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ANALYTICAL FINANCE II

Pure Bootstrapping

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In this paper we are going to show how to use linear bootstrapping to calculate and find the yield curve. We are going to build an excel application and find the yield curve of the Swedish government securities with linear interpolation and extrapolation between the nodes.

We are given the following data for the Swedish treasury bills and the Swedish government bonds:

Maturity, Yield, coupons, next coupon dates, bond (bill) prices, bond (bill) clean prices, and the Duration.

Namn	Maturity	Yield	Coupon	Next coupon	Price	Clean P.	Duration
01M	1/4/2007	2.000	0.00	1/4/2007	99.839	99.839	0.08
02M	2/3/2007	2.000	0.00	2/3/2007	99.679	99.679	0.16
03M	3/5/2007	2.070	0.00	3/5/2007	99.485	99.485	0.25
06M	6/3/2007	2.160	0.00	6/3/2007	98.943	98.943	0.49
09M	9/1/2007	2.275	0.00	9/1/2007	98.347	98.347	0.74
12M	11/30/2007	2.415	0.00	11/30/2007	97.674	97.674	0.99
1037	8/15/2007	2.685	8.00	8/15/2007	106.023	103.579	0.69
1040	5/5/2008	3.030	6.50	5/5/2007	108.510	104.718	1.36
1043	1/28/2009	3.250	5.00	1/28/2007	107.828	103.564	2.01
1034	4/20/2009	3.345	9.00	4/20/2007	118.298	112.673	2.15
1048	12/1/2009	3.430	4.00	12/1/2007	101.637	101.593	2.88
1045	3/15/2011	3.605	5.25	3/15/2007	110.188	106.396	3.82
1046	10/8/2012	3.730	5.50	10/8/2007	109.997	109.126	5.15
1041	5/5/2014	3.810	6.75	5/5/2007	122.596	118.658	6.01
1049	8/12/2015	3.875	4.50	8/12/2007	105.930	104.518	7.32
1050	7/12/2016	3.915	3.00	7/12/2007	93.970	92.778	8.33
1047	12/1/2020	3.875	5.00	12/1/2007	112.029	111.974	10.61

The bond prices can be calculated by the following relations for instruments which pay no coupons and also for instruments which pay coupons.

$$P = \frac{100}{1 + ytm.\frac{d}{360}}$$

$$P = \frac{N}{(1+ytm)^{T}} + \sum_{i=1}^{n} \frac{C}{(1+ytm)^{t_i}}$$

The Spot Rates

When coupon bonds mature in less than a year, there is only one coupon payment at maturity (since the coupons are paid annually. The spot rate for this time interval can be calculated by linear interpolation.

We used the following steps for our bootstrapping

We calculated the rates for all the instruments, which have only one cash flow until maturity. For no coupon bonds we used the formula

$$P = Fv \bullet d$$

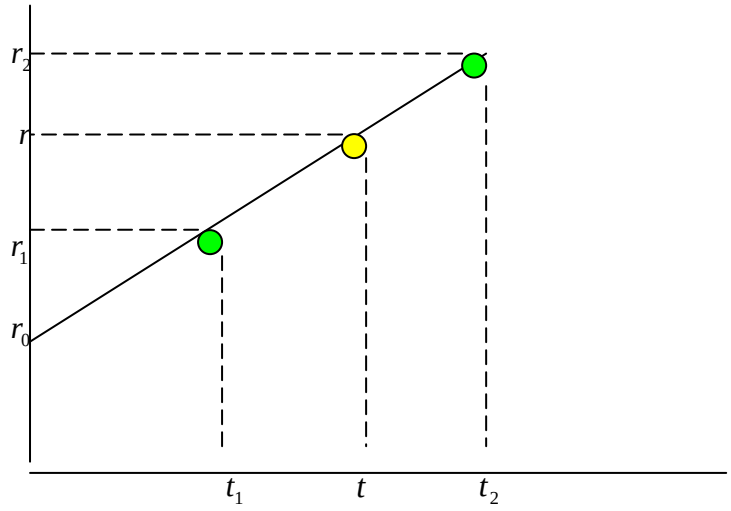
Where is P bond price, Fv is Face value and is d discount factor.

$$P = (Fv) \bullet e^{-rt}$$

$$r = \frac{In\frac{Fv}{P}}{t}$$

For the case with coupon payments we used the formula

Every time step (t_i, r_i) has a discount factor $e^{-r_i t_i}$



We interpolate to find the rates in between the two known rate r_1 and r_2. r_0 is the spot rate at time zero . The slope of the line was calculated using the following formula

$$\frac{\Delta y}{\Delta x} = \left(\frac{r_2 - r_1}{t_2 - t_1} \right)$$

To find r_0

$$\left(\frac{r_1 - r_0}{t_1} \right) = \left(\frac{r_2 - r_1}{t_2 - t_1} \right)$$

$$r_1 (t_2 - t_1) = r_0 (t_2 - t_1)$$

$$r_0 = \left(\frac{r_1 t_1 - r_1 t_2}{t_1 - t_2} \right)$$

We calculate the next interpolation point (rate) by multiplying the time and the slope we obtained above and adding the r_0 we found above to the product we obtain.

To find the cumulative discount factor we multiply all the discount factors for each time step upto the point e.g

$$Cd_3 = d_1 \times d_2 \times d_3$$

where is Cd_i cumulative discount at i time step and d_i is discount at each individual time step.

To calculate the Present Value of all coupons with payout before the maturity of the bond, we multiplied the dividend with the respective cumulative discount factor.

We then calculated the zero coupon price using the following formula:

$$ZCP = [P - PV(coupons)] \frac{100}{100 + C}$$

Since we now have the price of the zero coupon bond we can find the rate that corresponds to this price

$$ZCP = (Fv) \cdot CDF$$

$$ZCP = (Fv) \cdot e^{-rT}$$

$$r = \frac{In \frac{Fv}{ZCP}}{T}$$

We repeat the process above again by taking the next bond that has the least number of coupons. The rates between the bonds with the longest maturity have been calculated with both interpolation and extrapolation using solver. The results are as follows

Maturity	coupon	market price	PV dividends	PV(FV+C)	ZCP	Yield
1/4/2007	0.00	99.839	0.000	99.839	99.839	2.000%
2/3/2007	0.00	99.679	0.000	99.679	99.679	1.996%
3/5/2007	0.00	99.485	0.000	99.485	99.485	2.065%
6/3/2007	0.00	98.943	0.000	98.943	98.943	2.149%
9/1/2007	0.00	98.347	0.000	98.347	98.347	2.256%
11/30/2007	0.00	97.674	0.000	97.674	97.674	2.387%
8/15/2007	8.00	106.023	0.000	106.023	98.169	2.660%
5/5/2008	6.50	108.510	6.444	102.066	95.836	3.002%
1/28/2009	5.00	107.828	9.859	97.969	93.304	3.228%
4/20/2009	9.00	118.298	17.651	100.647	92.337	3.357%
12/1/2009	4.00	101.637	7.720	93.917	90.305	3.412%
3/15/2011	5.25	110.188	20.053	90.135	85.639	3.624%
10/8/2012	5.50	109.997	25.397	84.600	80.190	3.779%
5/5/2014	6.75	122.596	42.757	79.839	74.791	3.917%
8/12/2015	4.50	105.930	31.732	74.198	71.003	3.942%
7/12/2016	3.00	93.970	23.450	70.520	68.466	3.945%
12/1/2020	5.00	112.029	51.875	60.154	57.290	3.983%