MT1410 Analytical Finance

Hedging with Options

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Introduction

The application built in VBA to use Black-Scholes to find the optimal hedge for a given number of shares. There is the possibility to choose one or two derivative securities, call and put options, and therefore create a delta or a delta-gamma hedge. The shares are going to be hedged through forming a portfolio, that is why, the portfolio may consists of a number of shares, either one or two derivative securities with their possible combinations and transactions costs, if any exist. The graphics are provided to show the result of the total portfolio.

Delta – hedging

Δ – delta of an option

Δ_p – delta of a new portfolio, consisting of stocks and option

N_S – number of stocks

N_1 – number of options

The idea of delta-hedging implies creating a delta-neutral portfolio, consisting of stocks and options as one derivative security on those stocks. In order to find the optimal number of options needed for hedging we will use the following formula:

$$N_1 = \frac{-N_S}{\Delta}$$

Indeed, the delta of a new portfolio will equal to zero:

$$\Delta_P = N_S + N_1\Delta = N_S + (-\frac{N_S}{\Delta})\Delta = 0$$

which means that any instantaneous change in the stock price will not lead to huge losses.

Delta – Gamma hedging

Instead of delta hedging, the investors somestimes prefer delta-gamma hedging in order to obtain more effective hedging strategy. The difference between delta and delta-gamma is; delta-gamma hedging provides a solution for two derivative securities written for a number of share as delta hedging is just for one derivative security.

Δ_1 – delta of an option of the first type

Δ_2 – delta of an option of the second type

Δ_p – delta of a new portfolio, consisting of stocks and options of two types

Γ_1 – gamma of an option of the first type

Γ_2 – gamma of an option of the second type

Γ_p – gamma of a new portfolio, consisting of stocks and options of two types

N_S – number of stocks

N_1 – number of options of the first type

N_2 – number of options of the second type

Delta-gamma neutralisation of the portfolio would create a more accurate hedge, since it requires that both delta and gamma coefficients of a new portfolio equal zero.

In other words, we need to solve the following system of linear equations with respect to N_1 and N_2:

$$\begin{cases} N_S + \Delta_1 N_1 + \Delta_2 N_2 = 0 \\ \Gamma_1 N_1 + \Gamma_2 N_2 = 0 \end{cases}$$

It follows, that

$$N_1 = -N_S \frac{\Gamma_2}{\Gamma_2 \Delta_1 - \Delta_2 \Gamma_1}$$

$$N_2 = -N_S \frac{\Gamma_1}{\Gamma_1 \Delta_2 - \Delta_1 \Gamma_2}$$

Calculation of Delta and Gamma for different option types:

1. **European call option**

$$\Delta_C = e^{-qT} N(\frac{\ln\frac{S}{K} + T(r-q+\frac{\sigma^2}{2})}{\sigma\sqrt{T}}; \qquad \Gamma_C = \frac{N'(d_1)e^{-qT}}{S\sigma\sqrt{T}}$$

2. **European put option**

$$\Delta_P = -e^{-qT} N(-\frac{\ln\frac{S}{K} + T(r-q+\frac{\sigma^2}{2})}{\sigma\sqrt{T}}; \qquad \Gamma_P = \frac{N'(d_1)e^{-qT}}{S\sigma\sqrt{T}}$$

3. **American call option**

$$\Delta_C = \frac{C_1(1) - C_1(0)}{S(u-d)}; \qquad \Gamma_C = \frac{2(\frac{C_2(2) - C_2(1)}{u^2 - 1} - \frac{C_2(0) - C_2(1)}{d^2 - 1})}{S^2(u^2 - d^2)}$$

where;

$$C_k(i) = \max\left\{ Su^i d^{k-i} - K; \frac{1}{(1+r)^h}(pC_{k+1}(i+1) + (1-p)C_{k+1}(i)) \right\}, \text{ k = 0,1,2,… n-1;}$$

i = 0,1,2,… k.

$$C_n(i) = \max\left\{ Su^i d^{n-i} - K; 0 \right\}.$$

$$p = \frac{e^{(r-q)T} - d}{u - d}; \qquad u = e^{\sigma\sqrt{h}}; \qquad d = e^{-\sigma\sqrt{h}}$$

n denotes the number of steps in the binomial model

h = T/n denotes the length of one step in the binomial model

4. American put option

$$\Delta_P = \frac{P_1(1) - P_1(0)}{S(u - d)}$$

$$\Gamma_P = \frac{2\left(\frac{P_2(2) - P_2(1)}{u^2 - 1} - \frac{P_2(0) - P_2(1)}{d^2 - 1}\right)}{S^2(u^2 - d^2)}$$

$$P_k(i) = \max\left\{Su^i d^{k-i} - K; \frac{1}{(1+r)^h}\left(pP_{k+1}(i+1) + (1-p)P_{k+1}(i)\right)\right\}, \text{ k = 0,1,2,… n-1;}$$

i = 0,1,2,… k.

$$P_n(i) = \max\{Su^i d^{n-i} - K; 0\}.$$

Calculating C1, C2, P1, P2

```
Dim C(0 To 10)
  For i=0 To 10
    k=10
    C(k,i)= max{S*u^i*d^(10-i) – K,0}
  end

Dim C1(1 To 9, 1 To 10)
  For i=0 To 9
    k=9
```

$$C1(k,\ i) = \max\{S\ * u\text{^} i * d\text{^}(k - i) - K,\ \frac{1}{(1+r)^{h}}\{\Pi * C(k\ + 1, i + 1) + (1 - \Pi) * C(k\ + 1, i)\}\}$$

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  end
  For k=8 to 0
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$$C1(k,\ i) = \max\{S\ * u\text{^} i * d\text{^}(k - i) - K,\ \frac{1}{(1+r)^{h}}\{\Pi * C1(k\ + 1, i + 1) + (1 - \Pi) * C1(k\ + 1, i)\}\}$$

Calculation of total portfolio value as the function of the stock price

$$PotrVal(S) = N_S \cdot S + N_1 \cdot O_1 + N_2 \cdot O_2 + TC$$