqrt{dt}\epsilon$$

Where:

ϵ - random number from standard normal distribution

We will obtain $dW = \sqrt{dt}\varphi_i$ where φ_i's are correlated normally distributed variables [N(0,1)]

φ_i's are values in the *i-th* row of the column vector $\boldsymbol{\varphi} = \boldsymbol{M\epsilon}$

$\boldsymbol{M}$ **–** matrix satisfying $\boldsymbol{MM}^T = \boldsymbol{\Sigma}$**,** where $\boldsymbol{\Sigma}$ is correlation matrix

$\boldsymbol{\epsilon}$ **–** column vector of uncorrelated normally distributed variables [N(0,1)]

3. Final model

i. Formulas for valuation part:

According to arguments above, and switching from continuous to discrete time, value of the asset *i* at day τ equals to:

$$S_i(t_\tau) = S_i(0)e^{\left(r_{fi} - q_i - cov(S_i, Ex_i) - \frac{\sigma_i^2}{2}\right)\Delta t\tau + \sum_{j=1}^{\tau} \sigma_i \sqrt{\Delta t}\varphi_{ij}}$$

$S_i(t_\tau)$ - value of *i-th* asset on day τ

r_{fi} - daily risk free rate for currency of asset *i*

q_i – daily dividend yield for asset *i*

$cov(S_i, Ex_i)$ –covariance of log returns on the *i-th* asset with log returns on the *i-th* currency

σ_i - variance of log returns on the *i-th* asset

t_τ – number of the day when measurement takes place

Δt - length of the time step [counted in days, set to one day]

φ_{ij} - random number generated by means of Cholesky decomposition for *i-th* asset on *j-th* day

Processing further, average value of the *i-th* asset on day *T* is calculated as

$$\bar{S}_i(T) = \frac{1}{T}\sum_{\tau=1}^{T} S_i(0)e^{\left(r_{fi}-q_i-cov(S_i,Ex_i)-\frac{\sigma_i^2}{2}\right)\Delta t\tau+\sum_{j=1}^{\tau}\sigma_i\sqrt{\Delta t}\varphi_{ij}}$$

$^{-}$- average value of *i-th* asset from *T* days at day *T*

Defining relative value as $R_i(T) = \frac{\bar{S}_i(T)}{S_i(0)}$

$R_i(0)$ - relative value

We obtain:

$$R_i(T) = \frac{\bar{S}_i(T)}{S_i(0)}\frac{1}{T}\sum_{\tau=1}^{T} e^{\left(r_{fi}-q_i-cov(S_i,Ex_i)-\frac{\sigma_i^2}{2}\right)\Delta t\tau+\sum_{j=1}^{\tau}\sigma_i\sqrt{\Delta t}\varphi_{ij}}$$

Then defining payoff indicator as an average of relative values of *n* assets

$$S^* = \frac{1}{n}\sum_{i=1}^{n} R_i(T) = \frac{1}{n}\sum_{i=1}^{n}\frac{\bar{S}_i(T)}{S_i(0)}\frac{1}{T}\sum_{\tau=1}^{T} e^{\left(r_{fi}-q_i-cov(S_i,Ex_i)-\frac{\sigma_i^2}{2}\right)\Delta t\tau+\sum_{j=1}^{\tau}\sigma_i\sqrt{\Delta t}\varphi_{ij}}$$

S^* - payoff indicator

Hence, the payoff of the option from the simulation is

$$\Pi(T) = \begin{cases} 1 \; if \; S^* \geq 0 \\ 0 \; if \; S^* < 0 \end{cases}$$

After discounting payoff with the risk free rate, we obtain option value:

$$ValOpt = e^{-rT} \times \Pi(T)$$

$ValOpt$ - value of the option obtained from a single simulation

Simulating option value K times we obtain our estimate of the option fair price with the expression:

$$\Pi(0) = e^{-rT} \times E^Q[\Pi(T)] \approx \frac{1}{K}\sum_{k=1}^{K} ValOpt_k$$

$\Pi(0)$ - fair price of an option as obtained from K simulations

$E^Q[]$ – expectation value under risk-neutral probability measure

K - total number of performed simulations

k - simulation number

$ValOpt_k$ - Value of the option obtained in *k-th* simulation

We calculate variance of obtained results as:

$$V_K = \frac{1}{K}\sum_{k=1}^{K}(ValOpt_k - \Pi(0))^2 = \frac{1}{K}\sum_{k=1}^{K} ValOpt_k^2 - \Pi(0)^2$$

V_K - Variance of option values obtained in *K* simulations

Hence, standard error of *K* simulations is

$$SE_K = \sqrt{\frac{V}{K}}$$

SE_K - standard error of *K* simulations

ii. Probabilities

Additionally to pricing option contract, we obtained estimates for probabilities of reaching the strike under both physical and risk-neutral measure. Drift rates used to approximate physical probability, were estimated on the same data set as variances and correlation matrix.

Such an estimate of real probability can be used by both issuer and buyer of the option:

- Issuer of the option can assess what is the probability that option will eventually expire in the money. Moreover issuer can perform simulation for his entire portfolio with and without issued option. Thus, examine influence of this contract on the entire portfolio, which is of higher importance than gain or loss realized on a single instrument.
- Buyer of the option can conduct similar simulation for his portfolio. Obtained results could help to evaluate optimal amount of the instrument to be included in the portfolio, accordingly to buyer risk and return objectives.

We calculated estimates of the real drift rate according to the formula

$$\mu_i = \frac{1}{N-1}\sum_{j=1}^{N} R_{j,S_i}$$

μ_i – estimate of the drift rate of the *i-th* asset under physical probability measure

R_{j,S_i} - logarithmic rate of return for *j-th* observation of *i-th* index

N - number of observations (here 24)

iii. Greeks

Greek parameters were by conducting additional simulations. Use of parameters obtained from Monte Carlo simulation should be always done with care.

Approximation error is implicitly included in numerical procedures. Thus, for instance, running two sets of simulations to calculate delta, we obtain two results with two approximation errors. In extreme cases, it can lead to result such as negative delta for a call option.

However, there are some advantages of simulating exponential Brownian motion related to greeks. First, we can obtain values necessary to calculate delta and gamma just by multiplying the price obtained in the last step by change factor (1+ϵ) or (1-ϵ). Hence, we can include their calculation into the main simulation. It saves computational time. Second advantage of geometric

Brownian motion is even more important. To minimize simulation errors while estimating Greeks, we calculate both values S(1+ϵ) and S(1-ϵ) for one set of random numbers. This method results in simulation errors offsetting each other. This leads to more accurate estimates. We employed this procedure in our calculations of delta and gamma.

Formulas used for calculation of Greeks are presented below. In the formulas we used shorthand notation $FairPrOpt(S_{1,0}(1+\epsilon))$ for

$$\Pi(S_{1,0}(1+\epsilon), S_{2,0}, S_{3,0}, r, r_{f1}, r_{f2}, r_{f3}, K, \sigma_1, \sigma_2, \sigma_3, \Sigma, cov(S_1, Ex_1), cov(S_2, Ex_2), cov(S_3, Ex_3), q_1, q_2, q_3, t)$$

It means that other parameters are held constant.

Delta:

Delta is calculated for ϵ=1 unit of index value

$$\Delta_i = \frac{\Pi(S_{i0}+\epsilon) - \Pi(S_{i0}-\epsilon)}{2\epsilon}$$

Δ_i – delta with respect to *i-th* asset /sensitivity of the option's price to change in *i-th* underlying asset price

Gamma:

Gamma is calculated for ϵ=1 unit of index value

$$\Gamma_i = \frac{\Pi(S_{i0}+\epsilon) + \Pi(S_{i0}-\epsilon) - 2\Pi(S_{i0})}{\epsilon^2}$$

Γ_i – gamma with respect to *i-th* asset /sensitivity of the option's *i-th* delta to change in *i-th* underlying asset price

Vega:

Vega is calculated for ϵ=1%, but in a yearly volatility. That is $\epsilon_d = \frac{1\%}{\sqrt{252}}$ where

ϵ_d – daily change in volatility

$$\nu_i = \frac{\Pi(\sigma_i + \epsilon) - \Pi(\sigma_i - \epsilon)}{2\epsilon}$$

ν_i - vega with respect to *i-th* asset /sensitivity of the option's price to change in *i-th* underlying asset volatility

iv. Inputs:

Stated model requires several inputs.

Risk free rates

Risk free rates corresponding to Nikkei 225, FTSE 100 and DJIA in the model are respectively: Japanese 3 month government bond yield, British 6 month government bond yield and US 3 month Treasury bill.

Risk free rates were converted to continuous compounding, and scaled to one day time step by employing formula

$$r = \frac{1}{252} \ln\left(1 + r^o\right)$$

r – continuously compounded rate after scaling

r^o – rate observable on the market, used as a proxy for instantaneous rate

Rates were scaled to trading days, as the simulation is entirely performed with time steps equal to one trading day. Weekends and so called effect of Monday were not modeled in the simulation. The only rate treated accordingly to typical market manner was Swedish risk free rate used for discounting option payoff. Those calculations were performed with 30/360 day count convention.

The rates used in the project are rates at 05.10.2008.

r_{fi} - daily risk free rate for currency of asset *i*

Dividend yield

www.bloomberg.com. The values in the stated sources are yearly dividend yields which were scaled to daily values by dividing them by assumed number of 252 trading days.

q_i – daily dividend yields for these assets are quoted from

Covariances

First logarithmic returns were calculated for DJIA, Nikkei 225, FTSE 100 and US Dollar, Japanese Yen and British Pound respectively. Then the covariance between returns on the corresponding indexes and exchange rates were computed according to the formula

$$cov(S_i, Ex_i) = \frac{1}{N-1}\sum_{j=1}^{N}(R_{j,S_i} - \bar{R}_{S_i})(R_{j,Ex_i} - \bar{R}_{Ex_i})$$

$cov(S_i, Ex_i)$ –covariance of log returns on the *i-th* asset with log returns on the *i-th* currency

N - number of observations (here 24)

R_{j,S_i} - logarithmic rate of return for *j-th* observation of *i-th* index

R_{j,Ex_i} - logarithmic rate of return for *j-th* observation of *i-th* currency

$\bar{R}_{S_i}$ - average of logarithmic rates of return for *i-th* index

$\bar{R}_{Ex_i}$ - average of logarithmic rates of return for *i-th* currency

Averages were calculated according to the formula:

$$\bar{A} = \frac{1}{N-1}\sum_{j=1}^{N} A_j$$

Variances

To calculate sample variances of logarithmic rates of return, we employed following formula:

$$\sigma_i{}^2 = Var(S_i) = \frac{1}{N-1}\sum_{j=1}^{N}(R_{j,S_i} - \bar{R}_{S_i})^2$$

$\sigma_i{}^2$ - variance of log returns on the *i-th* asset

Correlation matrix

Elements in the correlation matrix calculated from sample data were obtained as:

$$\rho_{ik} = \frac{1}{N-1}\sum_{j=1}^{N}(R_{j,S_i} - \bar{R}_{S_i})(R_{j,S_k} - \bar{R}_{S_k})$$

Where *i* denotes the number of the row, and *k* denotes the number of the column in the matrix.

Remark. Quotes used to calculate variances, correlation matrix and drift rates for indices were taken for period starting 27.08.2008, ending 03.10.2008. Choice of the sample period is based on the following reasons:

- *Sampling period consists of 24 trading days, so it is equal to the duration of the valued contract*
- *Older data have little predictive value, due to major changes on the global financial markets in recent weeks.*

Values of the inputs are listed in the table below.

	Nikkei 225	FTSE 100	DJIA
i	1	2	3
r_{fi}	0,15%	0,63%	3,76%
r		4,41%	
σ_i	0,022319733	0,031466583	0,028170129
$Cor(S_i, S_1)$	1	0,537666544	0,06287788
$Cor(S_i, S_2)$	0,537666544	1	0,649144246
$Cor(S_i, S_3)$	0,06287788	0,649144246	1
$cov(S_i, Ex_i)$	-0,00030796	-9,76919E-05	-4,6102E-05
q_i	3,84921E-05	0,000163492	0,000126587
μ_i	-0,0064	-0,00435	-0,0045

4. Results

Results presentation:

Option value	0,4834
Number of simulations	10000
Variance of results	0,2480
Standard error of simulation	0,00498
Probability of expiring in the money (P)	0,1456
Probability of expiring in the money (Q)	0,4851
Confidence interval	0,4717-0,49494
Confidence level	99%
Width of confidence interval	0,023170831
Width of confidence interval (% of price)	0,049114347

P – physical probability measure

Q – risk-neutral probability measure

Greeks:

Greek	Value
$\Delta_{1-Nikkei\ 225}$	0,000149
$\Delta_{2-FTSE\ 100}$	0,000399
Δ_{3-DJIA}	0,000149
$\Gamma_{1-Nikkei\ 225}$	0,005879
$\Gamma_{2-FTSE\ 100}$	-0,00139
Γ_{3-DJIA}	-0,00847
$\nu_{1-Nikkei\ 225}$	1,245513
$\nu_{2-FTSE\ 100}$	0,109605
ν_{3-DJIA}	0,388600

Noticeable facts

- All the values are very low but we should remember that the whole value of the contract is less than 0,5, while indices are quoted in thousands of units. Thus, change of 1% is more than 100 points for Nikkei 225. This translates to approximately 0,0149 value change of the option. That is more than 3% of options initial value in relative terms!
- Gammas with respect to FTSE 100 and DJIA are negative. This could be due to approximation error or specific construction of the contract
- Contract seems to be relatively stable with respect to volatilities of indices (especially with regards to FTSE 100 and DJIA). This leads to conclusion, that small misestimation of parameters would result in just minor changes of option price. This stability of the model is very desirable property. As stated in the previous parts of the report, there is no accurate method of volatility estimation. Therefore, decreases in importance of volatility parameters will improve accuracy of simulated results.

5. Summary

Analytical finance is a fast growing area. As the models become more and more complicated, Monte Carlo simulation often becomes the only method to evaluate the prices of complicated derivatives and the risk measures for complex portfolios. In this paper, we solved our problem with Monte Carlo simulation. Although we constructed model employing all of our financial knowledge, there are still areas for further improvement.

We should admit that as it is in case of every model, accuracy of results is limited by the quality of underlying assumptions and inputs. In our problem, areas of highest uncertainty are volatilities and correlations used as inputs. Those, together with the approximation error, lead to unstable estimates of greek parameters. Still formula with sound theoretical basis can yield results that are going to be critically verified by the market. Even though, analysis based on the outputs of the model, gives us valuable knowledge necessary to act in the financial environment. What we should do, is to be aware of drawbacks of the employed method and adjust obtained result. Financial markets nowadays are based on quantitative analysis more than ever, but still there is no method that could replace human flexibility to act in volatile environment. Hence, what is needed is analytical tools but also an analyst that will use them properly and adjust results for qualitative factors.

References:

1. “Lecture notes in Analytical Finance I” 2008 Jan Röman
2. “Paul Wilmott on Quantitative Finance” 2006 Paul Wilmott
3. “Options, Futures and Other Derivatives.6th edition” 2005 John C. Hull
4. “Analysis of Derivatives for the CFA® Program” 2003 Don M. Chance
5. “PRM Handbook” editors: Prof. Carol Alexander, Prof. Elizabeth Sheedy
6. “Equity Asset Valuation: Valuation” John D. Stowe, Thomas R. Robinson, Jerald E. Pinto, Dennis W. McLeavey
7. “Stochastic Processes with Application to Finance” 2002 Masaaki Kijima