

# Binomial Model

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# Project Goals

**Excel/VBA application to compare different Binomial Models**

- Cox-Ross-Rubenstein Model**
- The Tian Model**
- The Leisen Reimer Model**

# Description

$$S(t) = 1/R(p \times uS + q \times dS)$$

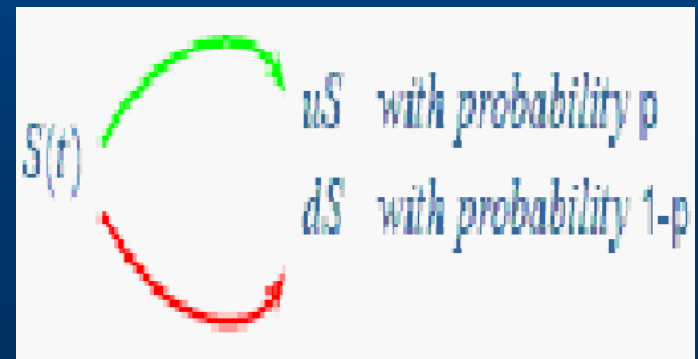
$$- R = \exp\{r \times dt\}$$

For risk-neutral probabilities

$$- P + q = 1$$

$$- P \times u + q \times d = R$$

So  $p = (R - d) / (u - d)$ ,  $q = 1 - p$



# Lognormal Distribution

$$S(T)=[S(T)/S(T-\Delta)][S(T-\Delta)/S(T-2\Delta)]\dots[S(2\Delta)/S(\Delta)][S(\Delta)/S(0)]S(0)$$

$$S(T)=S(0)\exp\{z_1+z_2+\dots+z_T\}$$

$$- Z(T)=z_1+z_2+\dots+z_T$$

$$- Z(T)=\ln[S(T)/S(0)]$$

# Lognormal Distribution -2

$$\text{Ln}[S(t)/S(t-\Delta)]=$$

- $\mu\Delta + \sigma \text{sqr}(\Delta)$
- $\mu\Delta - \sigma \text{sqr}(\Delta)$

$$S(t)=S(t-\Delta)U \text{ or } S(t)=S(t-\Delta)D$$

$$U=\exp\{\mu\Delta + \sigma \text{sqr}(\Delta)\}$$

$$D=\exp\{\mu\Delta - \sigma \text{sqr}(\Delta)\}$$

# Assumption

The returns  $\{z(t)\}$  are independently distributed

The returns  $\{z(t)\}$  are identically distributed

$$E[z_t] = \mu \Delta$$

$$\text{Var}[z_t] = \sigma^2 \Delta$$

# Multi-period Binomial Price

## Stock prices at T with n intervals

- $S(0)u^n$   $p^n$
- $S(0)(u^{n-1}d)$   $n(p^{n-1})q$
- $S(0)(u^{n-2}(d^2))$   $n(n-1)/2(p^{n-2})(q^2)$
- .....
- $S(0)d^n$   $q^n$

# Compare with Diff. Models

**Tian model used 2<sup>nd</sup> order moments for normal distribution, so has better accuracy**

**L-R model has quadratic convergence, accuracy is much better**

**Binomial approaches often use complex numerical analyses involving large amounts of computer time. It is used more frequently in the real world**

**Black-Scholes model works well on only a narrow set of problems**



# Example – Wal-Mart (WMT)

## Volatility from Robert's Web Page

1 month : 15.30%

2 months: 15.02%

3 months: 17.66%

6 months: 17.82%

9 months: 16.75%

1 year: 16.89%

# Wal-Mart 2

**Stock price: 48.31**

**Strike price: \$45, \$50, \$55**

## Example 2 – Alcoa (AA)

**Stock price: \$26.99**

**Volatility (1 month): 25.16%**

**Nov 27.50 call @0.60**

**Nov 27.50 put @1.10**

# Complicated Real World

**Model Choose**

**Continuous Trading**

**Continuously Changing Prices**