Analytical Finance II Project

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Hull-White Trinomial Trees

AIM

Build an application in Excel/VBA to valuate Contracts as Hull-White with trinomial trees.

Stage1.Building the Tree

- **The Hull-White mode**

$$dr = [\theta(t) - ar]dt + \sigma dz$$

Where

r ---instantaneous short rate

$\theta(t)$---some function of t

a,σ--constants

Stage1.Building the Tree

- Construct trinomial trees for R^* and followed the process:

$$dR^* = -aR^* dt + \sigma dz$$

when $R^* = 0$, the variable $R^* \;\; t + \Delta t \;\; - R^* \;\; t$ is normally distributed.

Stage1.Building the Tree

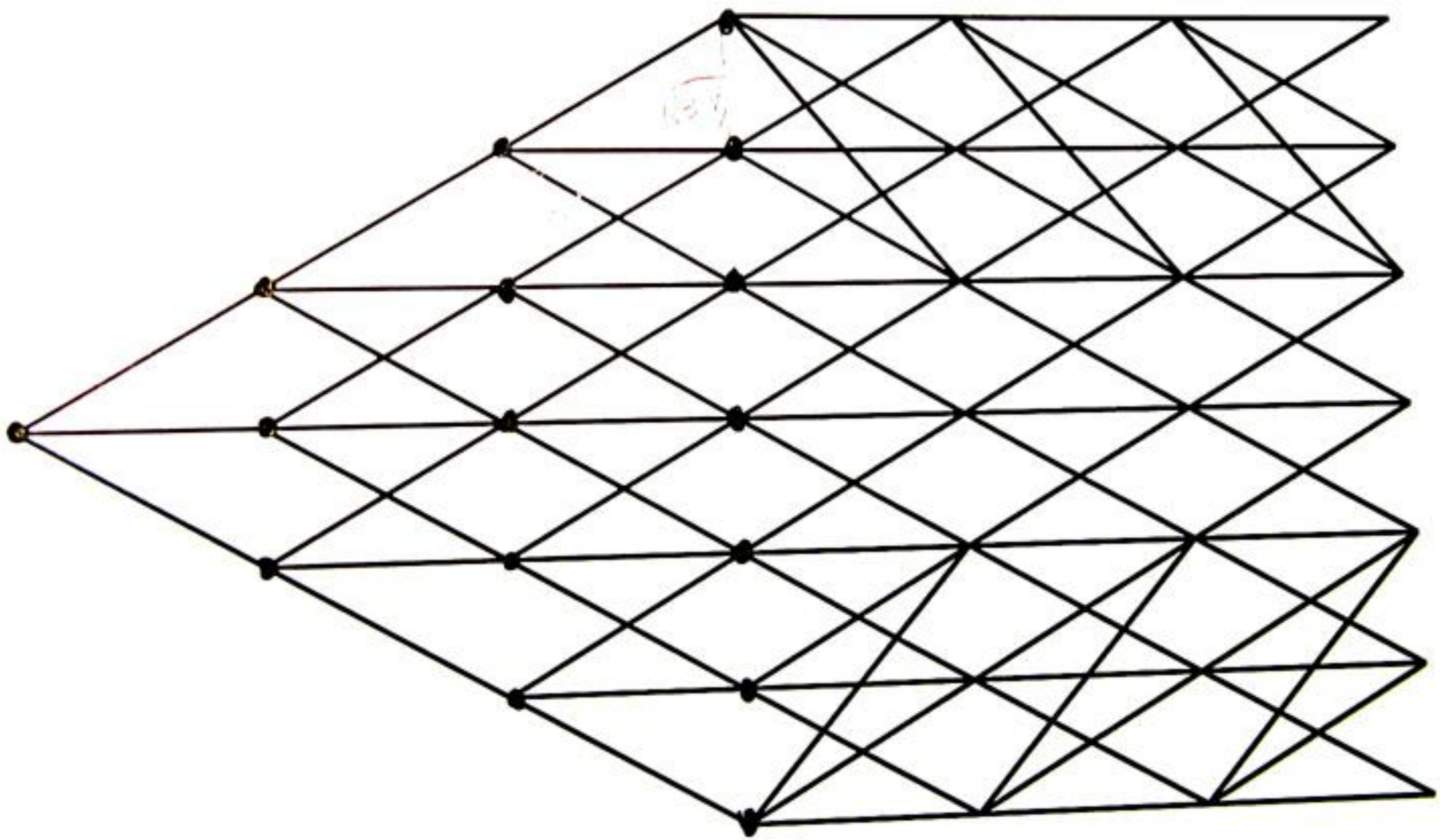
$$E\left[R^{*}\ \ t+\Delta t\ \ -R^{*}\ \ t\ \ \right]=-aR^{*}\ \ t\ \ \Delta t$$

$$Var\left[R^{*}\ \ t+\Delta t\ \ -R^{*}\ \ t\ \ \right]=\sigma^{2}\Delta t$$

The initial Tree will be built like this

Figure 1

The initial tree ($\theta(t) = 0$ and $x(0) = 0$)



Stage1.Building the Tree

- ***First***, establish $\Delta t, a, \sigma$ from the data.
- ***Second***, calculate $\Delta R, j^*_{\max}, j^*_{\min}$

$$\Delta R = \sigma\sqrt{3\Delta t}$$

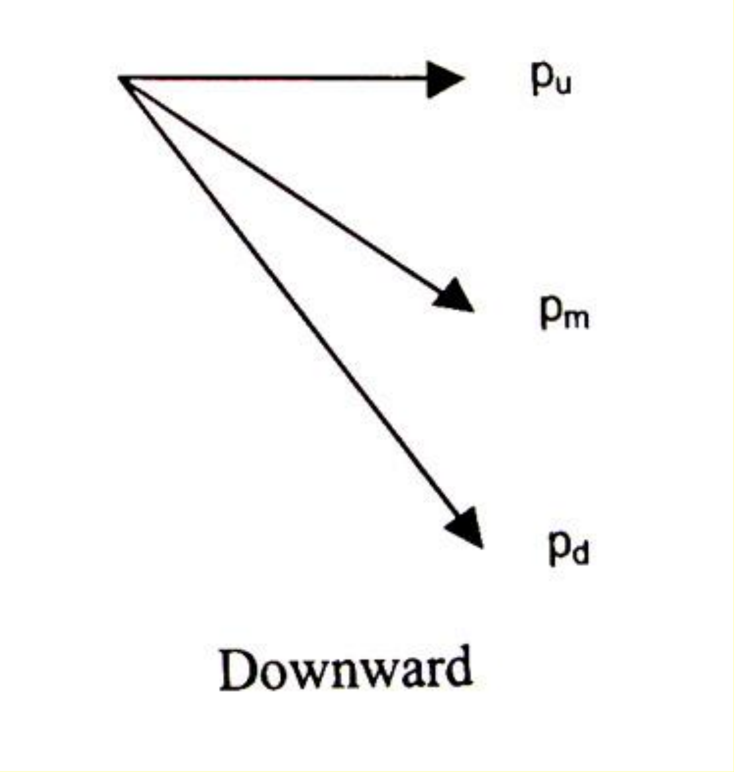
$$j^*_{\max} = \text{the smallest integer greater than} \left[\frac{-0.184}{a\Delta t}\right]$$

$$j^*_{\min} = -j^*_{\max}$$

Stage1.Building the Tree

Third, calculate the transition probabilities in the tree, and we need some formulas.

Stage1. Building the Tree



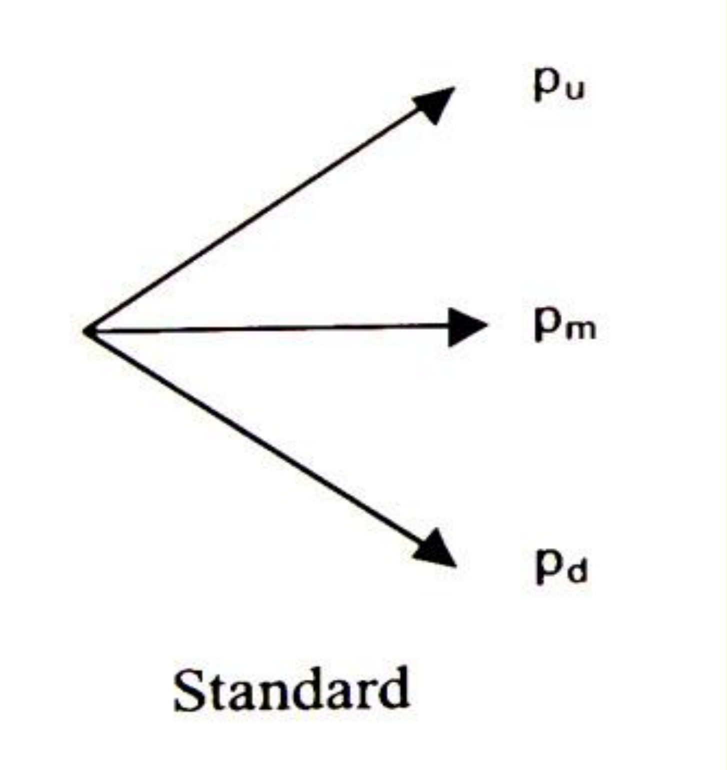
At the top node $j_{\max}$ the transition probabilities are:

$$P_u = 1/6 + \frac{a^2 j^2 \Delta t^2 + aj\Delta t}{2}$$

$$P_m = -1/3 - a^2 j^2 \Delta t^2 - 2aj\Delta t$$

$$P_d = 7/6 + \frac{a^2 j^2 \Delta t^2 + 3aj\Delta t}{2}$$

Stage1.Building the Tree



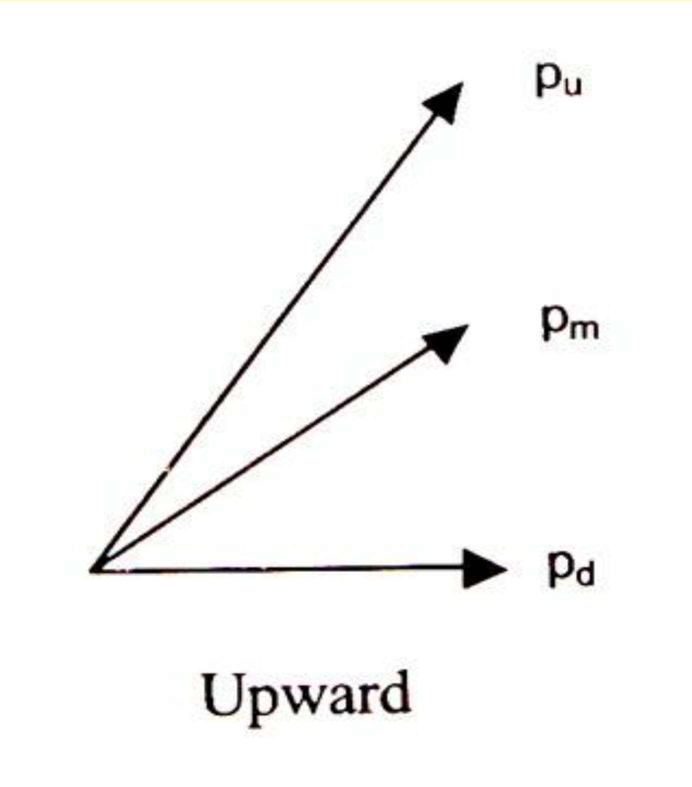
At intermediate node the transition probabilities are:

$$P_u = 1/6 + \frac{a^2 j^2 \Delta t^2 - aj\Delta t}{2}$$

$$P_m = 2/3 - a^2 j^2 \Delta t^2$$

$$P_d = 1/6 + \frac{a^2 j^2 \Delta t^2 + aj\Delta t}{2}$$

Stage1. Building the Tree



At the bottom node $j_{\min}$ the transition probabilities are:

$$P_u = 7/6 + \frac{a^2 j^2 \Delta t^2 - 3aj\Delta t}{2}$$

$$P_m = -1/3 - a^2 j^2 \Delta t^2 + 2aj\Delta t$$

$$P_d = 1/6 + \frac{a^2 j^2 \Delta t^2 - aj\Delta t}{2}$$

Stage2. Fitting the Tree

objective

Covert the tree for R^*
to the tree for R.

Define

$$\alpha \;\; t \; = R \;\; t \; - R^* \;\; t$$

where

α_0 =initial Δt period interest rate

$$\alpha_m = \frac{\ln \sum_{j=-n_m}^{n_m} Q_{m,j} e^{-j\Delta R \Delta t} - \ln P_{m+1}}{\Delta t}$$

Stage2. Fitting the Tree

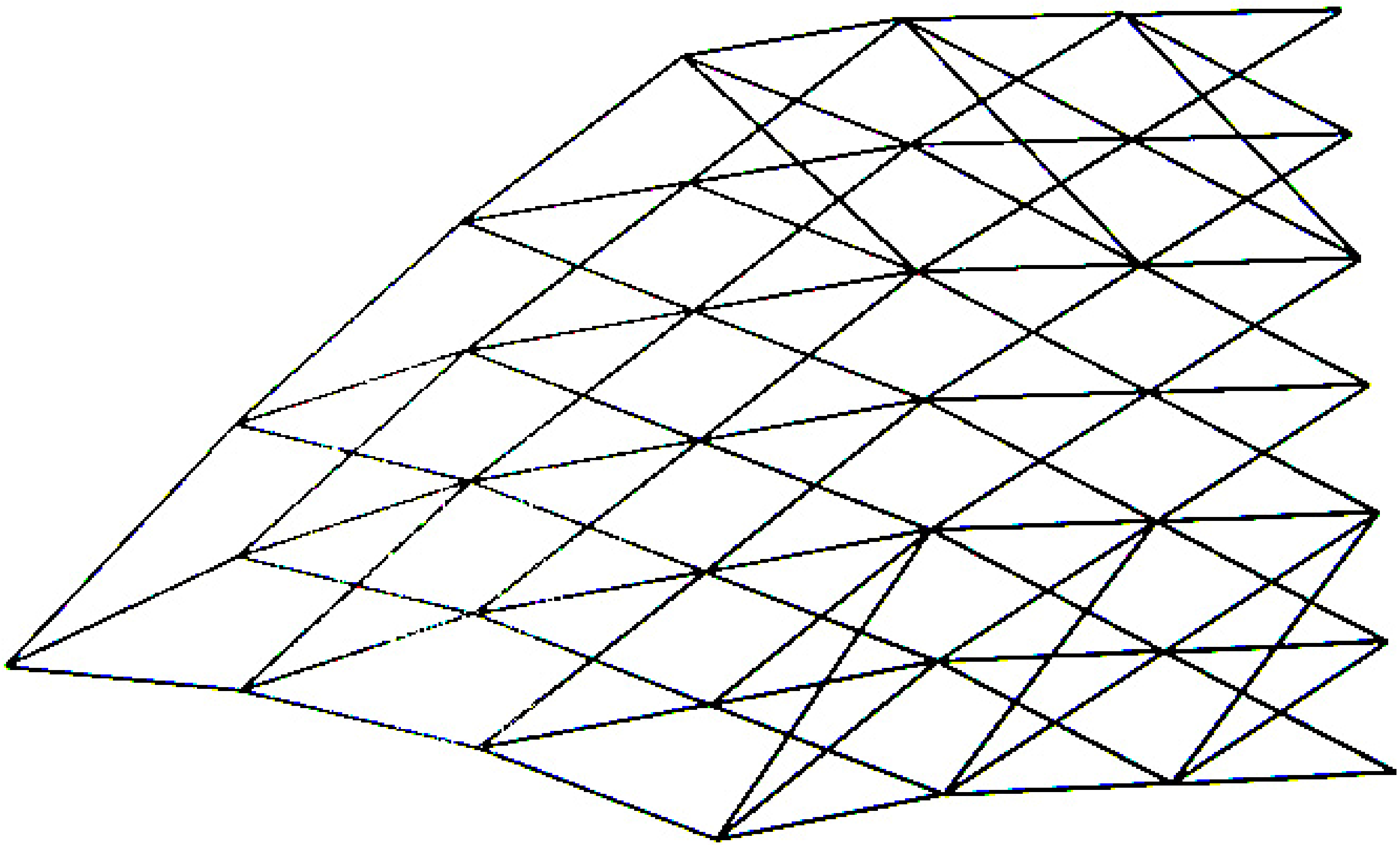
$$Q_{m+1,j} = \sum_{k} Q_{m,k}\, q \;\; k, j \;\; \exp\left[- \;\; \alpha_m + k\Delta R \;\; \Delta t\right]$$

$$p_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j} \exp\left[- \;\; \alpha_m + j\Delta R \;\; \Delta t\right]$$

The Final Tree will be like this

Figure 2

The final tree for x



Thank you!