Convertible Bonds

- Characteristics of Convertible Bonds
- Convertibles as Bonds
- Who invests in Convertibles?
- A Simple pricing model for convertible bonds

Characteristics of Convertible Bonds

A convertible bond is a security issued by a company for the purposes of borrowing money. In addition to the usual features of a bond (coupon rate, payment interval, face value, maturity date), a convertible bond has been endowed with the added benefit of being converted into common (sometimes preferred) stock. This leads to new features which the buyer must be well aware of before buying this instrument.

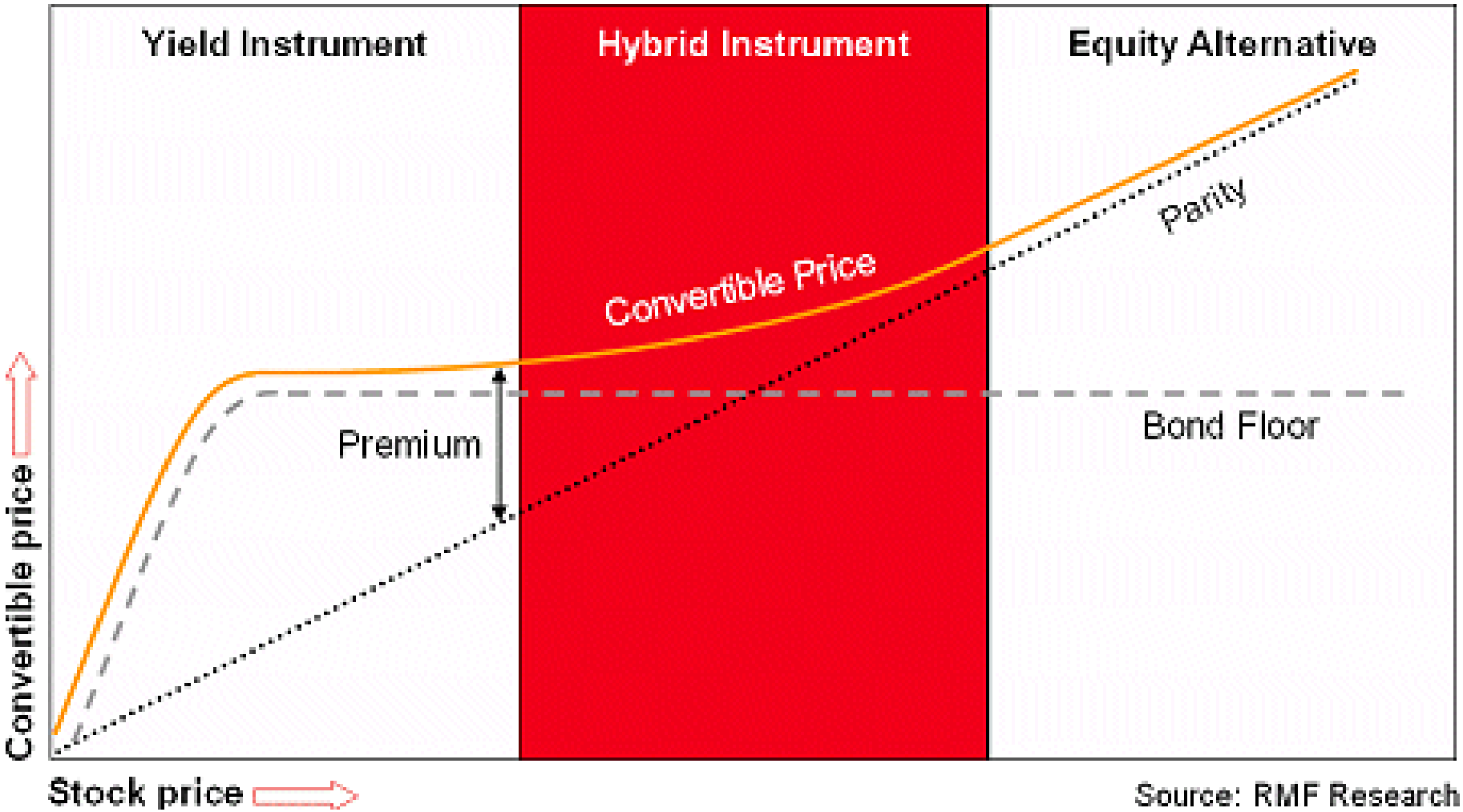
First is the **conversion price**; this is the price at which a convertible bond can be converted into stock. The conversion price goes hand in hand with the **conversion ratio** which is simply calculated as the face value of the bond divided by the conversion price. To go a step further, we can determine the value of doing a conversion simply by multiplying the conversion ratio by the current stock price. If that result is greater than the amount paid for the bond, then the bond is said to be ‘in-the-money’ (same expression and sentiment as in the options world). When a convertible bond is in-the-money, a certain part of its trading price is derived from this fact. That amount is referred to the **conversion premium** and is obviously exclusive to convertible bonds.

Obviously, a portion of the bond's value remains fixed; this is the traditional value derived from face value and coupon rate and is typically called **investment value** or **bond value**. This is sometimes also referred to as the floor value, since the bond should never trade below this value. The conversion option on the bond is exactly that, an option, so the minimum value of it is 0. If the option part of the bond is worthless, (unlikely since there should be some time value, unless we are very close to maturity) it still has the value equivalent to any other bond of equal credit rating paying that coupon and face value. For that reason we state again that a convertible can never trade below its bond value.

Convertibles as Bonds

In general, a convertible bond is issued at a lower coupon rate than a regular bond in the same debt issue. This is due to the fact that its value can derived from the bond yield or from the equity price. In addition, unlike a normal bond, a convertible is less sensitive to changes in the market interest rate because the stock value is there to offset the fluctuations in the interest rate.

However, in most cases, the behaviour of the bond price is an after-thought in the discussion of a convertible's value. What matters the most (as you'll see in the chart below) is the stock price of the underlying company.



Source: RMF Research

The stock price in the first region is such that it is not profitable to convert the bond. In this case we see that the convertible bond's value is the bond value plus a small premium due to the potential of a profitable conversion in the future. In the middle region, the conversion becomes mildly profitable and we start to see the change in the valuation of the convertible using solely the bond price towards using just the stock price. By the time we reach the third region, the conversion premium is the only thing driving the price of the convertible.

Who invests in Convertibles?

Generally these are people seeking (relatively) safe income yet would like to share in capital appreciation. The convertible bond is the perfect way to do that. It is much more direct than the new so-called "index-linked notes" (which are, at the core, just a fancy name for a portfolio which is long a zero-coupon bond and long some index options). The advantage with the convertible is that you get the interest payments (whereas index-linked notes give no interest) and as we see in the third region of the graph, you are fully participating in the capital appreciation of the company. The only advantage the index-linked notes provide is that their zero-coupon bond is a government bond, thus has the advantage of being risk-free compared to the corporate convertible bond.

A Simple pricing model for convertible bonds

We will now explain via an example, how a convertible bond can be priced.
This example is taken from the Hull book.
The structure of the model is as follows:

The issuer's stock price follows a Brownian motion with the added feature that. . .

There is a probability of $1-e^{-\lambda\Delta t}$ that there will be a default in the period Δt
.
According to Hull, if there is a default, the stock price falls to zero and there is a recovery of the bond.

The variable λ is the risk neutral default intensity.

We represent the stock price using a binomial tree where at each node:

There is an up tick of size u over the next time period of length Δt with probability p_u

There is a down tick of size d over the next time period of length Δt with probability p_d

.

With a probability of $1-e^{-\lambda\Delta t}$, there will be a default so that the stock price moves to zero over the

next time period Δt

.

The parameters are as follows:

$$p_u = \frac{a - de^{-\lambda\Delta t}}{u-d}, p_d = \frac{ue^{-\lambda\Delta t} - a}{u-d}, u = e^{\sqrt{\left(\sigma^2-\lambda\right)\Delta t}}, d = \frac{1}{u}$$

And $a = e^{(r-q)\Delta t}$, *r* is the risk free rate, and q is the dividend yield on the stock.

<u>Constructing the tree:</u>

The life of the tree is just the life of the convertible bond.

For simplicity we assume there is no threshold for the holder to convert the bond.

How do we calculate the value of the convertible bond at each node?

The values at the final nodes are easy, we just calculate the value base on any conversion options that the **holder** has at that time.

Working out the values at the intermediate nodes is a little trickier…..

The formula for the value at each node is:

$$\max\left[\min\left(Q_1, Q_2\right), Q_3\right] \qquad \ldots.(*)$$

Where

Q_1 is the value given by the 'rollback' (i.e.: using Feynman Kac),

Q_2 is the (issuers) call price and

Q_3 is the value if conversion takes place. (i.e.: you now have a stock)

Why does this make sense!?!?!

Well, let's consider 2 cases: $Q_1 \le Q_2$ and $Q_1 > Q_2$

If $Q_1 \le Q_2$, then this means that the value of the bond with 'rollback' is less than the issuers call price. The issuer will obviously not want to exercise the call (the holder will have a free lunch). So then we know the value at the node by deciding if the value at roll back is greater or smaller than the value of the bond if conversion takes place. i.e.:
if Q_1 is bigger or smaller than Q_3. This clearly ties in with equation (*): Q_1 being less than or equal to Q_2 led us to compare the greater between Q_1 and Q_3

If $Q_1 > Q_2$, then this means that the value of the bond at roll back is greater than the issuers call price. Therefore the issuer will obviously want to exercise his right to call the bond. Now the holder has to make a decision, since he always has the right to convert the bond once it has been called by the issuer, he has to consider if the call price gives him more than if he was to convert the bond to stock. I.e.: he has to consider whether or not Q_2 is bigger or smaller than Q_3. Again, we see that this ties in exactly with equation (*): Q_2 being less than Q_1 led us to compare the greater between Q_2 and Q_3.

In either case we end up evaluating:

$$\max\left[\min\left(Q_1, Q_2\right), Q_3\right]$$

Time for an example:

We consider a model with these characteristics:

One 9-month zero-coupon bond issued by company XYZ, face value $100.

It can be converted to two shares of XYZ's stock at any time.

The bond is callable at $113 at any time.

Initial stock price is $50, the volatility is 30% per annum, there are no dividends, λ is 1% per year and all risk free rates for all maturities are 5%.

We assume that in the event of a default the bond is worth $40. (recovery rate)

We consider 3 time steps so that Δt = 0.25.

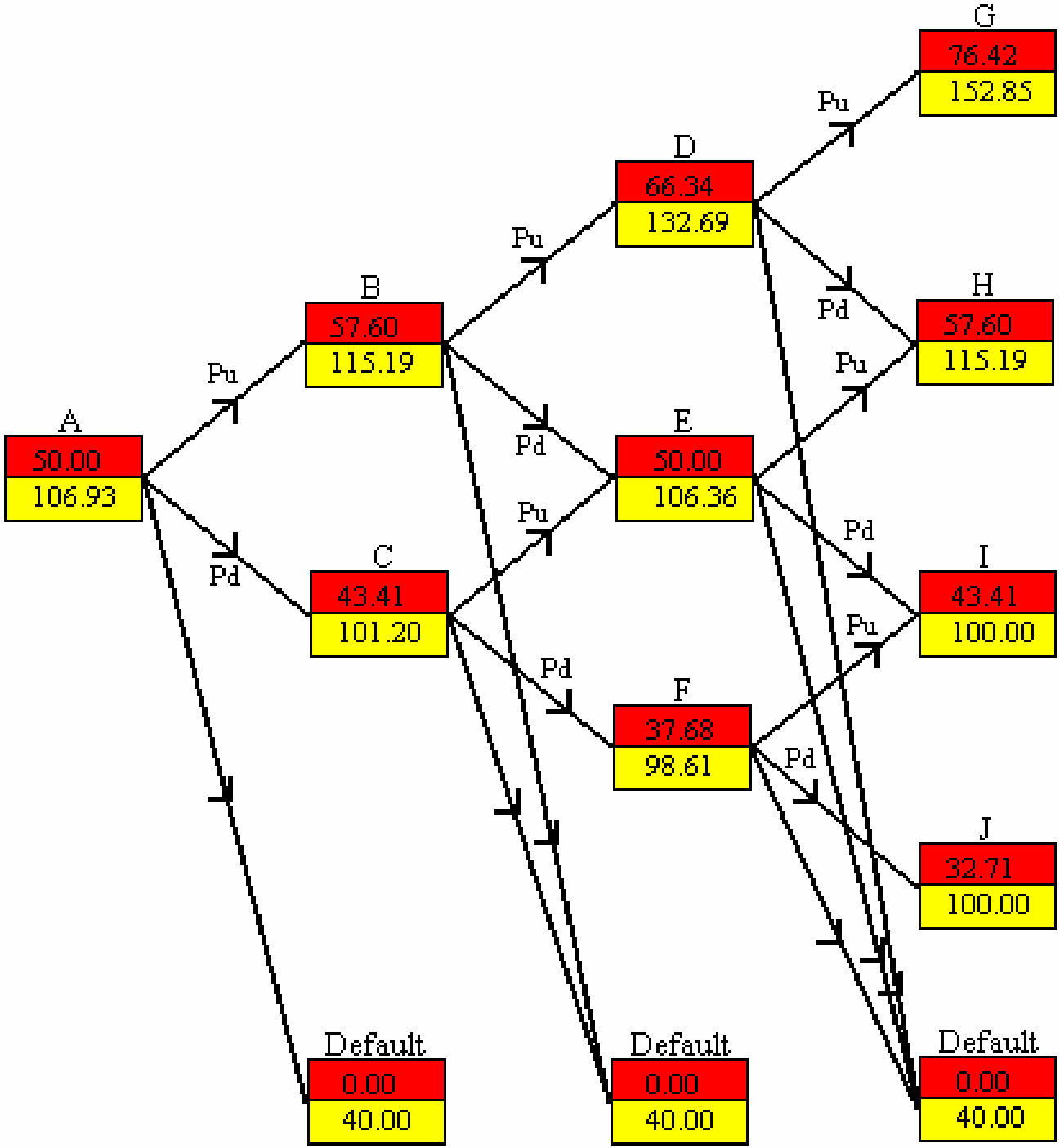
The tree parameters work out as:

$$p_u = 0.5167,\ p_d = 0.4808, u = 1.1519, d = 0.8681$$

The probability of a default is $1 - e^{0.05\times 0.25} = 0.002497$ at each node. At the default nodes, the stock price is 0 and the bond is worth $40.

We plot the tree by first writing all the stock prices in the tree (the red boxes) and then work back from the end nodes, filling in the prices of the convertible bond (the yellow boxes).

Look at the following diagram:


A
50.00
106.93
B
57.60
115.19
C
43.41
101.20
D
66.34
132.69
E
50.00
106.36
F
37.68
98.61
G
76.42
152.85
H
57.60
115.19
I
43.41
100.00
J
32.71
100.00
Pu
Pd
Default
0.00
40.00

So the stocks are easy to work out.

First let's see how the final nodes are worked out.
By inspecting the stock prices at nodes G and H, we see that twice the value in the red box is over 100, so we should convert, hence the result in the yellow boxes.

We then move back through the tree.
Let's look at node E for example:
recall the formula:

$$\max\left[\min\left(Q_1, Q_2\right), Q_3\right]$$

So, at node E,

$$Q_1 = (0.5167 \times 115.19 + 0.4808 \times 100 + 0.002497 \times 40) \times e^{-0.05 \times 0.25} = 106.36$$ (The issuers call price)

$$Q_2 = \$103$$ (Conversion)

$$Q_3 = 2 \times 50 = \$100$$

$$\Rightarrow \max\left[\min\left(Q_1, Q_2\right), Q_3\right] = \max\left[\min\left(106.36, 113\right), 100\right] = \$106.36$$

This formula applied at every node leads to the fair price at node A being $106.93

REGISTERED

REGISTERED

1000

1000

OTIS ELEVATOR COMPANY

6½% CONVERTIBLE SUBORDINATED DEBENTURE DUE 1995

RM 7676

RM 7676

Otis Elevator Company, a New Jersey corporation (hereinafter called the "Company", which term includes any successor corporation under the Indenture hereinafter referred to), for value received, hereby promises to pay to

6½%
DUE 1995

6½%
DUE 1995

.BLACK & COMPANY INC

CUSIP 689002 AA 1

or registered assigns, the principal sum of

ONE THOUSAND DOLLARS

on October 1, 1995, and to pay interest thereon from October 14, 1970 or from the most recent Interest Payment Date to which interest has been paid or duly provided for, semi-annually on April 1 and October 1 in each year, commencing April 1, 1971, at the rate of 6½% per annum, until the principal hereof is paid or made available for payment. The interest so payable, and punctually paid or duly provided for, on any Interest Payment Date will, as provided in said Indenture, be paid to the Person in whose name this Debenture (or one or more Predecessor Debentures, as defined in said Indenture) is registered at the close of business on the Regular Record Date for such interest, which shall be the March 15 or September 15 (whether or not a Business Day), as the case may be, next preceding such Interest Payment Date. Any such interest not so punctually paid or duly provided for shall forthwith cease to be payable to the registered Holder on such Regular Record Date, and may be paid to the Person in whose name this Debenture (or one or more Predecessor Debentures) is registered at the close of business on a Special Record Date for the payment of such Defaulted Interest to be fixed by the Trustee, notice whereof shall be given to Holders of Debentures not less than 10 days prior to such Special Record Date, or may be paid at any time in any other lawful manner not inconsistent with the requirements of any securities exchange on which the Debentures may be listed, and upon such notice as may be required by such exchange, all as more fully provided in said Indenture. Payment of the principal of (and premium, if any) and interest on this Debenture will be made at the office or agency of the Company maintained for that purpose in the Borough of Manhattan, The City of New York, in such coin or currency of the United States of America as at the time of payment is legal tender for payment of public and private debts; provided, however, that payment of interest may be made at the option of the Company by check mailed to the address of the person entitled thereto at such address shall appear on the Debenture Register.

Reference is hereby made to the further provisions of this Debenture set forth on the reverse hereof, which further provisions shall for all purposes have the same effect as if set forth at this place.

Unless the certificate of authentication hereon has been executed by the Trustee referred to on the reverse hereof by manual signature, this Debenture shall not be entitled to any benefit under the Indenture, or be valid or obligatory for any purpose.

In Witness Whereof, the Company has caused this Debenture to be duly executed under its corporate seal.

Dated: JUL 9 1971

TRUSTEE'S CERTIFICATE OF AUTHENTICATION.
This is one of the Debentures referred to in the within-mentioned Indenture.

FIRST NATIONAL CITY BANK, as Trustee,

By

Authorized Officer

Attest:

Secretary

Otis Elevator Company

By

President

By

Treasurer