CIR Term Structure Model

According to interest rate process:

$$dr = a(b-r)dt + \sigma\sqrt{r}dz$$

First we need to set the values of a, b and σ which are variable, and dz is standard Wiener process.

As we know the value of coupon bond:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

In this function we need to know A and B, so we have

$$A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma+a)\left(e^{\gamma(T-t)}-1\right)+2\gamma}\right]^{2ab/\sigma^2}$$

$$B(t,T) = \frac{2\left(e^{\gamma(T-t)}-1\right)}{(\gamma+a)\left(e^{\gamma(T-t)}-1\right)+2\gamma}$$

Then we also know $\gamma = \sqrt{a^2+2\sigma^2}$

Infinitely-long Rate (Y_∞): $Y_\infty = 2ab/(a+\gamma)$

$$Discount\_factor = A(t,T)e^{-B(t,T)r(0)}$$

Then we use discount factor to get CIR Zero Rate.

CIR volatility of zero rate σ Y(t,T):

$$\sigma_{Y(t,T))} = \sigma\sqrt{r_0}\,\frac{B(t,T)}{(T-t)}$$

Long-term distribution of r (Steady State Probability Density Function):

$$P_\infty = \frac{\left(\frac{2a}{\sigma^2}\right)^k}{\Gamma(k)} r^{k-1} e^{-2ar/\sigma^2} = \left(\frac{2a}{\sigma^2}\right)^k r^{k-1} e^{-2ar/\sigma^2 - \ln(\Gamma(k))}$$

$$with\ k = \frac{2ab}{\sigma^2}$$

This is gamma distributed.

Γ(.) is Gamma Function. Mean & standard deviation gamma distribution:

$$\Gamma(Mean) = k\frac{\sigma^2}{2a} = b$$

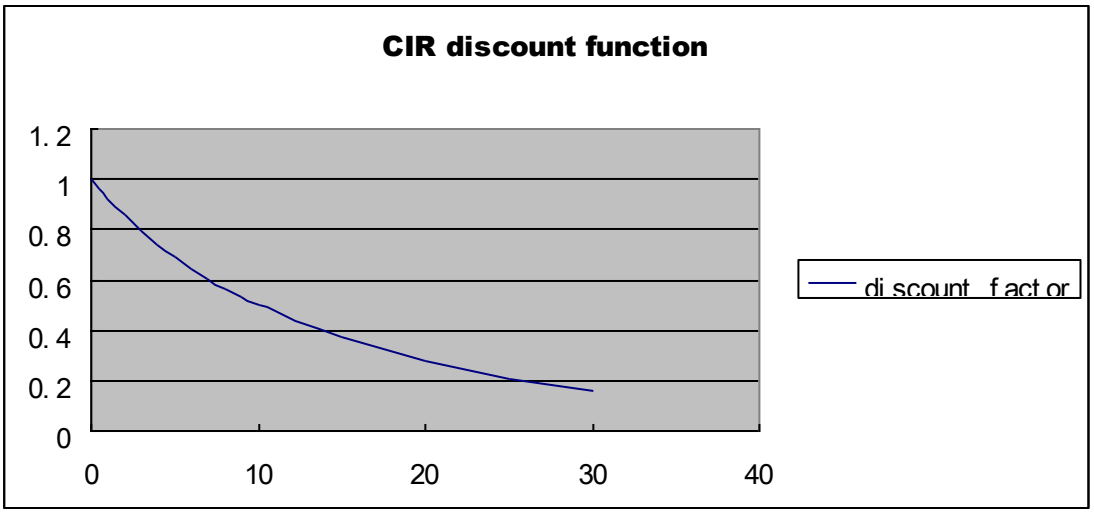
$$\Gamma(Stdev) = \sqrt{k}\,\frac{\sigma^2}{2a} = \sqrt{\frac{b}{2a}}\sigma$$

Gamma distribution for probability, we use in excel notation

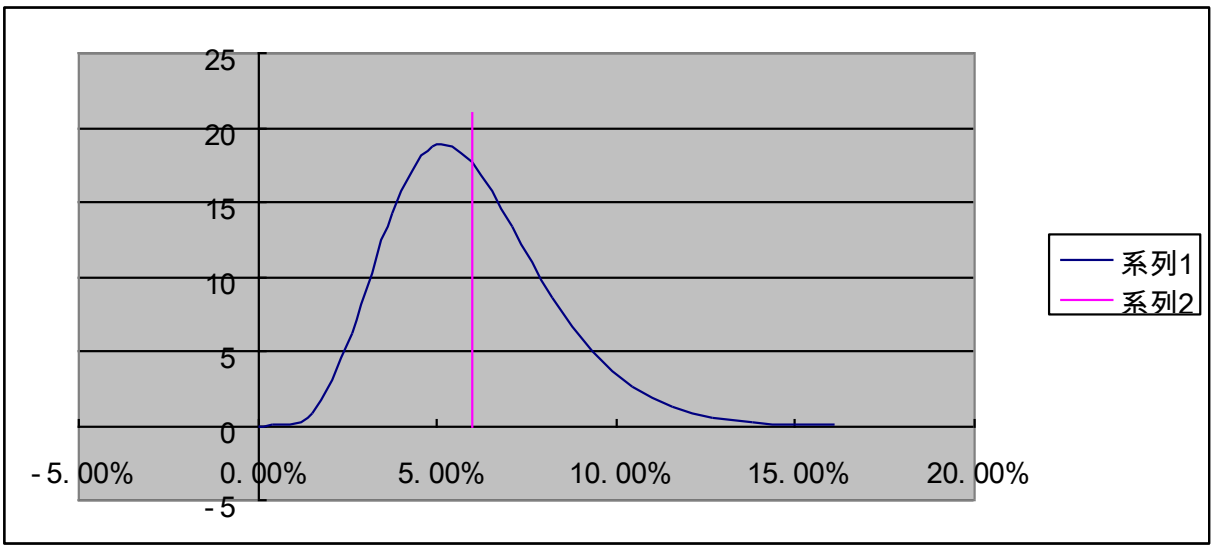
$$f(x,\alpha,\beta)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}$$

α =**K and** β = σ **/2a**

For graph of discount function, we set X is the column of periods and Y is the column of discount factor.



For graph of PDF, first we need to draw a probability density function as we set X is the column of spot rate and Y is the column of probability. And we also set the mean as X is the column of Mean = mean of P_{∞} **and Y is the column of Mean which is the value times 1.2, which is corresponding to 0.**



For CIR zero rate, first we need to draw CIR zero rate, we set X is the column of periods and Y is CIR zero rate, secondly we draw the r(0) point we set X as 0 and Y as r(0) = r_0 , finally we need to draw infinitely long rate, we take long term equilibrium rate 0, 35 as X and theta as Y.