Analytical Finance

MT1410: Analytical Finance

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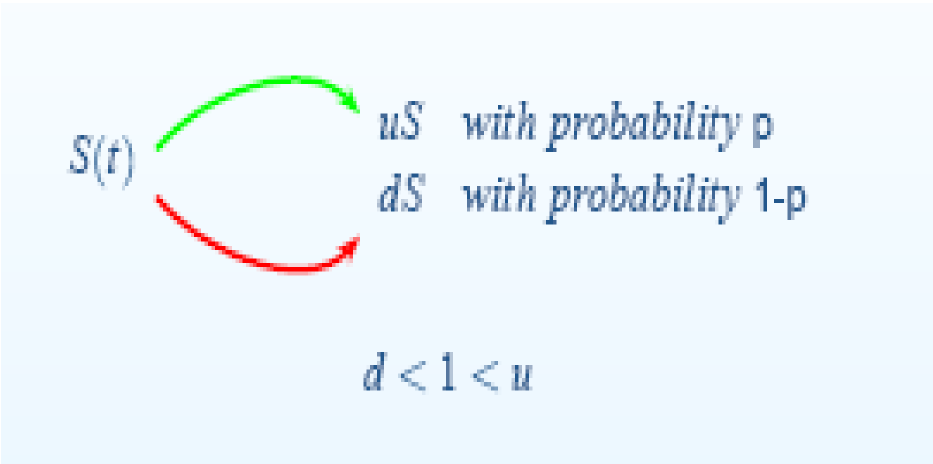
Binominal Model



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Introduction

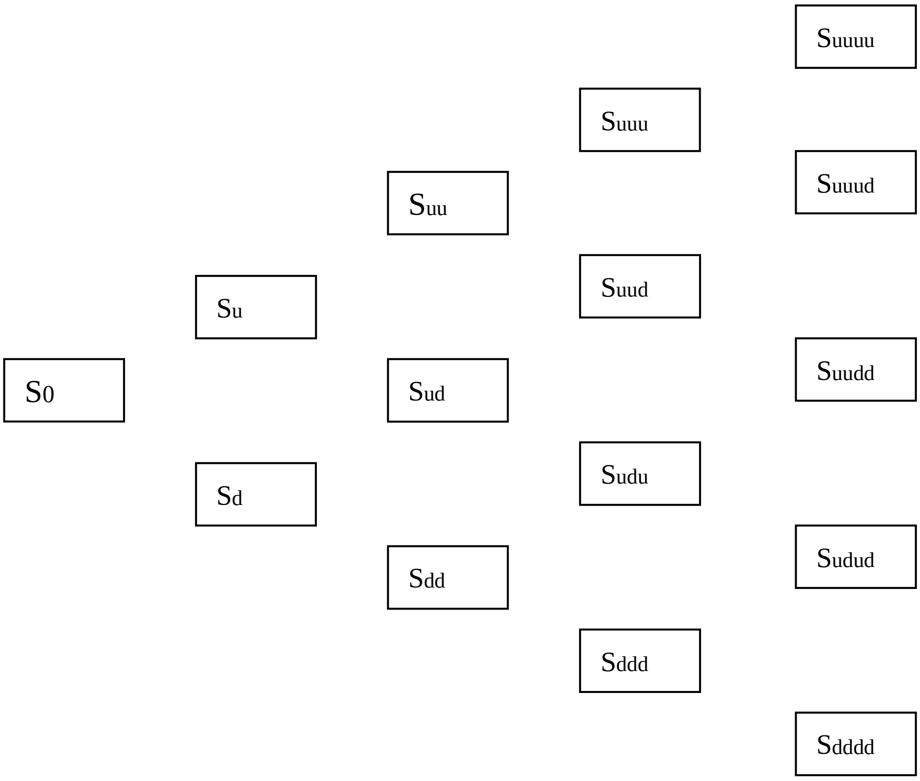
Build an application in Excel to compare different Binominal models. Use the following models: CRR, Tian and Leisen-Reimer. Also include Black-Scholes smoothing and Richardson extrapolation and a possibility to compare with Black-Scholes. Then calculate the standard error.

Part one   Theorem

Binominal tree

In this part , we will go through three different binominal models and also the black-schole smoothing and Richardson extrapolation as well.

First ,we establish a binominal tree as follows:



S_0 ----The initial value of the stock

u,d----the upper and lower coefficient respective

r------- risk free interest rate compounded continuously

$p = \frac{e^{rdt} - d}{u - d}$ ----risk neutral probability measure

Now we start from the last period , then backward:

$$S_{uuu} = e^{-rdt} \quad p \cdot u \cdot S_{uuuu} + q \cdot d \cdot S_{uuud}$$

$$S_{uud} = e^{-rdt} \quad p \cdot u \cdot S_{uuud} + q \cdot d \cdot S_{uudd}$$

$$S_0 = e^{-rdt} \quad p \cdot u \cdot S_u + q \cdot d \cdot S_d$$

i. CRR model

The CRR model (Cox, Ross, Rubinstein, 1979) can be regarded as a discretization of the more popular Black –Scholes model but much simpler.

$$u = e^{\sigma\sqrt{\Delta t}}, d = e^{-\sigma\sqrt{\Delta t}}$$

$$p = \frac{e^{r\Delta t} - d}{u - d}, q = 1 - p$$

The CRR model can be written as

$$c = e^{-r\Delta t}\left[\left(\frac{e^{r\Delta t} - d}{u - d}\right)C_u + \left(1 - \frac{e^{r\Delta t} - d}{u - d}\right)C_d\right]$$

$$= e^{-r\Delta t} \quad p \cdot C_u + q \cdot C_d$$

T----- expiration date

K-----strike price

r------- risk free interest rate compounded continuously

C--------the value of the option at the beginning

C_u -----the value of the option at timeT if the stock price goes up

C_d ----- the value of the option at timeT if the stock price goes down

ii. The Tian model

$$u = \frac{M \cdot V}{2}\left[V + 1 + \sqrt{V^2 + 2V - 3}\right]$$
$$d = \frac{M \cdot V}{2}\left[V + 1 - \sqrt{V^2 + 2V - 3}\right]$$

Where

$$M = e^{r \cdot \Delta t}, V = e^{\sigma^2 \cdot \Delta t}$$

$$p = \frac{e^{r \cdot \Delta t} - d}{u - d}$$

Therefore, the Tian model can be written as:

$$c = e^{-r\Delta t}\left[\left(\frac{e^{r\Delta t} - e^{r\Delta t + \sigma^2 \Delta t}/2\left[e^{\sigma^2 \Delta t} + 1 - \sqrt{e^{\sigma^4 \cdot \Delta t^2} + 2e^{\sigma^2 \Delta t} - 3}\right]}{e^{r\Delta t + \sigma^2 \Delta t}\sqrt{e^{\sigma^4 \cdot \Delta t^2} + 2e^{\sigma^2 \Delta t} - 3}}\right)C_u + \left(1 - \frac{e^{r\Delta t} - e^{r\Delta t + \sigma^2 \Delta t}/2\left[e^{\sigma^2 \Delta t} + 1 - \sqrt{e^{\sigma^4 \cdot \Delta t^2} + 2e^{\sigma^2 \Delta t} - 3}\right]}{e^{r\Delta t + \sigma^2 \Delta t}\sqrt{e^{\sigma^4 \cdot \Delta t^2} + 2e^{\sigma^2 \Delta t} - 3}}\right)C_d\right]$$
$$= e^{-r\Delta t}\left[p \cdot C_u + q \cdot C_d\right]$$

iii. The Leisen Reimer model

$$u = a \cdot \frac{\overline{p}}{p}$$
$$d = a \cdot \frac{1 - \overline{p}}{1 - p}$$
$$a = e^{r \cdot \Delta t}$$
$$d_1 = \frac{\ln\left(\frac{s}{k}\right) + \left(r + \frac{1}{2}\sigma^2\right) \cdot \; T - t}{\sigma\sqrt{T - t}}$$
$$d_2 = d_1 - \sigma\sqrt{T - t}$$

B------the inverse of the binonmial distribution

N------the number of time steps(should be odd)

d_1 and d_2 are from Black-scholes equations

$$p = B(d_2, N)$$

$$\bar{p} = B(d_2 + \sigma \cdot \sqrt{T-t}, N)$$

$$p = B(z, n) = \frac{1}{2} \mp \left[\frac{1}{4} - \frac{1}{4} \cdot \exp\left\{ -\left(\frac{z}{n+1/3} \right)^2 \cdot \left(n + \frac{1}{6} \right) \right\} \right]^{\frac{1}{2}}$$

Replace z with d_2 and n with N, we get

$$p = B(d_2, N) = \frac{1}{2} \mp \left[\frac{1}{4} - \frac{1}{4} \cdot \exp\left\{ -\left(\frac{d_2}{N+1/3} \right)^2 \cdot \left(N + \frac{1}{6} \right) \right\} \right]^{\frac{1}{2}}$$

Replace z with $d_2 + \sigma \cdot \sqrt{T-t}$ and n with N, we get

$$\bar{p} = B(d_2 + \sigma \cdot \sqrt{T-t}, N) = \frac{1}{2} \mp \left[\frac{1}{4} - \frac{1}{4} \cdot \exp\left\{ -\left(\frac{d_2 + \sigma \cdot \sqrt{T-t}}{N+1/3} \right)^2 \cdot \left(N + \frac{1}{6} \right) \right\} \right]^{\frac{1}{2}}$$

Then we can calculate C by the following formula

$$c = e^{-r\Delta t} \left[\left(\frac{e^{r\Delta t} - e^{r\Delta t} \cdot \frac{1-\bar{p}}{1-p}}{e^{r\Delta t} \cdot \frac{\bar{p}}{p} - e^{r\Delta t} \cdot \frac{1-\bar{p}}{1-p}} \right) C_u + \left(1 - \frac{e^{r\Delta t} - e^{r\Delta t} \cdot \frac{1-\bar{p}}{1-p}}{e^{r\Delta t} \cdot \frac{\bar{p}}{p} - e^{r\Delta t} \cdot \frac{1-\bar{p}}{1-p}} \right) C_d \right]$$

$$= e^{-r\Delta t} \left[\left(\frac{1 - \frac{1-\bar{p}}{1-p}}{\frac{\bar{p}}{p} - \frac{1-\bar{p}}{1-p}} \right) C_u + \left(1 - \frac{1 - \frac{1-\bar{p}}{1-p}}{\frac{\bar{p}}{p} - \frac{1-\bar{p}}{1-p}} \right) C_d \right]$$

$$= e^{-r\Delta t} \left[p \cdot C_u + q \cdot C_d \right]$$