Pricing cancellable swaps using tree models calibrated to swaptions

Master's Thesis carried out at the
Department of Management and Engineering,
Linköping Institute of Technology
and at Swedbank

by

Karolin Friberg

and

Matilda Rappe

LIU-IEI-TEK-A-[11/01000]—SE

Supervisors

Angelo Corelli

Jan Röman

Abstract

This master's thesis has been carried out at Linköping Institute of Technology at the Department of Management and Engineering. It has been performed at the request of Swedbank by two students from the program Industrial Engineering and Management – International, within the financial profile of the program. The project has been carried out at the bank's headquarters in Stockholm during the autumn 2010.

The assignment was to price cancellable swaps by programming and calibrating two interest rate models. The models were the Hull-White model (trinomial tree) and the Black-Derman-Toy model (binomial tree), which were calibrated to the current interest rate curve and to actively traded swaptions. This calibration method is rather unusual, since interest rate models most often are calibrated to the interest rate and the interest rate volatility.

These models are currently used by the bank, but they have been programmed completely independent of the bank in the programming language C++. For the calibration and pricing, methods for pricing swaptions and cancellable swaps have been built. To enable for the calibration to swaptions, a numerical search method that was available at the bank has been implemented.

The calibration method was constructed in order to make the tree models price swaptions correctly. By using market data on ATM swaptions that matures at each time step in the models, it is possible to let the models price the swaptions, to be able to find the volatility that for each time step will evolve the interest rate trees that will price the cancellable swaps correctly. In this way, the models will be calibrated to actively traded swaptions at the same time as they are calibrated to the interest rate, since the interest rate is always used in the construction of the trees.

In this study, a simple example was carried out where series of European cancellable swaps were to be priced. This showed to be difficult to achieve for the two models, since the calibration gave insufficient results. For the Hull-White model it turned out that the volatilities from the calibration process were too high which resulted in too wide interest rate trees and in prices of cancellable swaps that were not good enough. When comparing to using a constant volatility of 1 % for the whole model, it could be seen that both interest rate trees and prices of the cancellable swaps were considerably better. This lead to the conclusion that the calibration to swaptions performed in this study could not be preferred to using a constant volatility.

For the second model, the Black-Derman-Toy model, the pricing of swaptions was conducted in an alternative way. This was necessary to enable any pricing at all. The results from the calibration showed that the swaptions could not be priced exactly, which makes the model unsuitable to be used for pricing since it cannot be considered reliable. Furthermore, several solutions could be received using the same input data, which was due to the fact that not yet calibrated volatilities were used. To solve this problem it is recommended that another numerical search method is implemented that can find the value of all volatilities at the same time instead of searching for them one by one. This was unfortunately not possible to accomplish within the scope of this master's thesis, but is a recommendation for further studies.

Sammanfattning

Detta examensarbete har skrivits på institutionen för ekonomisk och industriell utveckling under avdelningen för produktionsekonomi vid Linköpings Tekniska Högskola. Det har genomförts av två studenter från civilingenjörsprogrammet Industriell ekonomi – Internationell, inom programmets finansiella inriktning. Uppdragsgivaren för examensarbetet har varit Swedbank och examensarbetet har utförts på bankens huvudkontor i Stockholm under hösten 2010.

Uppgiften avsåg att prissätta stängningsbara swappar genom att programmera och kalibrera två räntemodeller. Modellerna var Hull-White (trinomialträd) och Black-Derman-Toy (binomialträd), vilka har kalibrerats mot aktuell räntekurva samt mot aktivt handlade swaptioner. Detta kalibreringssätt är relativt ovanligt, då räntemodeller oftast kalibreras mot aktuell räntekurva och dess volatilitet.

Dessa modeller används av banken idag, men har programmerats helt fristående i programmeringsspråket C++. För kalibrering och prissättning har metoder för prissättning av swaptioner och stängningsbara swappar byggts. För att möjliggöra kalibreringen mot swaptioner har en numerisk sökmetod som fanns tillgänglig på banken implementerats.

Kalibreringsmetoden har konstruerats för att trädmodellerna ska kunna prissätta swaptioner korrekt. Genom att använda marknadsdata på ATM swaptioner som förfaller vid varje tidssteg i modellerna, kan man låta modellerna prissätta swaptionerna för att finna den volatilitet som för varje punkt i tiden ska spänna upp ränteträden för att kunna prissätta de stängningsbara swapparna korrekt. På detta sätt blir modellerna kalibrerade mot aktivt handlade swaptioner samtidigt som de är kalibrerade mot aktuell räntekurva, då denna alltid används vid uppbyggnaden av träden.

I denna studie har ett förenklat exempel genomförts där en serie europeiska stängningsbara swappar har blivit prissatta. Detta visade sig vara svårt att genomföra för båda modellerna, då kalibreringen gav otillfredsställande resultat. För Hull-White modellen visade det sig att volatiliteterna från kalibreringen var allt för höga, vilket resulterade i allt för uppspända ränteträd och otillräckligt bra priser på de stängningsbara swapparna. Vid jämförelse med konstant volatilitet på 1 % för hela modellen, visade det sig att både ränteträd samt priser på de stängningsbara swapparna blev betydligt bättre. Detta gjorde att slutsatsen kunde dras att kalibrering mot swaptioner i denna studie inte var fördelaktig framför att använda en konstant volatilitet.

För den andra modellen, Black-Derman-Toy, gjordes prissättningen av swaptionerna på ett alternativt sätt. Detta var nödvändigt för att överhuvudtaget möjliggöra prissättning. Resultatet av kalibreringen visade att swaptionerna inte kunde prissättas exakt, vilket gör att denna modell inte lämpar sig för prissättning då den inte kan anses tillförlitlig. Flera lösningar kunde dessutom fås utifrån samma indata, vilket berodde på att ännu inte kalibrerade volatiliteter användes. För att lösa detta problem rekommenderas att en annan numerisk sökmetod implementeras som kan lösa ut värdet på alla volatiliteter samtidigt istället för en i taget. Detta var tyvärr inte möjligt inom ramen för examensarbetet, men är ett förslag för framtida studier.

Table of contents

1 Introduction

This project was carried out at Swedbank during the autumn 2010 within the area of financial valuation methodology. This report presents the project, which starts with a theory background and a description of the implemented calibration method. This is followed by the results of the study and a discussion which leads to the final conclusions.

In this first chapter, the background of the project is described and the project intention is defined. To be specific about the project, limitations, methods and source criticism is needed.

1.1 Background

Swedbank is one of the largest banks in Sweden and its roots go back to 1820. The bank is also present in Estonia, Latvia and Lithuania. Swedbank's main customers are the private sector and small- and medium-sized companies. It has 9.5 million private customers and 669 000 corporate customers and the group employs about 17 500 people. (Swedbank, 2010)

Today Swedbank is offering cancellable swaps to their corporate customers. These are offered to customers who want to pay a lower fixed rate than what a usual swap contract would imply. In exchange for this lower fixed rate, the bank has the right to cancel the contract at points in time specified in the contract. These contracts have been difficult to price and different models have given different results in different market situations. Currently the bank uses two different models for pricing, where one has proven to be better than the other. The bank uses one software product for pricing, containing both models.

1.2 Intention

The intention of this project is to price cancellable swaps using the two different tree models that are currently used by the bank. The first model is the Hull-White model, which is a model with normal distributed interest rates and will be built using a trinomial tree. The second model is the Black-Derman-Toy model, which is a model with lognormal distributed interest rates and will be built using a binomial tree. In this study, these models will be programmed in C++ independent of the models used by the bank.

Once the models are built, they will be calibrated to the interest rate and actively traded swaptions. This calibration method is an alternative way of calibrating, compared to most studies where the calibration is done to interest rates and interest rate volatilities. A small theoretical example will be performed in order to study the calibration method together with each of the two models. The models will then be compared and the calibration method analyzed in order to price the cancellable swaps as close as possible to reference prices from Bloomberg.

1.3 Limitations

The models will be developed purely to price the type of interest rate derivatives called cancellable swaps. Other kinds of exotic instruments will not be investigated. How contracts are written and how the instruments are traded will not be a part of this study.

The accuracy of the calibration will of course depend on whether matching instruments are available, the number of calibrating instruments used, as well as the accuracy of the market data. The models will take a decided, unchanging set of data as input which will not be fetched using real time updates and day count conventions will not be incorporated in the models. If further generalizations or limitations are made, this is stated explicitly in the report.

1.4 Methods

To price a cancellable swap it is necessary to get an understanding of the motive for using the instrument and how it is constructed. This overall understanding is built up at the beginning of the report and makes it possible to look closer into the two models that have been chosen for this study.

The models have first been studied in depth and then programmed in C++. They have also been tested by pricing some simple instruments for verification. Thereafter, methods for pricing swaptions and cancellable swaps were programmed and an optimization algorithm was implemented in order to calibrate the models. All market data was collected from Bloomberg at the same day, 26th of November in 2010.

After the modeling and calibration, the models have been compared with each other by pricing the same instruments and the results have been analyzed. Conclusions about the calibration in combination with each of the models have been drawn and are presented at the end of the report.

1.5 Source and method criticism

The results from the models can of course never be exact in any way, since both of them are numerical models. The concept of exact is also somewhat undefined, dependent on what is seen as reference. Since market prices from Bloomberg are seen as references in this report, the aim is of course to make the models come as close as possible to these prices.

Reasons why a match in prices is difficult to get are dependent on many factors. The calibration method used covers a large area of possible reasons. Possible is that the chosen method for calibrating is not the most optimal or that the wrong type of calibration instruments, or too few, have been used.

All code written to perform this study has been developed and tested carefully. However, there is still a risk that some small error remains despite the thorough bug searches.

The parameters in the models, to be varied in the calibration, control the degree of freedom of the models. The choice of making parameters constant or time dependent is of course an important matter that has a large effect on the interest rate trees as well as on prices.

The instruments used for the calibration can also be a possible source of error. While selecting how the market data on instruments from Bloomberg is to be obtained, many parameters have to be set. If some parameters are set differently, different prices are obtained. The change in any parameter can have a small or a large influence on the price but the parameters are chosen as good as possible.

Since prices and interest rates on the market changes constantly, a model that does not have real time updates of the input data can hardly get output in form of prices corresponding to the prices on the market.

It is of course doubtful to use Bloomberg as only source for market data, but this is anyway chosen in this study for consistency and furthermore Bloomberg is considered sufficiently reliable.

2 Frame of references

To be able to understand the construction of the financial instrument called cancellable swap, this section first gives a brief description of related basic instruments. Some background information on pricing the cancellable swap is more closely described as well as its properties in order to highlight what has to be dealt with while pricing the instrument. Finally the two models chosen for the study as well as their characteristics are closely described.

2.1 Financial instruments: option, swap, swaption, cancellable swap

An **option** is a financial instrument that gives the owner the right, but not the obligation, to do something in the future. Common options are call or put options that give the owner the right to buy or sell some underlying asset, specified in the contract, in the future. The simplest options are called plain vanilla options. Those can be priced either analytically, using Black-Scholes option pricing formula or numerically, using for example binomial trees.

Options can be used to hedge against a change in value of some asset or just for speculation purposes. Many much more complicated options can be constructed that cannot be priced analytically with the Black-Scholes option pricing formula. Instead they need numerical methods for pricing. One example is American put options that can be exercised at many points in time and must therefore be priced by evaluating this opportunity in each time step of the tree model.

A **swap** is a contract between two parts that binds them to exchange interest rate payments, where one is paying a fixed rate and the other a floating rate. This can be useful for example when a company has a loan where the interest is paid at a floating rate. By entering the swap contract the company can instead make fixed interest rate payments. Then the cash flows can be known in advance and the risk of increasing interest rates can be eliminated.

The floating and the fixed interest rate payments must not be made at the same frequency. The fixed interest rate to be paid is calculated at the beginning to give the same initial value of the floating part of the contract as of the fixed part of the contract. By doing this no payment has to be made when entering the contract. The contract will change in value for both parts after some time, depending on the changes in the interest rate.

A **swaption** is an option to enter a swap contract at a future point in time. This will be exercised if the interest rates move in such direction that is profitable for the holder of the option.

Swaptions on the market are normally quoted in volatilities instead of prices. The volatilities are quoted as implied volatilities and are in Bloomberg implied from prices using Black's formula for pricing swaptions. Since they differ from other volatility quotes, they cannot simply be used in calculations where other volatilities are required.

A **cancellable swap** is a swap instrument with an added possibility to cancel the contract at a point in time prior to maturity. This possibility will be used if the holder of the cancellation possibility benefits from terminating the contract. There exist both putable and callable contracts on the market. When the holder of the cancellable swap will receive fixed cash flows and pay floating it is putable and the opposite is callable.

The contract can be cancelled at one or many points in time. If it has one cancellation point it is of European style and is sometimes referred to as a simple cancellable swap. If it has many cancellation points it is of Bermudan style. A cancellable swap can also be constructed to be cancellable in a time interval. It is then an American style cancellable swap.

When pricing the instrument, a cancellable swap can be replicated with a swap and one or many European swaptions, where the number of swaptions is dependent on the number of cancellation points. The cancellable swap can also be replicated by a swap and one Bermudan swaption. The exercise dates of the Bermudan swaption must then be the same as for the cancellable swap. Even if a cancellable swap can be constructed as a sum of one swap and one or many swaptions, there is a demand for cancellable swaps, since the buyer prefer entering one contract instead of two or more.

The contract could not be valued by pricing the swap and the swaptions separately and summing them up, since the cancellable swap has optionality built in. This optionality makes the swaptions interdependent, so the swaptions with maturity after the termination of the cancellable swap will be useless. The optionality makes no analytical way of pricing possible. Instead, some numerical way of pricing is needed, such as a tree model.

The value of the cancellable swap must be higher than the value of a plain vanilla swap, since there is an optionality built in. This makes the fixed rate of the cancellable swap higher than the fixed rate in a plain vanilla swap contract if the holder of the optionality is paying fixed rate. The counterparty paying at a floating rate pays at a lower level as compensation for the other part having the optionality.

When the bank's customers want to have known and fixed interest rate payments instead of floating on a loan, they can enter a swap contract. This gives them the right to pay at a fixed level and receive floating from the bank. To be able to pay at a lower fixed rate than the market swap rate, the customers can instead enter a cancellable swap which in return gives the bank the right to cancel the contract at points in time specified in the contract.

2.2 Building a trinomial tree with the Hull-White model

The Hull-White model can be solved either by solving the partial differential equation or by building a trinomial tree for the interest rates. The stochastic process for the Hull-white model is

$$dr = [\theta(t) - ar]dt + \sigma dz$$

where r is the instantaneous short rate, $\theta(t)$ is a function of time and the parameters a and σ are constants.

To construct a final tree that is calibrated to interest rates, it is necessary to first build a symmetric R^*-tree that evolves from $R^* = 0$. By shifting it to exactly match the initial term structure the final R-tree is constructed.

The stochastic process for the R^*-tree is

$$dR^* = -aR^*dt + \sigma dz$$

The spacing between the interest rates in the nodes at one time step of the tree is $\Delta R = \sigma\sqrt{3\Delta t}$. This is constant for constant value of Δt, otherwise dependent on Δt. In the following description Δt is assumed to be constant for the model.

For each node in the tree there is one index for the time period (i), starting at zero for the first node, and one index for the level in the tree (j) that is zero for the middle level. The time at one node is then $t = i\Delta t$ and the value of the node in the R^*-tree is then $R^* = j\Delta R$.

If the tree would expand without limits as time evolves, negative interest rates as well as very high interest rates will occur. To avoid this it is necessary to build in a maximum level and a minimum level of nodes in the interest rate tree. Since the R^*-tree is symmetric the maximum and minimum level will be at the same distance from the middle level. The R^*-tree will then assume the shape presented in Figure 2.1, where the maximum level is two and the minimum level is minus two.

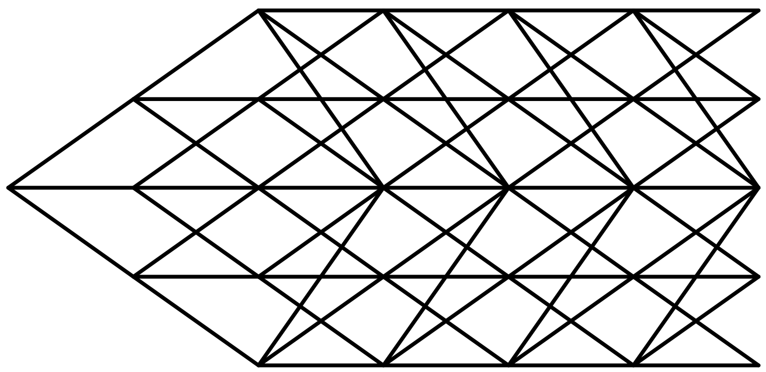

Figure 2.1 The symmetrical R*-tree with maximum and minimum level
Source: Hull, 2009

This phenomenon of limiting the tree is a mean-reverting characteristic of the Hull-White model, which makes very high interest rates revert down towards the medium level and very low interest rates revert up towards the medium level as the tree evolves.

When the maximum or minimum level is reached, the probabilities of moving from one node to a node in the next time step will look a bit different. Therefore one of the three branching possibilities presented in Figure 2.2 must be chosen at each node.

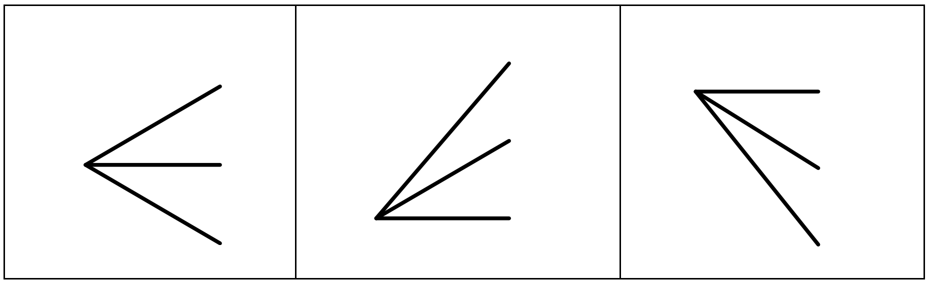

Figure 2.2 Picture of the three branching possibilities: normal branching, upward branching and downward branching
Source: Hull, 2009

The probabilities of moving from one node at one time step to another in the next time step differ dependent on the type of branching and must be calculated for each node. For each node three probabilities are calculated. For the situation with normal branching, the three probabilities are

$$p_u = \frac{1}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 - aj\Delta t)$$

$$p_m = \frac{2}{3} - a^2 j^2 \Delta t^2$$

$$p_d = \frac{1}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 + aj\Delta t)$$

For the situation with upward branching, the probabilities are

$$p_u = \frac{7}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 - 3aj\Delta t)$$

$$p_m = -\frac{1}{3} - a^2 j^2 \Delta t^2 + 2aj\Delta t$$

$$p_d = \frac{1}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 - aj\Delta t)$$

For the situation with downward branching, the probabilities are

$$p_u = \frac{1}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 + aj\Delta t)$$

$$p_m = -\frac{1}{3} - a^2 j^2 \Delta t^2 - 2aj\Delta t$$

$$p_d = \frac{7}{6} + \frac{1}{2}(a^2 j^2 \Delta t^2 + 3aj\Delta t)$$

When the probabilities are calculated, the final tree that is fitted to the initial term structure can be developed. At each time step, the equation for converting the R^*-tree into an R-tree is

$$\alpha(t) = R(t) - R^*(t)$$

The α's are the prices of a zero coupon bond maturing in the next time period. This has to be calculated iteratively to make sure to find the correct α for each time period that makes the R-tree correspond to the initial term structure. The first α, $\alpha(0)$, is the price of a zero coupon bond maturing at the end of the first time step and is equal the initial short rate for this period.

To calculate this, $Q_{i,j}$ is defined to be the value of a security that pays one cash unit if the node (i, j) is reached and zero otherwise. For each time step the α is calculated based on the Q's for the time step. The formulas for the α's and the Q's are

$$\alpha_m = \frac{ln \sum_{j=-n_m}^{n_m} Q_{m,j} e^{-j\Delta R \Delta t} - \ln P_{m+1}}{\Delta t}$$

$$Q_{m+1,j} = \sum_k Q_{m,k} q(k,j) \exp[-(\alpha_m + k\Delta R)\Delta t]$$

where $q(k,j)$ is the probability of moving from node (m,k) to node $(m+1,j)$. This is done iteratively since the prices of zero coupon bonds always, at each time step, must match the interest rates from the initial term structure.

The initial values at the beginning of the tree are set so that $R(0,0)$ equals the initial interest rate at the first Δt period and $Q(0,0)$ must equal one. (Hull, 2009)

The final tree of interest rates that is fitted to match the initial term structure is shifted and will look something like in Figure 2.3 below.

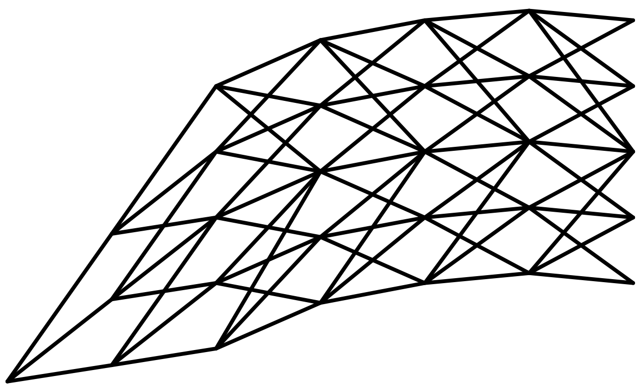

Figure 2.3 Picture of a final Hull-White tree
Source: Hull, 2009

2.3 The Black-Derman-Toy model

The Black-Derman-Toy model is a single-factor short-rate model. To build a tree that models the evolution of the short-term interest rate, today's interest rate term structure is used as input to price zero coupon bonds that matures at each time step of the tree. The model was originally described algorithmically by modeling the evolution of the term structure in a discrete time binomial lattice framework. It has been shown that when the time step tends to zero, the model implies the following continuous stochastic differential equation

$$d\ln r(t) = \left[\theta(t) + \frac{\sigma'(t)}{\sigma(t)} \ln r(t)\right] dt + \sigma(t) dz$$

where $\sigma(t)$ is the instantaneous short-rate volatility and $\theta(t)$ is the value of the underlying asset. They are chosen to adapt the model to the observed term structure of interest rates and volatilities. When these parameters are chosen, the forward volatilities can be calculated. One downside of the model is that for certain choices of $\sigma(t)$, the short rate can become mean-fleeing instead of mean-reverting. The changes in the underlying short rate are log-normally distributed in the model, which implies that the interest rate can never become negative. (Clewlow and Strickland, 2000)

2.3.1 The tree building procedure

Since the Black-Derman-Toy model is lognormal, no analytical way of pricing exists. Instead, the model requires a numerical procedure for pricing interest rate derivatives. This means that a short-rate tree have to be built with time steps until maturity of the derivative that is to be priced. The discrete time calculation requires a discrete time representation of the Brownian path independent model. This is made as a binomial tree and such a representation is shown in Figure 2.4 below.

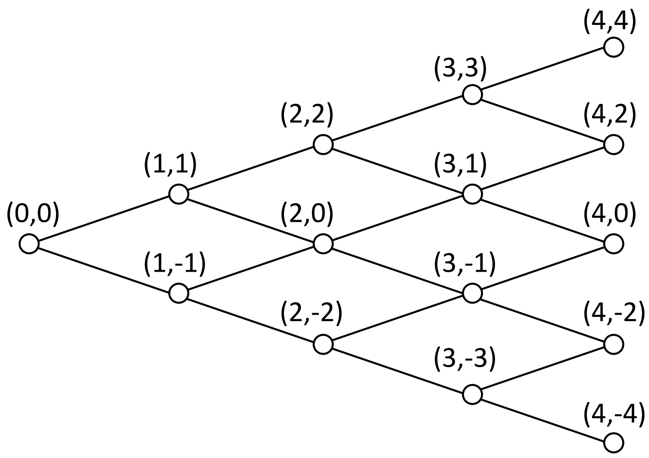

Figure 2.4 Binomial tree
Source: Clewlow and Strickland, 2000

The interest rate tree will be calibrated to observed interest rates and matched to volatilities. The observed interest rates are yields on bonds with different maturities. The first bond, $B(1)$, matures after one period and the last bond, $B(N)$, matures at time N, which is the maturity of the derivative that is to be priced with the tree model.

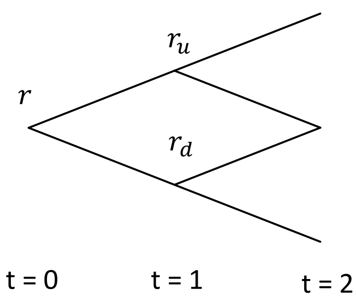

Figure 2.5 Short-rate tree
Source: Röman, 2010

The one period long interest rate r in the first time step of the tree can be seen as the yield of a bond maturing one time period later in the tree, in time $t = 1$, illustrated in Figure 2.5. If the length of the time step in the tree is one year, the price of this bond is

$$B(1) = \frac{N}{(1+r)^1} = \frac{N}{(1+y_1)^1}$$

where y_1 is the observed interest rate from the market. The equation can easily be solved to find the interest rate r for the first time step in the tree.

The rates at the two nodes at $t = 1$, r_u and r_d, are one period interest rates that can be seen at time $t = 1$ as yields of bonds maturing at time $t = 2$, where *u* denotes an upward move in the tree and *d* a downward move. For all nodes in the tree, the probability of moving up is 0.5 as well as the probability of moving down. These bonds are named $S_u(1)$ and $S_d(1)$ to distinguish them from the observed bond prices. They and are always denoted with one, since they always matures one period later in the tree, and are priced as

$$S_u(1) = \frac{N}{(1 + r_u)^1} \qquad S_d(1) = \frac{N}{(1 + r_d)^1}$$

The interest rates in the next time step of the tree also have to match the initial yield of a bond maturing in $t = 2$, seen from time $t = 0$. The price of this bond, $B(2)$, is

$$B(2) = \frac{N}{(1 + y_2)^2}$$

where y_2 is the observed interest rate on the market for a bond maturing after two years. Calculating the bond price using the interest rates in the tree must give the same price and therefore the following must hold

$$B(2) = \frac{0.5 * S_u(1) + 0.5 * S_d(1)}{(1 + r)^1} = \frac{0.5 * \frac{N}{(1 + r_u)^1} + 0.5 * \frac{N}{(1 + r_d)^1}}{(1 + r)^1}$$

In order for the model to match the given volatility structure one other equation has to be satisfied, which describes the relation between the volatility and the interest rates in one time step

$$\sigma_n = \frac{1}{2\sqrt{\Delta t}} \ln\left(\frac{r_u}{r_d}\right)$$

Substituting $r_u = r_d e^{2\sigma_n\sqrt{\Delta t}}$, the equation for $B(2)$ can easily be solved and the interest rates at nodes r_u and r_u can be calculated.

For the following time periods, the calculations have to be performed to find the $t + 1$ interest rates which shall be matched to the given interest rates and volatilities. This becomes heavier to compute for each further step in the tree, since more parameters are unknown for each time step. A way of avoiding this is to build the binomial tree using forward induction, which is presented below. (Röman 2010)

2.3.2 Forward induction

Forward induction can be applied to models described as Brownian path independent. Jamshidian showed that the level of the short rate at time *t* in the Black-Derman-Toy model is

$$r(t) = U(t)e^{\sigma(t)z(t)}$$

where $U(t)$ is the median of the lognormal distribution for $r(t)$, $\sigma(t)$ is the level of the short-rate volatility and $z(t)$ is the level of the Brownian motion. The $U(t)$ has to be determined for each time step in order to fit the model to the initial yield curve.

Each node in the tree in Figure 2.6 is represented by (i,j) where i is the time step of the node in the tree and j is the level in the tree indicating the distance from the center level. The initial node is (0,0) which means that $t = 0$ and the center level is zero. The nodes below the center level will have negative j and the ones above will have positive j.

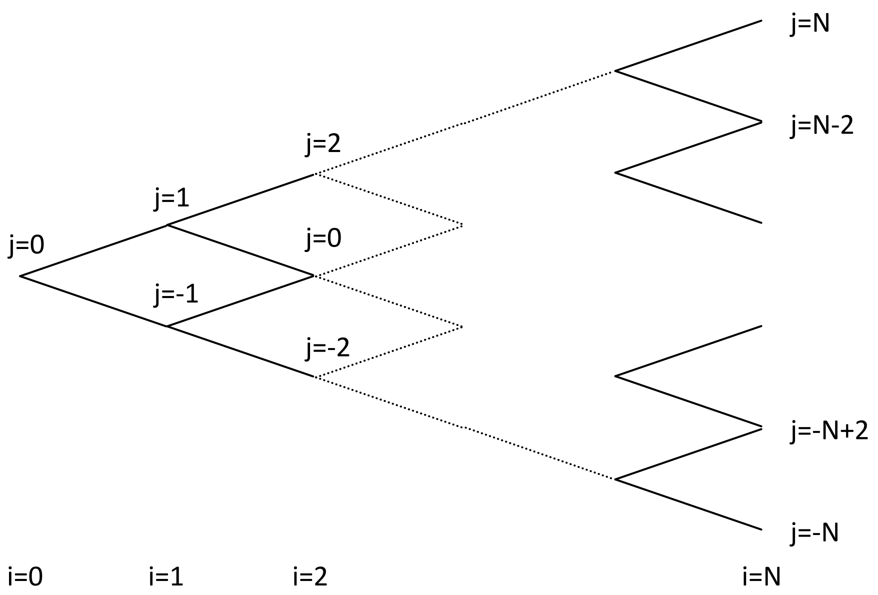

Figure 2.6 Discrete representation of a BPI model
Source: Clewlow and Strickland, 2000

Since j has a centralized binomial distribution with mean 0 and variance *N* at the last time step *N*, $j\sqrt{\Delta t}$ is distributed with mean 0 and variance t. When $\Delta t \rightarrow 0$, the binomial process $j\sqrt{\Delta t}$ converges to the Wiener process $z(t)$. A discrete version of the continuous equation for the short rate above is

$$r_{i,j} = U(i)e^{\sigma(i)j\sqrt{\Delta t}} \qquad \text{(Clewlow and Strickland, 2000)}$$

After calculating all the bond prices from the observed interest rates for each time period of the tree, the next step is to calculate the Arrow-Debreu prices. These represent the value today of a security that pays one cash unit if a point is reached and zero otherwise. These are in each time step of the tree calculated using the previous Arrow-Debreu prices and discount functions as

$$G(i+1,j) = \frac{1}{2}[p(i,j-1)G(i,j-1)]$$ for the top nodes,

$$G(i+1,j) = \frac{1}{2}[p(i,j+1)G(i,j+1)]$$ for the bottom nodes and

$$G(i+1,j) = \frac{1}{2}[p(i,j-1)G(i,j-1) + p(i,j+1)G(i,j+1)]$$ for the nodes in the middle.

To do this, initial values needs to be set for the first time step in the tree. These are $G(0,0) = 1$ and $p(0,0) = B(1)$.

To calculate the discount function for each time step, the following equation needs to be solved to find the $U(i)$.

$$p(0,i+1)=\sum_j G(i,j)p(i,j)=\sum_j G(i,j)\cdot\frac{1}{\left(1+U(i)e^{\sigma(i)j\sqrt{\Delta t(i)}}\right)^{\Delta t(i)}}$$

The equation can be solved using a numerical method such as a one dimensional Newton-Raphson as is described below

$$x_i^{n+1}=x_i^n-\frac{f(x_i^n)}{f'(x_i^n)}$$

A function $f(x_i^n)$ of the unknown variable x_i^n and the derivative $f'(x_i^n)$ of the function are used to find a solution to the Newton-Raphson equation, starting with an educated guess of x_i^n. The formula is iterated until a good enough solution is found, or until a maximum number of iterations is reached.

When building a binomial Black-Derman-Toy tree with forward induction, one period discount factors are used together with Arrow-Debreu prices to adapt $U(i)$ so that the interest rates observed on the market are matched. $U(i)$ is the unknown variable in each time step and is found using the Newton-Raphson method. When it has been found for one time step, the interest rates in that time step can be calculated using

$$r(i,j)=U(i)e^{\sigma(i)j\sqrt{\Delta t(i)}}$$

This calculation gives the final values of the interest rates in that time step. Using those interest rates, the discount functions that will be used in the next iteration can be calculated by

$$p(i,j)=\frac{1}{\left(1+r(i,j)\right)^{\Delta t(i)}}$$

When the discount functions are found, the Arrow-Debreu prices can be calculated for the next time step and the procedure is iterated in this way until all interest rates are found and the tree is complete. (Röman, 2010)

3 Calibration

The reason why an interest rate model should be calibrated is to make the model exactly match an initial term structure. This is done by adjusting the values of some parameters in the model. When the model is calibrated it could be used for pricing. It can then be used to price instruments correctly, with respect to the market data used in the calibration process.

3.1 What to calibrate a model to

Calibration makes it possible to capture expectations for the future of some instrument that the model is calibrated to. This is done by using market data on those instruments in the calibration process. When pricing interest rate derivatives it is of course necessary to somehow calibrate the model to the interest rate curve. This could be done by calibrating to bonds maturing at different points in time. Except from matching the interest rate term structure, it is also important to match the volatility of the interest rate. This is important when pricing interest rate derivatives, since the interest rate is the underlying asset. This implies that the volatility of the interest rate should be used.

To be able to price cancellable swaps, it is not enough to calibrate the model to the interest rate and the interest rate volatility. The volatility of the interest rate does not completely capture the price movements of the cancellable swap, since there is a larger risk involved in the swap rate than in the interest rate. The volatilities of the cancellable swap do not only depend on today's expectations, but are also dependent on future volatilities. These expectations on future volatilities can be seen on instruments on the market that matures where there are possible cancellations in the cancellable swap that is to be priced.

Hagan has shown that it is better to calibrate a model to the volatility of actively traded swaptions instead of the volatility of the interest rate. The expectations of future volatilities will then be incorporated when pricing the cancellable swaps. The reasons why it is not possible to calibrate to the volatility of traded cancellable swaps is that the instruments are not liquid enough to generate sufficiently reliable market data.

3.2 How calibration to swaptions works

The calibration to swaptions works by letting the interest rate model price a swaption, whose price is dependent on some volatility. By varying the value of the volatility, the swaption price can be matched to some reference price. When the prices are exactly matched, the volatility that makes the prices match can be found. This is done for each time step through the model, one at a time, until all volatilities have been found. The volatility that is to be found in one time step is then dependent on volatilities in previous time steps, but independent of those in latter time steps. The volatilities that are the result of the calibration can then be used for pricing the cancellable swaps.

The swaptions that are used for the calibration should have the opposite payment direction to the cancellable swap that is to be priced. The reason is that a cancellation could be seen as entering an offsetting position. The cancellable swap is then seen as an ordinary plain vanilla swap where the cancellation point is an option to enter a swaption with the same fixed rate and with cash flows that are offsetting the rest of the cash flows in the swap.

This way of calibrating to swaptions has not been seen in many articles, but has been developed to give better prices than only calibrating a model to the interest rate and the interest rate volatility.

The Hull-White model has been calibrated to swaptions in the paper "Methodology for callable swaps and Bermudan 'exercise into' swaptions" by Hagan but no such study has been found on the Black-Derman-Toy model. Since the calibration method to calibrate to swaptions has been seen working for some models it is interesting to try to apply it to the Black-Derman-Toy model as well.

Even if the models in this study are calibrated to swaptions, they are still also calibrated to the interest rate term structure. This is done in both models in the tree building procedure, as described in chapter 2.2 and 2.3.

3.3 Calibrating the Hull-White model

When calibrating the Hull-White model, there are two parameters that can be used to match the initial term structure. Both a and σ are variables that can either be constant or time-dependent. These are set in order to make the model exactly price instruments maturing at each time step of the model. The number of time steps chosen for the model will be an arbitrary number in order to build the tree in a satisfactory way.

When more parameters are used, the degree of freedom when fitting the model is higher. By making both a and σ time dependent, the model has the highest degree of freedom but at the same time there is a risk of over parameterization. The model might then be over calibrated, which leads to a more complex model that could result in less good prices. This could happen when pricing instruments that depend on the future volatility term structure, since the volatility structure then is non-stationary. When fixing the parameters in the calibration process, the model will exactly match the initial volatility structure and then make an assumption about how the volatility term structure will evolve to the end of the tree. According to Hull and White (1996) there should not be more than one parameter dependent on time. This means that either a or σ could be time dependent, but not both at the same time.

3.4 Parameters in the Hull-White model

One parameter to vary at each time step should be enough since there is a one-to-one relationship between the volatility and the price of some instrument. If a is fixed as a constant for the whole model, it is always possible in each time step to find a volatility that makes the price of a swaption in the tree exactly equal to the price of the swaption on the market. This makes it reasonable to have a constant for the model and σ time dependent.

The tree is built with some fixed value for a and some assumed values for σ. The swaption prices are then calculated using the tree model and compared with prices on swaptions taken from market data. By varying the value of the σ parameter in one time step in the Hull-White model, the minimum price difference is searched for. According to the one-to-one relationship between the price and the volatility, the value for σ that makes the prices exactly match can always be found. This procedure is done for each time step in the model.

If both a and σ are constant for the whole model, the model could not be calibrated to swaption prices in this way anymore. It will then only be calibrated to the initial term structure of interest rates.

3.5 Calibrating the Black-Derman-Toy model to swaptions

The parameters that can be changed in the Black-Derman-Toy model are the volatility and the median for the interest rates. These could be changed at each time step since both of them are time dependent. When the interest rate tree is calibrated to only the interest rate, the interest rate volatility is taken as an input. The parameter that is changed during the calibration procedure is then the interest rate median. The interest rate tree is then built around it.

When the Black-Derman-Toy model is calibrated to swaptions, the volatilities are changed until the swaption prices equal some reference prices of swaptions. This calibration method requires swaptions to be priced one by one. This can be done in different ways, both by pricing swaptions as usual and by pricing them as bond options. The calibration of the model will be performed for both cases.

3.6 Pricing swaptions as bond options

When pricing a swaption as a bond option some steps are required. The first step is to price the swap part and the second is to price the option part. This is done only for the Black-Derman-Toy model.

When pricing a swap, the cash flows from the fixed side can be priced separately from the floating side. The fixed side will in that case be priced in the tree and the floating side will be priced outside of the tree as if it was a floating rate note (FRN). At each node at maturity of the swaption, a swap for the remaining time of the tree will be priced as if the interest rate at that node was realized.

The second step is the option part which starts when the swaption starts and ends at maturity. The searched values are the present value of the swaps priced at maturity. These can be found by using a sort of discount function. The discount function gives the value at the start node, of receiving one cash unit at a later point if this point is reached, and zero otherwise. This later point is one of the nodes at maturity. When the discount function is multiplied with the value of a swap at the node in question, the result is the value at the start node of receiving this swap value. By summarizing for all nodes at maturity, the value of the bond option is given.

4 Implementation

In this part of the report the implementation of the study is presented. It is described how the study is carried out and which assumptions that are made. Also the market data used as input for the study is presented and explained in this part.

4.1 Presentation of the theoretical example

To show some results on how the two models can price interest rate derivatives as cancellable swaps, a simple test example is constructed where some simplifications are made compared to actual contracts. This example is a theoretical test that illustrates some results that can be compared with prices from Bloomberg. The prices from Bloomberg are in this report seen as reference prices for the models, with the purpose to be matched. All market prices used as input are taken from instruments traded in SEK.

To make a theoretical example that is comparable with prices from Bloomberg, prices on European cancellable swaps with a maturity of seven years have been chosen. This is a reasonably long time period where interest rates over a relevant long future time period could be reflected. The time period should not be too long since the market on calibration instruments then would not be sufficiently liquid.

All cancellable swaps are chosen with the same maturity but with different cancellation dates, where one instrument is cancellable in each year. The time step in both tree models has in this example been set to one year, so annual interest rates from one to seven years are needed as input to the models. Since the models aim to price derivatives as close to Bloomberg as possible, it is important to use the same input as Bloomberg does, as far as it is possible. The interest rates in Table 4.1 are calculated from an interest rate curve taken from Bloomberg the same day as all other market data were collected. Unfortunately it could not be retrieved at the same time, so some deviation on the input data may therefore occur.

Table 4.1 Interest rates used as input
Source: Bloomberg

Time in years	Interest rate in %
1	2. 194
2	2. 543
3	2. 567
4	2. 592
5	2. 780
6	2. 935
7	3. 055

4.2 Types of cancellable swaps and replicating contracts

The chosen cancellable swaps do only have one cancellation point, which makes them European style. European style cancellable swaps are also possible to price by replicating the contract with one plain vanilla swap and one swaption with maturity corresponding to the cancellation date. When the cancellable swap is of Bermudan style, with more than one cancellation point, the contract could not be replicated in this way by adding one swap and one swaption for each cancellation date. This could not be done since the premiums of the cancellation opportunities in a Bermudan style cancellable swap are not just to add and since the cancellation dates are correlated. If the contract is cancelled at one cancellation date, the following cancellation opportunities are worthless. Therefore some other method must be used for pricing this type of contracts, such as a tree model.

The Bermudan style cancellable swap could be replicated with one plain vanilla swap and one Bermudan swaption. The Bermudan swaption must then have the same dates for exercise as the cancellable swap has cancellation dates. This way of replicating is not preferred since the liquidity is not as high in Bermudan style instruments as in plain vanilla instruments. Calibrating models to vanilla instruments seems to be a standard procedure and is also chosen for the calibration method in this study.

Even if the tree models should be able to price any cancellable swap with more than one cancellation point, cancellable swaps with only one cancellation point have been chosen for this study. European cancellable swaps could, as known, be priced much simpler using replicating contracts but are chosen as test instruments for this study anyway. Mainly since they are the simplest possible contracts and the purpose for this test only aims to see that the tree models work for pricing. The fact that they only have one cancellation point does not affect the pricing, even if the models are built for pricing cancellable swaps of Bermudan style.

4.3 Further specifications on the theoretical example

To further construct a test that is as simple as possible, the payment frequencies in both directions, from the fixed and the floating side, have been set to be on a yearly basis. All market data used in the study are also taken when prices on the instruments are at the money (ATM), since the liquidity should be as high as possible. This means for swaptions that the strike rate equals the forward swap rate.

The prices of actively traded swaptions, to be used for the calibration, were also retrieved from Bloomberg on the same day as all other market data. They are presented in Table 4.2 and Table 4.3 below, where m is the time of the option and n the time of the swap. Table 4.2 shows swaptions that allows the holder to receive the fixed rate and Table 4.3 shows swaptions that allows the holder to pay the fixed rate.

Table 4.2 Prices of swaptions to receive the fixed rate
Source: Bloomberg

Swaption $(m * n)$ maturing in year	Price from Bloomberg in %
(1*6) 2011	2.03188
(2*5) 2012	2.41801
(3*4) 2013	2.33632
(4*3) 2014	1.92174
(5*2) 2015	1.39216
(6*1) 2016	0.753747

Table 4.3 Prices of swaptions to pay the fixed rate
Source: Bloomberg

Swaption $(m * n)$ maturing in year	Price from Bloomberg in %
(1*6) 2011	2.03188
(2*5) 2012	2.41802
(3*4) 2013	2.33632
(4*3) 2014	1.92174
(5*2) 2015	1.39216
(6*1) 2016	0.753748

The contracts chosen for the study are either for the holder of the cancellable swap to pay the fixed rate and receive the floating, or to receive the fixed rate and pay floating. The fixed rate is also retrieved from Bloomberg at the same day and is observed at 3.048 % when the contract is to pay the fixed rate and 3.051 % when the fixed rate is to be received. The contract that is to be priced is in both cases priced for the holder of the cancellation opportunity. To calculate the price for a holder of a contract where the counterparty has the cancellation opportunity, first the value of the contract for the counterparty is calculated and then the negative value of this is taken to get the value for the holder.

When the final prices on the cancellable swaps are to be calculated, there are two different ways of calculating. Either to let the tree model price both the fixed and the floating cash flows and discount them through the tree, or to let the tree model be used only when pricing the fixed side of the contract and let the floating side be priced outside the tree seen as a FRN. The cancellation possibilities must in both cases be evaluated in the tree, since they are dependent on the level of interest rates in the tree. Both ways of calculating should result in the same prices, but the way of pricing all cash flows in the tree is in this report referred to as the usual way since it is the most intuitive and is used if nothing else is stated.

When the final prices on cancellable swaps are presented for both models, they are given with a corresponding interval. The intervals are calculated by adding and subtracting five times the price of risk to the reference prices of each cancellable swap. The price of risk is the change in price, given in SEK, when the interest rates changes one basis point. This is a guideline at the bank for getting reasonable intervals where sufficiently good prices could be found.

4.4 Specific for the Hull-White model

The time step in the Hull-White model is, according to the arguments above, chosen to be constant and equal to one year and the parameter a is set to be constant at 0.1 for each time step. This is a good value for a according to Hull and White (1996).

To further analyze whether the chosen calibration method is good for pricing or not, a simple test is constructed for comparison purposes. In the test, both parameters in the Hull-White model, a and σ, are chosen to be constant. The interest rate tree, built on this constant volatility, is then used for pricing cancellable swaps as well. The results are then compared with the prices calculated for the tree built on the volatilities from the first calibration process.

According to Hull and White (1996) a good value for the constant parameter σ is 1 %. This value is also seen in many articles and seems to be a representative value to use for the model. This is also the value used in this simple test together with the same value for the parameter a as in the studied calibration method.

When the cancellable swaps are to be priced, it is important to choose the right pricing method for the cash flows. When interest rates become very negative, the usual way of pricing all cash flows in the tree gives very strange values. The reason is that the negative interest rates do not only make the tree use discount factors larger than one, but they do also define some negative cash flows. This has a large impact when the floating side is priced in the tree, since these cash flows are defined by the tree's interest rates. The cash flows then become negative. This can be avoided when the floating side is priced as a FRN.

5 Results

In this part, the results of the study are presented. The first results come from the calibration process and are presented in form of volatilities. By using these volatilities, calibrated interest rate trees could be built and finally the prices of the cancellable swaps could be calculated. All steps are presented for both the Hull-White model and the Black-Derman-Toy model. For both models also alternative ways of pricing are presented.

5.1 Results from the calibration of the Hull-White model

By calibrating the model, the volatilities that make the model exactly price the swaptions, could be found. The method does then only find solutions if the swaption prices from Bloomberg exactly match the prices from the model.

The volatilities that are the outcome of the calibration process are presented in the following two tables. The volatilities are both presented in the case of a cancellable swap where the fixed rate has to be paid and for the case when it is to be received, depending on the swaptions in the calibration. If the swaptions are constructed to receive the fixed rate, the volatilities that are found from the calibration are used to price the cancellable swap where the owner pays the fixed rate.

Time in years	Volatility in %
2011	29.01
2012	19.75
2013	14.12
2014	9.324
2015	4.939
2016	1.579

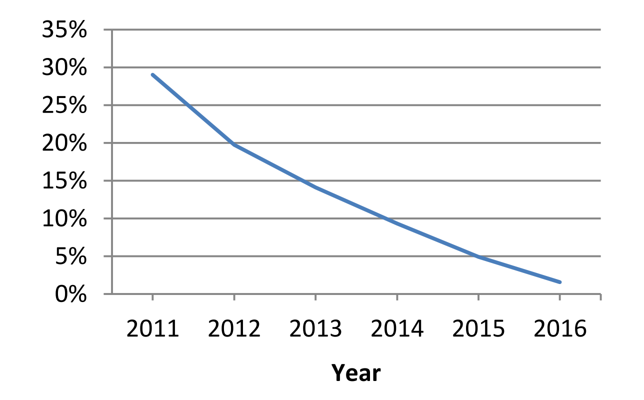

Figure 5.1 Volatilities from swaptions to receive the fixed rate, used to price cancellable swaps to pay the fixed rate

Analogously are the volatilities from the calibration, where the swaptions were to pay the fixed rate, used to price cancellable swaps where the owner has the right to receive the fixed rate.

Time in years	Volatility in %
2011	41.51
2012	25.24
2013	16.10
2014	8.452
2015	3.553
2016	0.6263

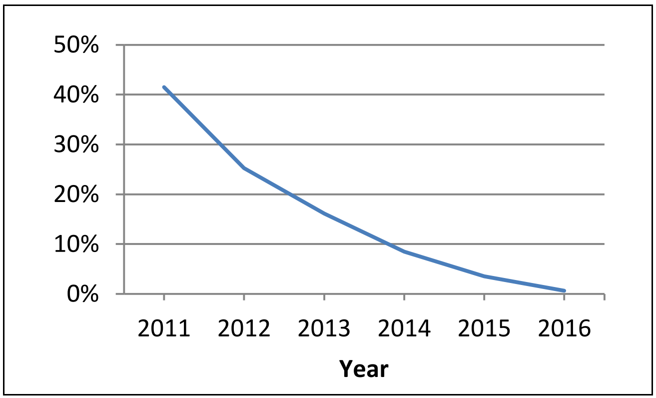

Figure 5.2 Volatilities from swaptions to pay the fixed rate, used to price cancellable swaps to receive the fixed rate

As could be seen in Figure 5.1 and Figure 5.2 above, the volatilities over time present a nice slope, which can be expected from a Hull-White one-factor model. (Hull, 2009)

5.1.1 Results from the Hull-White model while using the calibrated volatilities

By using the volatilities that were the outcome of the calibration process, the following interest rate trees are evolved.

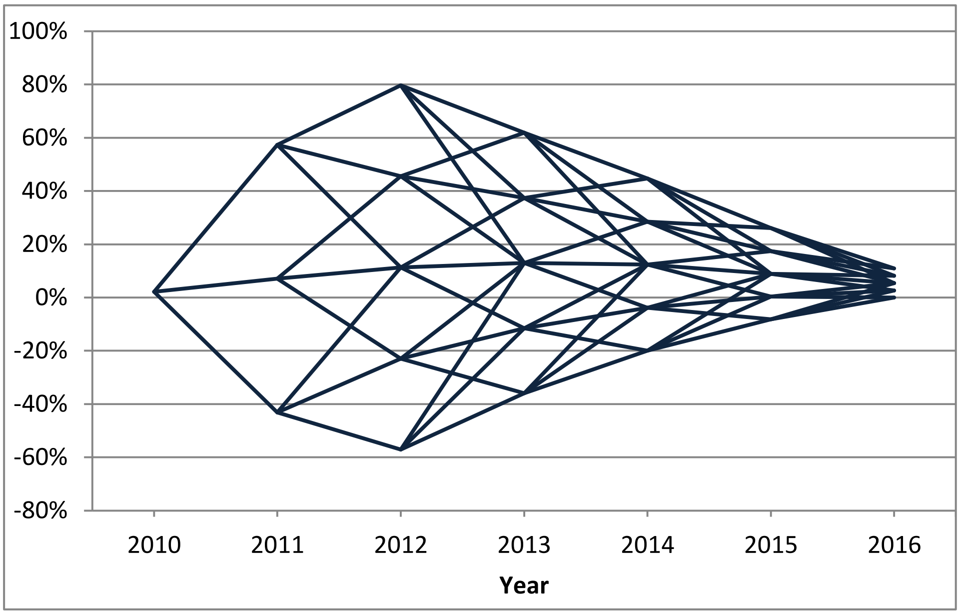

Figure 5.3 Interest rate tree, built on volatilities from Figure 5.1, used to price cancellable swaps to pay the fixed rate

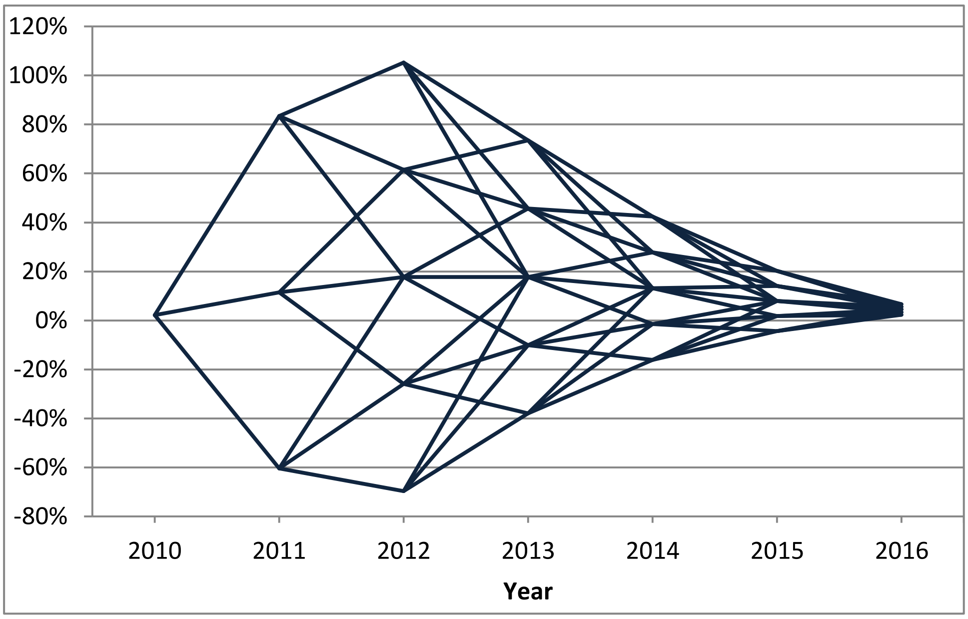

Figure 5.4 Interest rate tree, built on volatilities from Figure 5.2, used to price cancellable swaps to receive the fixed rate

As could be seen in Figure 5.3 and Figure 5.4 above, interest rates move up to very high levels as well as very low. These interest rate movements are of course not probable ways for the actual interest rate to move, especially not to the highest levels and also not to the most negative levels observed. These negative levels are a result from the normal distribution, since the model allows for negative interest rates. Even if the model allows for negative interest rates, the deviations from the central line are too large. The probabilities of moving to these extreme levels are though quite small, but not small enough to be neglected.

Since the interest rates take very negative values, the pricing of cancellable swaps will be made with only the fixed part in the tree. The floating part is calculated as a FRN outside the tree and is included in the final prices of the cancellable swaps.

By using these interest rate trees from the figures above when pricing the cancellable swaps the results presented in the tables below are obtained. The reference prices are, as earlier, prices from Bloomberg, here presented with corresponding intervals for sufficiently good prices. The results from the model are then compared to the intervals and are presented in the columns named result. There is a mark telling whether the result from the Hull-White model fits the corresponding interval or not.

Table 5.1 Prices of cancellable swaps to pay the fixed rate, compared with Bloomberg
Source: Bloomberg

Cancellable swap with cancellation point in year	Price from the Hull-White model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	2.140	1.509	1.340 - 1.679	-
2012	1.653	1.606	1.409 - 1.803	OK
2013	1.039	1.420	1.202 - 1.638	-
2014	0.4530	1.125	0.8889 - 1.360	-
2015	0.1534	0.8282	0.5794 - 1.077	-
2016	0.1120	0.4634	0.2025 - 0.7243	-

Table 5.2 Prices of cancellable swaps to receive the fixed rate, compared with Bloomberg
Source: Bloomberg

Cancellable swap with cancellation point in year	Price from the Hull-White model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	3.590	2.516	2.413 - 2.620	-
2012	2.607	3.212	3.087 - 3.338	-
2013	1.513	3.290	3.137 - 3.442	-
2014	0.5496	2.799	2.612 - 2.987	-
2015	0.08340	2.045	1.828 - 2.263	-
2016	-0.07466	1.104	0.8579 - 1.350	-

These results show that only one of the cancellable swap prices fits the corresponding interval. This would imply that the model could not price this type of instrument that well. This is not strange, since calculations are made in a couple of steps in order to get the prices. If a small change is made to some input data, a difference in the final prices is obtained. Even a very small change in the input data can result in much larger deviations in the final prices, since each step of the calculations will be affected. What could argue for the prices as being correct is the matter of fact that they are in the right dimension. Dependent on from which time the input interest rate curve is taken, more or fewer prices can be found in the corresponding intervals. Even if the input interest rates are taken from just a few hours later.

The fact that the last cancellable swap in Table 5.2, with cancellation possibility in 2016, has a negative value is not strange. The price is just the same as the price of a seven year ordinary plain vanilla swap with the same strike, which implies that a cancellation never will be preferable at this point in time in the model and the cancellable swap therefore should have the same value as a swap that is not cancellable. This is also true for the case in the first table, Table 5.1. The price of the cancellable swap with cancellation point in 2016 is also there the same as a seven year plain vanilla swap with the same strike. The fact that there is a negative value in the second table and not in the first depends on the choice of strike in combination with the input interest rates used by the model.

The prices of the cancellable swaps from the Hull-White model using the data in Table 5.1 and Table 5.2 are presented in Figure 5.5 and Figure 5.6. As could be seen, the prices from the calibration do not fit the intervals well at all.

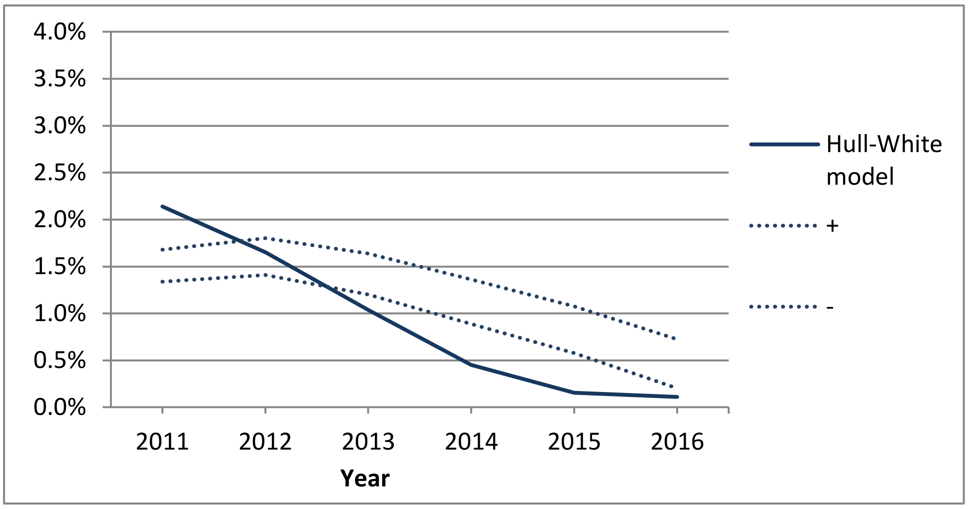

Figure 5.5 Prices of cancellable swaps with the right to pay the fixed rate, compared with Bloomberg

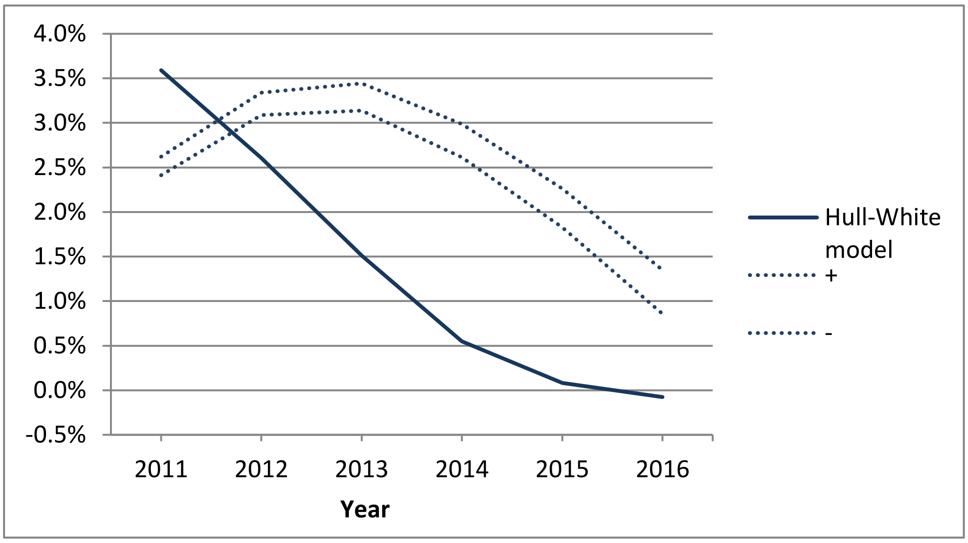

Figure 5.6 Prices of cancellable swaps with the right to receive the fixed rate, compared with Bloomberg

5.1.2 Results from the Hull-White model while using constant volatility

To be able to analyze whether the studied method for calibration is to be preferred or not, the simple test of using constant volatility is presented. When using a = 0.1 and σ = 0.01, the interest rate tree in Figure 5.7 is built. The reason why only one tree is presented is that the tree is no longer calibrated to swaptions and instead the same input volatility of 1 % is used.

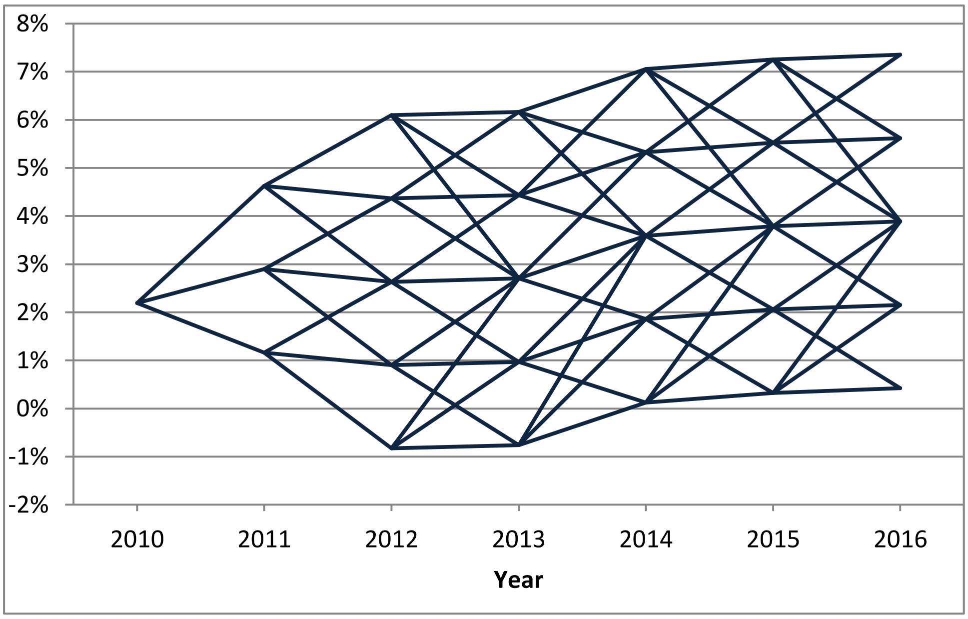

Figure 5.7 Interest rate tree built on constant σ, used to price both cancellable swaps to pay the fixed rate and cancellable swaps to receive the fixed rate

The tree now behaves more like an interest rate tree would be assumed to behave. The interest rates move up to 7-8 % as highest and are not very negative, even if the tree also evolves below the zero line. Since the tree do not evolve to very negative levels, the usual way of calculating all cash flows in the tree can be used when pricing the cancellable swaps. Prices on the cancellable swaps priced by this tree are presented in Table 5.3 and Table 5.4 below. The prices are as previously compared with prices from Bloomberg.

Table 5.3 Prices on cancellable swaps to pay the fixed rate, compared with Bloomberg
Source: Bloomberg

Cancellable swap with cancellation point in year	Price from the Hull-White model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	1.177	1.509	1.340 - 1.679	-
2012	1.659	1.606	1.409 - 1.803	OK
2013	1.622	1.420	1.202 - 1.638	OK
2014	1.330	1.125	0.8889 - 1.360	OK
2015	1.101	0.8282	0.5794 - 1.077	-
2016	0.7902	0.4634	0.2025 - 0.7243	-

Table 5.4 Prices on cancellable swaps to receive the fixed rate, compared with Bloomberg
Source: Bloomberg

Cancellable swap with cancellation point in year	Price from the Hull-White model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	2.259	2.516	2.413 - 2.620	-
2012	2.905	3.212	3.087 - 3.338	-
2013	3.284	3.290	3.137 - 3.442	OK
2014	3.353	2.799	2.612 - 2.987	-
2015	2.720	2.045	1.828 - 2.263	-
2016	1.874	1.104	0.8579 - 1.350	-

These results show that prices on cancellable swaps, when using a constant volatility to build the tree, have a better match to prices from Bloomberg. In this test, four prices are within the corresponding intervals, which means that they are sufficiently good. The other prices, outside the intervals, are here much closer to the intervals than the prices resulting from the studied calibration method. The prices resulting from 1 % volatility do also follow the same pattern over time as the reference prices from Bloomberg and could be observed in Figure 5.8 and Figure 5.9 below.

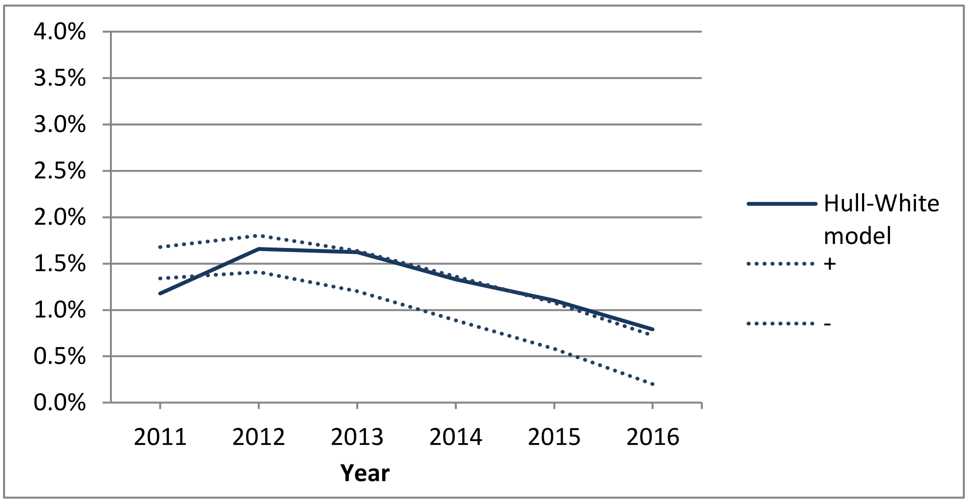

Figure 5.8 Prices of cancellable swaps with the right to pay the fixed rate, compared with Bloomberg

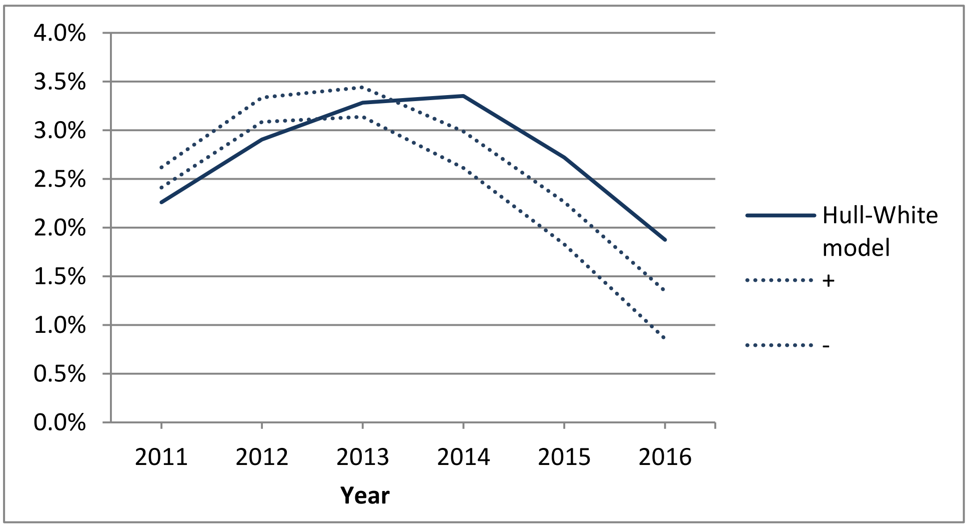

Figure 5.9 Prices of cancellable swaps with the right to receive the fixed rate, compared with Bloomberg

5.2 Results from the calibration of the Black-Derman-Toy model

When the original swaption pricing method was used in the calibration, only the volatility for year 2016 was given as result. When calibrating to swaptions with the right to receive the fixed rate, the resulting volatility was 40.29 % and when calibrating to swaptions with the right to pay the fixed rate, the volatility was 14.67 %. This lack of results led to the decision to price swaptions as bond options for the Black-Derman-Toy model.

The calibration should result in a series of volatilities. In order to get the calibrated values of the volatilities, they must have some initial values at the beginning in order to make the calibration work. These will be replaced one by one in the calibration process.

It was observed that different volatilities were given as result when different initial values of the volatilities were used, even though the input data was the same. Obtaining different solutions from the same input data suggests that other factors than the intended have an impact. The only differences between the received solutions were the initial values of the volatilities. It was then discovered that the first time the calibration was performed, all volatilities could not be found. In order to get solutions for all volatilities in the calibration process, the process had to be iterated more than one time. In some cases more than one different series of resulting volatilities could be found by iterating several times. This can be seen in Table 5.5, where the volatility solutions from the calibration method differ dependent on the number of iterations. This implies that the solutions are not unambiguous since the same input data is used in all cases. The impact of the initial volatilities indicated that the volatilities depended on not yet calibrated volatilities.

The reason behind this phenomenon is that the swaption pricing method used needs volatilities for all time steps until maturity of the instrument in order to price it. The calibration method, on the other hand, assumes that volatilities for each time period can be calculated without knowledge of the volatilities of the following time steps.

Results from the calibration with different values on the initial volatilities can be seen in Table 5.5 and Table 5.6 below. In Table 5.5, the initial volatility values are random and referred to as set 1. The two resulting series of calibrated volatilities are presented, as described above. Since the first iteration of the calibrating procedure could not provide a complete series of volatilities it was iterated again. The first complete series of calibrated volatilities required two iterations and is presented in Table 5.5 as series (1,2). Another complete series of volatilities was the result of the third iteration which is presented as series (1,3) below.

Table 5.5 Volatilities calibrated to receive fixed swaptions with set 1 of volatilities as initial values

Time in years	Initial volatilities set 1 in %	2 calibration iterations series (1,2) in %	3 calibration iterations series (1,3) in %
2011	29.16	2.717	1.337
2012	28.55	54.04	81.73
2013	27.34	25.78	45.97
2014	25.63	28.97	51.90
2015	24.93	37.27	38.38
2016	24.87	40.38	40.97

Table 5.6 shows the result when the initial volatilities are constant at 10 %. This set of initial volatilities is referred to as set 2. The first complete series of volatilities was found in the fifth iteration of the calibration process and will be referred to as series (2,5). No other complete series could be found when a reasonable amount of iterations was performed.

Table 5.6 Volatilities calibrated to receive fixed swaptions with set 2 of volatilities as initial values

Time in years	Initial volatilities set 2 in %	5 calibration iterations series (2,5) in %
2011	10.00	26.77
2012	10.00	66.74
2013	10.00	23.56
2014	10.00	50.78
2015	10.00	38.03
2016	10.00	40.79

The calibration is done for two cases. The first case is when the swaptions are to receive the fixed rate, as seen in Table 5.5 and Table 5.6 above. The second case is when the calibration is to swaptions with the right to pay the fixed rate, as seen in Table 5.7 and Table 5.8 below.

Table 5.7 Volatilities calibrated to pay fixed swaptions with set 1 of volatilities as initial values

Time in years	Initial volatilities set 1 in %	2 calibration iterations series (1,2) in %	4 calibration iterations series (1,4) in %
2011	29.16	22.21	0.4926
2012	28.55	89.06	85.67
2013	27.34	49.23	57.55
2014	25.63	23.33	12.59
2015	24.93	9.017	8.993
2016	24.87	12.28	12.25

Table 5.8 Volatilities calibrated to pay fixed swaptions with set 2 of volatilities as initial values

Time in years	Initial volatilities set 2 in %	2 calibration iterations series (2,2) in %
2011	10.00	13.17
2012	10.00	50.44
2013	10.00	49.35
2014	10.00	10.10
2015	10.00	8.913
2016	10.00	12.21

As explained earlier, the results from set 1 are not unambiguous since more than one series of volatilities can be retrieved from the same input. Therefore only series (2,5) from the calibration to receive fixed swaptions and series (2,2) from the calibration to pay fixed swaptions will be investigated further. These are not ideal either since the shapes of the volatility curves are being uneven and behaving in a non-intuitive way, which can be seen in Figure 5.10 and Figure 5.11 below. The expected appearance would instead be a more or less smooth curve.

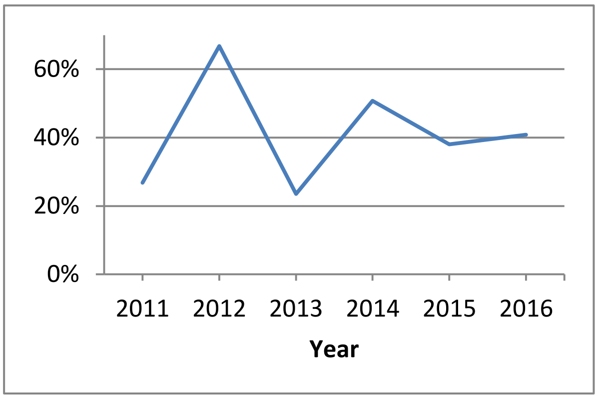

Figure 5.10 Calibrated volatilities from series(2,5)

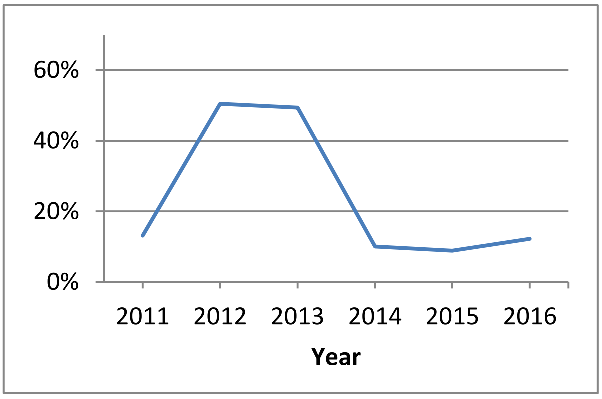

Figure 5.11 Calibrated volatilities from series (2,2)

To be able to move on with these volatilities it is necessary to try to verify them. The calibration can be verified by investigating whether the resulting series of volatilities can price swaptions as intended. If the swaption prices calculated by the model are the same as the reference prices, the result from the calibration will be verified.

5.2.1 Comparing swaption prices from the Black-Derman-Toy model

As described earlier, there are a lot of things arguing that the series of calibrated volatilities are not entirely correct. Below in Table 5.9 and Table 5.10 are the results from calculations of swaption prices that are performed to verify the volatilities that were the result of the calibration of the Black-Derman-Toy model. Each table contains the swaption prices from the model compared to the prices from Bloomberg and their corresponding intervals for reasonably good prices.

Table 5.9 Prices on swaptions to receive the fixed rate, calculated with series (2,5)
Source: Bloomberg

Swaption with maturity in year	Price from the Black-Derman-Toy model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	2.757	2.032	1.906 - 2.157	-
2012	2.584	2.418	2.317 - 2.519	-
2013	2.409	2.336	2.257 - 2.416	OK
2014	1.946	1.922	1.865 - 1.978	OK
2015	1.397	1.392	1.355 - 1.429	OK
2016	0.7537	0.7537	0.7356 - 0.7719	OK

Table 5.10 Prices on swaptions to pay the fixed rate, calculated with series (2,2)
Source: Bloomberg

Swaption with maturity in year	Price from the Black-Derman-Toy model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	2.260	2.032	1.883 - 2.181	-
2012	2.135	2.418	2.292 - 2.544	-
2013	2.207	2.336	2.236 - 2.437	-
2014	1.874	1.922	1.851 - 1.993	OK
2015	1.392	1.392	1.346 - 1.439	OK
2016	0.7537	0.7537	0.7310 - 0.7765	OK

In Table 5.9, four out of six swaption prices calculated from volatility series (2,5) are within the intervals and in Table 5.10, three out of six swaption prices calculated from volatility series (2,2) are within the intervals. The prices from Table 5.9 and Table 5.10 follow the Bloomberg prices over time, which could be seen in Figure 5.12 and Figure 5.13 below. It cannot be decided from these results if one of the series is better than the other.

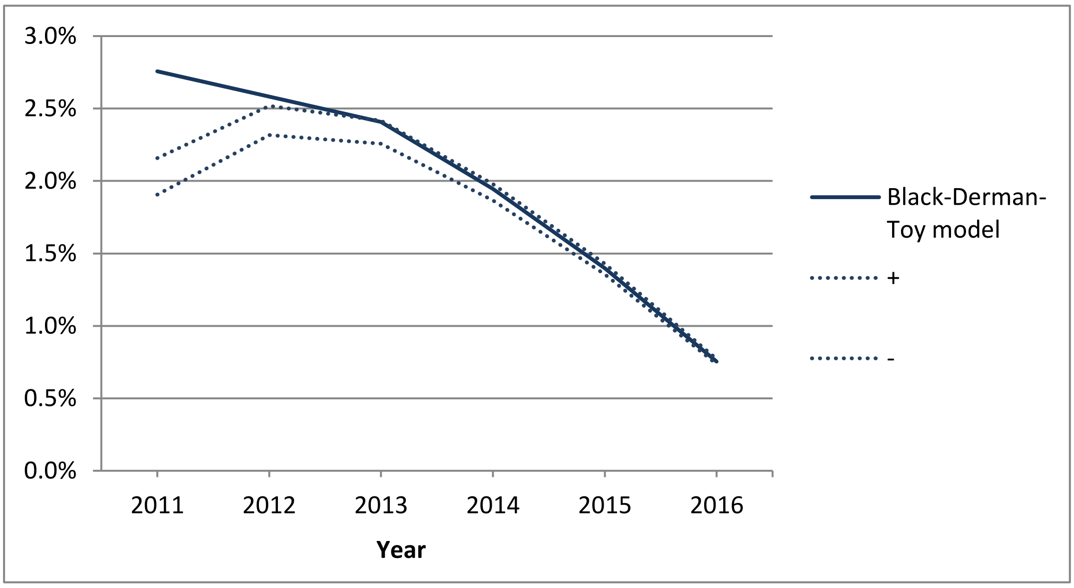

Figure 5.12 Prices of swaptions with the right to receive the fixed rate, calculated with series (2,5)

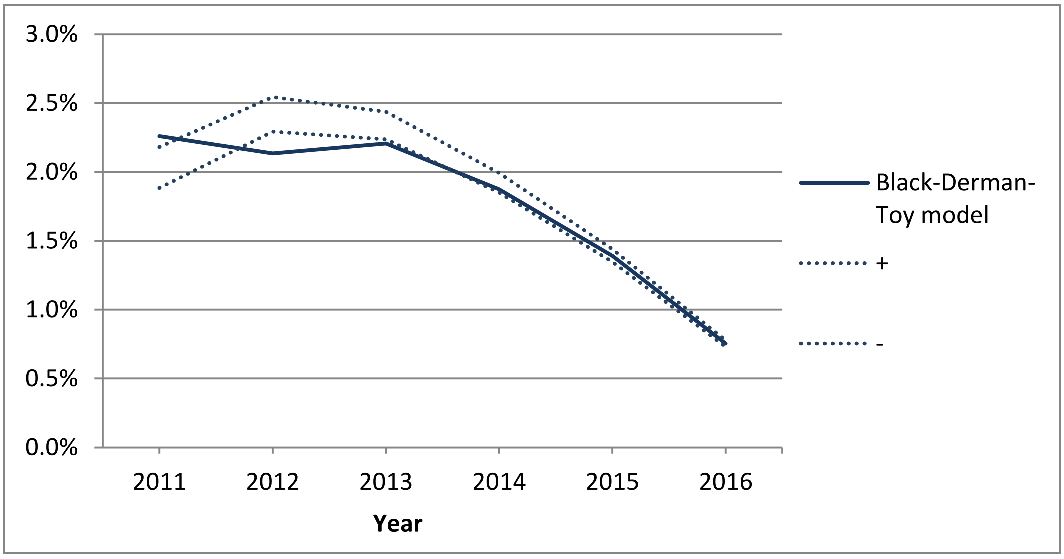

Figure 5.13 Prices of swaptions with the right to pay the fixed rate, calculated with series (2,2)

The calibration aims of course to match the prices from Bloomberg and if the calibration had been succeeded, the reference prices would have been exactly matched. The prices from the calibration are however considered as sufficiently good, so in lack of better results these calibrated volatilities will be used further.

5.2.2 Using the calibrated Black-Derman-Toy model

An interest rate tree needs a series of volatilities to be evolved and the series (2,5) and (2,2) of calibrated volatilities of course lead to two different interest rate trees pictured below. Figure 5.14 shows the interest rate tree created with series (2,5) of calibrated volatilities. Figure 5.15 shows the interest rate tree created with series (2,2) of calibrated volatilities.

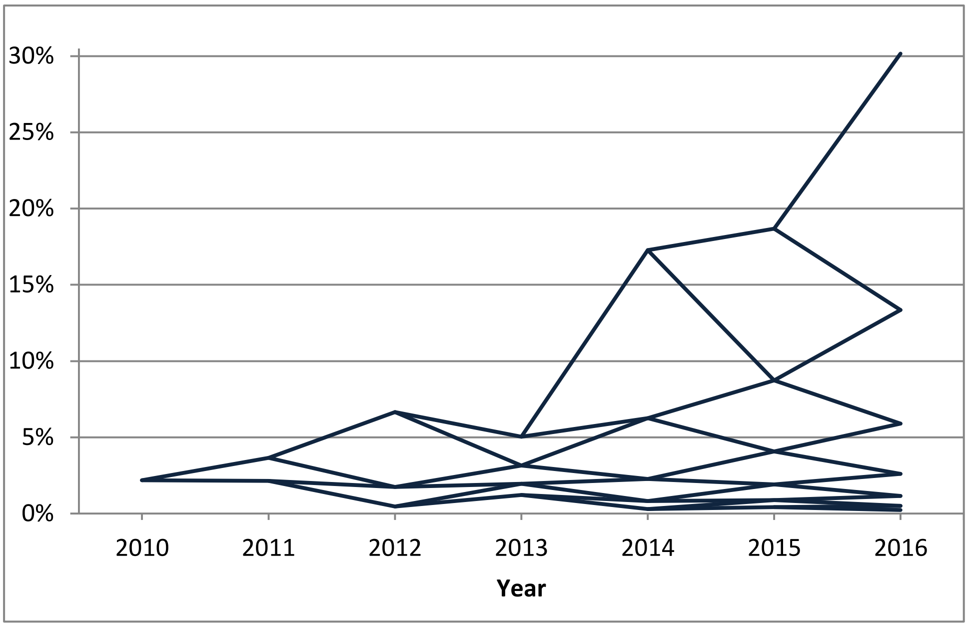

Figure 5.14 Interest rate tree on series (2,5) from calibration to swaptions with the right to receive the fixed rate

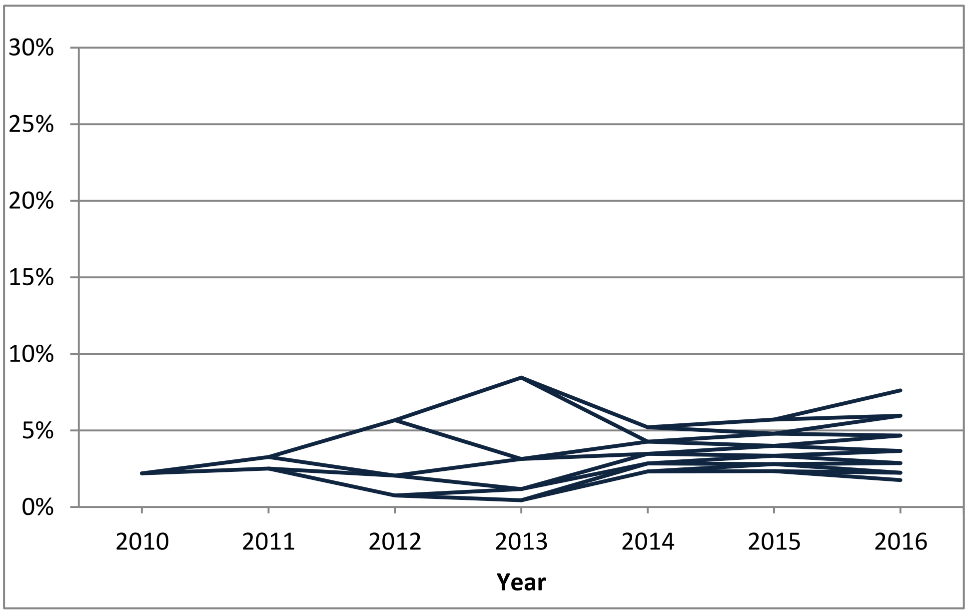

Figure 5.15 Interest rate tree on series (2,2) from calibration to swaptions with the right to pay the fixed rate

Looking at these trees it is clear that the volatilities used when building them have a large impact. Two trees that are evolved from the calibration to either swaptions to receive the fixed rate or swaptions to pay the fixed rate can be slightly different if the prices of the calibration swaptions are different. However, a difference as large as the one seen in the figures above cannot be explained by price differences in the swaptions calibrated to, since these prices were almost the same. The interest rate trees that are evolved reflect the volatilities that created them. The tree in Figure 5.14 is built with higher volatilities at the final time steps than the tree in Figure 5.15, which makes it wider. They do not present any negative interest rates, which is good since it is one of the implications of the log-normality. They also discount cash flows correctly, which could be proven and is one way of verifying an interest rate tree.

5.2.3 Pricing cancellable swaps using the Black-Derman-Toy model

The final step is to price the cancellable swaps. If the volatilities are correctly calibrated, these prices should be comparable to the prices from Bloomberg. The results in Table 5.11 are prices of cancellable swaps to pay the fixed rate and receive the floating, calculated using a tree built with series (2,5) of calibrated volatilities. The results presented in Table 5.12 are prices of cancellable swaps to receive the fixed rate and pay the floating, calculated using a tree built with series (2,2) of calibrated volatilities. Both tables also present the corresponding prices from Bloomberg and the intervals of sufficiently good prices. The price at year 2011 is the price of a seven year cancellable swap where the cancellation possibility is in year 2011.

Table 5.11 Prices of cancellable swaps to pay the fixed rate, calculated with series (2,5)
Source: Bloomberg

Cancellable swap with cancellation point in year	Price from the Black-Derman-Toy model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	2.832	1.509	1.340 - 1.679	-
2012	3.264	1.606	1.409 - 1.803	-
2013	2.480	1.420	1.202 - 1.638	-
2014	2.847	1.125	0.8889 - 1.360	-
2015	2.022	0.8282	0.5794 - 1.077	-
2016	1.561	0.4634	0.2025 - 0.7243	-

Table 5.12 Prices of cancellable swaps to receive the fixed rate, calculated with series (2,2)
Source: Bloomberg

Cancellable swap with cancellation point in year	Price from the Black-Derman-Toy model in %	Price from Bloomberg in %	Calculated price interval in %	Result
2011	2.456	2.516	2.413 - 2.620	OK
2012	2.949	3.212	3.087 - 3.338	-
2013	3.102	3.290	3.137 - 3.442	-
2014	2.193	2.799	2.612 - 2.987	-
2015	1.683	2.045	1.828 - 2.263	-
2016	1.282	1.104	0.8579 - 1.350	OK

It is easy to see that the prices in Table 5.11 above, calculated using series (2,5) of calibrated volatilities are far from the prices from Bloomberg. Series (2,5) is the series that gave the unreasonably high interest rates and since they are used to price a cancellable swap where the holder pays the fixed rate and receives the floating, there is no wondering why the prices in Table 5.11 above are so high.

The prices in Table 5.12 are somewhat better than the ones seen in Table 5.11 and the prices at year 2011 and 2016 are within the limits of reasonable deviations. This argues for the series (2,2) of calibrated volatilities being the better series for pricing cancellable swaps. On the other hand, the interest rate tree used to price these cancellable swaps is very narrow and by verifying the swaption prices it was indicated that the calibration was not perfectly done. The results from Table 5.12 are presented in the following graph, Figure 5.16.

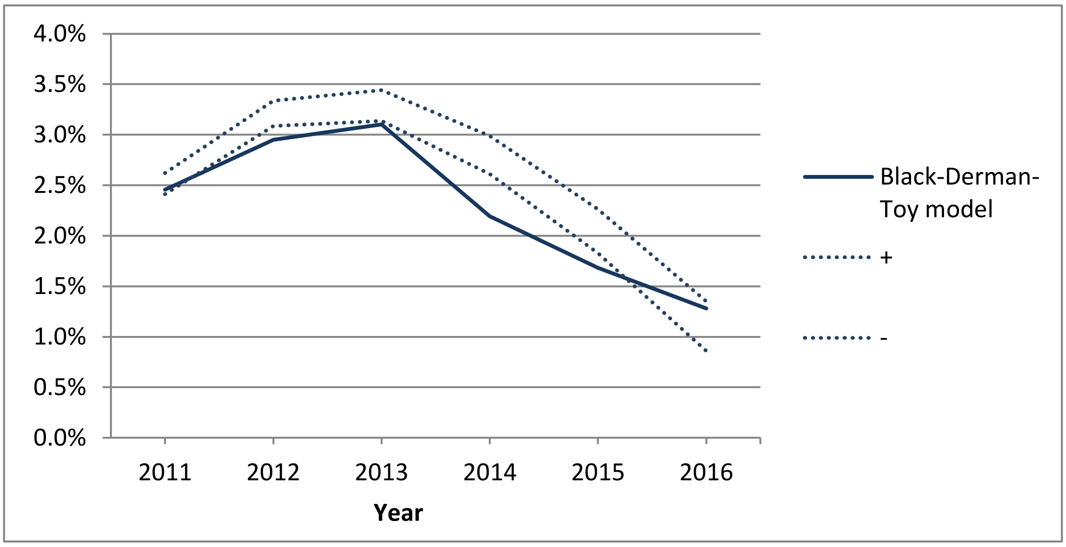

Figure 5.16 Prices of cancellable swaps to receive the fixed rate, calculated with series (2,2)

In Figure 5.16 we can see that the model prices follow the same pattern over time as the prices from Bloomberg and that prices outside the acceptable deviation interval anyway are quite near and of the same dimension.

6 Discussion

After presenting the results of this study, some discussion will be made about the results and the input used. The discussion will argue for and against the obtained results and a comparison will be made with the alternative ways of pricing. Finally the two models will be compared and the calibration method discussed. This will lead to the next part, the conclusion.

6.1 General discussion about input data for both models

If the aim is to make a model give prices on the instruments that matches the prices from Bloomberg, the interest rates that Bloomberg uses for pricing should also be used as input interest rates to the model. This could unfortunately not be done since all market data could not be generated at the same time. Even though all market data were collected on the same day, the movements in interest rates and prices during the day affected the results to a large extent. By using interest rates from just some hours later, the swaption prices as well as interest rates were noticeably different.

A small difference in the used interest rates leads to a mispricing transferred through the models. It first affects the volatilities, which are the result of the calibration, and then by evolving the new interest rate trees using these volatilities, it leads to even larger differences in the final prices of the cancellable swaps being priced using the trees. The deviations from the price intervals could be explained by the interest rates used as input not being correct. The fact that the models are very sensitive to interest rate movements is not strange since they are interest rate models where the interest rate is the underlying on which the instrument prices depend. This makes it understandable why the final prices differ. If the input interest rates show some small deviation from what Bloomberg uses, it is not strange if the prices of the cancellable swaps indicate a much larger price deviation.

6.2 Discussion about the results from the calibrated Hull-White model

The final prices of the cancellable swaps in this study do not fit very well in the corresponding intervals. This could speak for the model not being able to price this type of instrument correctly. An argument for the model although being correct is the fact that there are small deviations in the input data. When transferred through all calculations in the model, small deviations are enlarged on the way to its very last step of pricing the cancellable swaps, causing a much larger mispricing of the instrument. Another thing speaking for the model being correct is that a lot of code in the program is used for both pricing the swaptions and the cancellable swaps. Since the swaption prices are matched exactly using the code, it implies the code although being correct.

The interest rates from the Hull-White model, as are presented in the trees in Figure 5.3 and Figure 5.4, may seem unrealistic since they are both higher and lower than interest rates in reality are expected to be. Intuition would say they are wrong and that too large volatilities have been used to evolve the trees since they are very wide. This could be explained by the fact that the model uses normal distributed interest rates and that it allows for interest rates being negative. Again it could be argued that the interest rate trees are correct, since they can be used to price swaptions correctly. Whether the deviation in the input interest rates and the normal distribution of the model could explain the strange interest rates given by the model is not at all certain.

The reason why the interest rates take unreasonably high and low values is the direct connection to the volatilities used in the model to build the trees. High volatilities make the trees very wide, while smaller volatilities make all nodes in the trees much closer to the central nodes. If the interest rates are too high or low, it is mainly dependent on the volatilities used when building the tree.

Since there is a one-to-one relationship between swaption prices and volatilities, the volatilities that are the result of the calibration process in this study are for sure the right ones. The volatilities that are found make the prices from the model match the prices from Bloomberg exactly. So even if the volatilities could seem unreasonably high they are for certain the right ones according to this calibration method.

6.3 Discussion about the comparison test for the Hull-White model

To be able to analyze the calibration method studied in this report, some kind of comparison is needed. By comparing the results obtained when pricing cancellable swaps, the calibration method could be put in contrast to the test with the constant volatility.

By the simple comparison test presented in this report, the conclusion could be that constant volatility gives better prices than the prices resulting from the studied calibration method. The fact that different methods for pricing the cash flows of the cancellable swap have been used is of course not optimal for comparison purposes, but it was necessary in order to get any reasonable prices at all. The results from the two pricing methods should not differ under normal circumstances, but the very negative interest rates from the calibration have in this case a large effect on the prices. Other conclusions from using two ways of pricing cash flows are not possible to make.

Even when using a constant volatility, the model is still calibrated, but then only to the input interest rates. By using the calibration procedure studied in this report, the model is calibrated to swaptions as well as to interest rates. This way of making the model have a larger degree of freedom might result in the model becoming over-calibrated. If this is the case, the much simpler version of keeping both a and σ constant could preferably be used. A simple model can, as explained previously in the report, be better when it comes to pricing than a more complex model.

There is a quite large difference in size of the constant volatility of 1 % and the volatilities that are the result of the studied calibration method. To use 1 % constant volatility when evolving a tree in comparison to much higher volatilities makes huge differences in obtained interest rate levels. The interest rates trees evolved using the volatilities from the calibration have unrealistically high and low values, which is a direct effect of the volatilities used. The interest rate tree evolved with the constant volatility in Figure 5.7 is instead much more reasonable and realistic.

To continue pricing instruments using unrealistic interest rate trees cannot be recommended. Even if a model with both a and σ constant has a lower degree of freedom, it has by this comparison shown to be better. The main reasons are that it results in more reasonable interest rate trees as well as in better prices of the cancellable swaps.

6.4 Discussion about the results from the Black-Derman-Toy model

The differences in volatilities received through the calibration depend on the set of initial volatilities and how many times the calibration was iterated. This is not a desired behavior. A correct set of calibrated volatilities should be found independent of the initial volatilities and they should not change if the calibration is done again. Furthermore, when the reference prices that the calibration tries to match are almost the same for both kinds of swaptions, the outputted calibrated volatilities should also be almost the same as well. This means that the resulting volatilities calibrated to swaptions to pay the fixed rate and to swaptions to receive the fixed rate should resemble each other, but they do not. This could possibly come from the small difference in strike in the swaption contracts, but does mainly come from the calibration process. Since the calibration is failing to find unambiguous volatilities, it is hardly interesting to analyze prices calculated using a tree built on these volatilities. The rest of the results were studied out of pure curiosity.

The swaption prices from the model get closer to the prices from Bloomberg for each year in both examples, but the calibration process should make the trees price swaptions exactly. The calibrated volatilities should guarantee a correct pricing of swaptions, but since swaptions with early maturities are not correctly priced, the volatilities from the calibration process are not correct.

Interest rate trees evolved with the volatilities from the calibration process are acceptable when considering that they do not present any interest rates that are negative and that they discount correctly. The fact that some interest rates at the end of one tree are higher than what would be expected in reality indicates that the tree might not be correct. Then again, it is a model and therefore some extreme interest rates at the outmost nodes might not necessarily mean it is incorrect. But the interest rate trees are still considered incorrect since they are too different from each other. One of the trees enables for the interest rate to move up to 30 % which is not probable in reality. The other three presents no possibility to move beyond 10 % in a seven year long period. The trees are however adapted to today's interest rate curve so this narrow tree might be correct, since today's interest rates are relatively low.

Finally, the prices of the cancellable swaps were calculated. The interest rate trees used can have a great impact since they are both used for discounting and for defining the payments. The high prices on cancellable swaps that were priced with the wider tree can be explained by the fact that the cancellable swap is to pay fixed and receive floating. The holder of the instrument is profiting of high interest rates since the fixed amount paid is constant over time. This could be why these cancellable swap prices are so high.

Therefore, the wide interest rate tree gives a higher price of a cancellable swap than the narrower tree and the fact that the prices are far from the reference prices again indicates that the interest rate tree is erroneous and consequently, the volatilities are faulty. The narrower interest rate tree was used to price a swaption with the right to pay a fixed rate. The resulting prices were closer to the Bloomberg prices than the swaption prices calculated with the wider tree. In some cases the prices were in the acceptable intervals. This argues once again for this narrower tree being correct, since it prices swaptions almost as Bloomberg.

6.5 Discussion about the chosen source for input data

Since all market data is collected from Bloomberg, it could be discussed whether it is good or bad to rely on only one source. This would normally not be recommended, but for this study the input data as for example on interest rates should come from the same source as the prices of swaptions and cancellable swaps, especially since the prices should be comparable. Therefore it is considered good to use market data only from Bloomberg and furthermore Bloomberg is considered reliable.

6.6 Comparison of the two models and obtained results

An interesting idea would be to compare the two models. Since the Black-Derman-Toy model could not be calibrated with a good result in this study, a comparison of cancellable swap prices would not be relevant. What could be compared are instead the interest rate trees. What could be seen is that the levels of interest rates in the trees that are calibrated to swaptions are different between the models. While the interest rates in the Hull-White model take extreme levels, the interest rates in the Black-Derman-Toy model take more reasonable values, even if they most likely are unrealistic according to the failing calibration. When a constant volatility was tested for the Hull-White model, a much narrower interest rate tree was built. This seemed to be a very realistic tree presenting possible paths for actual interest rates to take, except for the negative values of course.

One explanation for negative interest rates only coming from the Hull-White model and not from the Black-Derman-Toy model is that the Hull-White model uses normal distributed interest rates while the Black-Derman-Toy model uses lognormal distributed interest rates. This is also why the Hull-White model, even with constant volatility, builds the tree with negative interest rates, but the Black-Derman-Toy model never does. The Black-Derman-Toy trees shows instead a more uneven distribution of the interest rates at maturity, since more paths are leading to middle and very low interest rates and only some interest rates at maturity are taking higher values. This distribution is a typical characteristic of this model, implied by the log-normality.

The swaption prices are not relevant to compare at all since there never is a problem to get the exact prices with the Hull-White model, while only about half of the prices are within the intervals using the Black-Derman-Toy model. The reason why the Hull-White model prices the swaptions better is a direct consequence of the swaption pricing method used. Unfortunately the Black-Derman-Toy model could not get any solutions at all using this method, so the bond option method for pricing swaptions had to be chosen in order to get any calibration results at all. Even with this method, the results were not satisfying.

The volatilities found through the calibration are not really comparable either, since different models require different types of volatilities to be used. Even if the calibration for one model works, the resulting volatilities cannot be used directly as an input to the other model. The reason is that the Black-Derman-Toy model uses lognormal distributed interest rates while the Hull-White model uses normal distributed interest rates.

Even if both models were tested for some constant volatility, they were tested in different ways. The Hull-White model was tested with the constant volatility as a direct input, while the Black-Derman-Toy model only was tested with some indirect constant volatility. This was necessary in order to get any calibration results at all from the Black-Derman-Toy model for pricing the cancellable swaps. The differences in swaption pricing method as well as the difference in calibration method are factors that make further comparison of the models difficult.

6.7 Generally comments on the calibration method

Some comments on the calibration method are needed in order to analyze the advantages and disadvantages of using it in combination with each of the two models. It is also interesting to discuss whether one model could be preferable to the other, to be used together with the calibration method.

The results of this study could not show that the calibration method, to calibrate to swaption volatilities, could be preferable. Instead a much simpler model, only calibrated to interest rates, could be used, at least for the Hull-White model. When using constant volatility the model resulted in better prices of the cancellable swaps, which at least for this study makes the studied calibration method unfavorable. Another argument against the calibration method is the fact that the Hull-White model, according to the one-to-one relationship between swaption prices and volatilities, always finds the correct volatilities. Since these with certainty are the right ones, according to the calibration method, the calibration method is either wrong or incorrectly implemented.

The calibration could possibly give better results if the model was implemented with more time steps and calibrated to more swaptions. This was not done in this study but is a recommendation for further studies before any definitive conclusion could be drawn about using this calibration method together with the Hull-White model.

When calibrating the Black-Derman-Toy model using the bond option pricing method, it assumes that all volatilities for the whole life of the instrument, except for the one that is to be found, are known. This makes the investigated calibration method unsuitable since it searches for the volatilities one by one and not all at the same time. The calibration concept of finding volatilities by changing parameters and thereby making the tree price traded swaptions correctly might work if a search method is used that can optimize several parameters at the same time. Then the volatilities for all time steps could be found at the same time. This could eliminate the use of initial volatilities and possibly result in correctly calibrated volatilities. This is, however, left as a suggestion for further studies.

7 Conclusions and recommendations

To conclude the results of this study and to give a suggestion for further studies, this part presents the main findings and most important areas of improvement.

7.1 Conclusions

The fact that the prices are very sensitive to interest rate movements speaks for the importance of using the "right" interest rate curve as input when pricing the instruments. The right curve was in this study defined as the curve that Bloomberg uses and the aim to compare prices from the models with prices from Bloomberg. Since the prices and interest rates change over the day, total conformity could only be achieved using real time updates. This could unfortunately not be used in this study, but is recommended for best possible price conformity.

The conclusions on the Hull-White model are that the model could be calibrated in this way to the interest rate and actively traded swaptions with reasonably good results, but the results are doubtful. The prices on the cancellable swaps are in the right dimension, but not as accurate as hoped. The fact that the interest rate trees built on the volatilities from this calibration are being as unrealistically high and low as in Figure 5.3 and Figure 5.4 makes it absurd to continue pricing instruments with these interest rate trees.

By using a constant volatility instead, more accurate prices on the cancellable swaps could be obtained. This argues for the model calibrated only to interest rates would be better than calibrating it to both interest rates and volatilities on swaptions. In the latter case the results might depend on the model being over-calibrated.

Since the pricing strategy used for swaptions together with the Black-Derman-Toy model required more information than could be given, unambiguous solutions could not be guaranteed, meaning that the results from the calibration were not reliable. There were also unexpected differences in volatilities over time, which is an additional argument for the calibration method not working correctly.

The succeeding computations also indicated that the trees used were not right. Prices of cancellable swaps were reasonable when the narrow tree in Figure 5.15 was used, but not when the wide tree in Figure 5.14 was used. Assuming that the pricing method for cancellable swaps is correct, the wide interest rate tree must be incorrect since it implies these faulty cancellable swap prices. Again this indicates that there could be an error in the calibration or in the calculations where results from the calibration are used. This does not mean that this kind of calibration is impossible or dissuaded for the Black-Derman-Toy model, but simply that this theoretical example could not provide enough good results to say anything definitive about this calibration method to be used together with the model.

Further studies might tell whether swaptions are suitable plain vanilla instruments to calibrate the Black-Derman-Toy model to, or if there are other calibration instruments better suited. They might also tell whether the calibration procedure investigated in this study is the most suitable one, or if there are other interesting ways of calibrating the model.

7.2 Recommendations

This theoretical study is of course just a sample and with more extensive models other results could be obtained. One way of making the models give better prices is to make them more accurate. If the interest rate trees would have smaller time steps, they would probably be more accurate by having more paths leading to the end. Incorporating day count conventions is another way of making the models more accurate, or at least more usable. Further extensions of the models could be to implement real time fetch of input interest rates and swaption prices.

All these extensions that would make the models more accurate are therefore recommended. But even if the models are made more accurate, they are still models, which means that they could never be able to price instruments exactly, if the definition of an exact price is the price on the market. Anyway, what could be said is that a calibrated model would do better than a model that is not calibrated, at least if it is calibrated in the right way and to the right instruments.

A recommendation for further studies is also to find a way to verify the interest rate trees, besides making sure they discount cash flows correctly, since this requirement is necessary but not sufficient.

To improve the results from the Black-Derman-Toy model, implementing a new search method that searches for all volatilities in all time steps of the cancellable swap at the same time could possibly be working if the bond option pricing method is applied.

References

Bloomberg. Retrieved November 26, 2010

Clewlow, L. and Strickland, C. (2000). *Implementing Derivatives Models*, Anthony Rowe LTD, Wiltshire

Hagan, P.S. *Methodology for callable swaps and Bermudan "exercise into" swaptions*, New York

Hull, J.C. and White, A. (1996). *Using Hull-White interest-rate trees,* Toronto

Hull, J.C. (2009). *Options, futures and other derivatives,* 7th edn, Pearson Education, Inc., New Jersey

Röman, J. Analytical Finance II. *Analytical Finance by Jan Röman.* Retrieved September 1, 2010 from http://janroman.dhis.org/index_eng2.html

Swedbank. Om Swedbank*. Swedbank.* Retrieved January 23, 2010 from http://www.swedbank.se/om-swedbank/index.htm