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Mathematics and Physics Department

Project in Analytical Finance II

Credit Default Swap

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1. Introduction

In our project, we will give an overview of swaps and the variety of their types. Then, we will concentrate on Credit Default Swaps in details. We will discus pricing and valuing models as well.

To answer how swaps are created and traded, first we have to analyze the swap market and the needs of its participants and then the benefits of such derivatives.

The first swap contract was negotiated in the early 1980s and since that time swap market has developed rapidly. At first, swap market was highly structured and special purpose, however, in the late 1990s and early 2000s it transferred to extremely liquid and flexible market.

A key reason for using swaps is hedging interest rate risk by financial institutions, corporations and large institutional investors. That is why, the most common type of swap is a “plain vanilla” interest rate swap.

In general swap is an agreement between two parties, in which they agree to make periodic payments to each other according to two different indices.

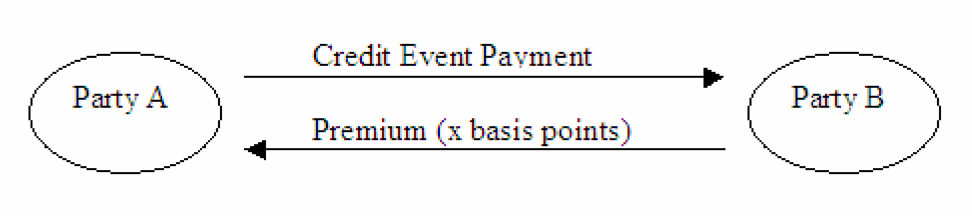
Credit swaps are recent innovation in swap market. The Credit Default Swap market consists of buyers and sellers of credit protection. Credit derivative can be described as a contract where the payoffs depend on the creditworthiness of one or more commercial entities. In other words, one party of the contract pays fixed fee and, in case of default, the party will receive a contingent claim.

We can point out some advantages provided by credit derivatives. The most important is the opportunity to transfer the credit risk to third party. It enables the accesses to some bank loans and taking the short positions in credits previously not possible in the underlying market. Credit derivatives also help to diversify the credit portfolio as well as hedge specific section of credit risk.

One of the most commonly traded credit derivatives is Credit Default Swap. In general, CDS are used to transfer credit risk from one company to another. This is a contract where party A has the right to sell a bond issued by a company C for its face value to company B, when default occurs. At the same time company A makes periodic payment to company B.

Example:

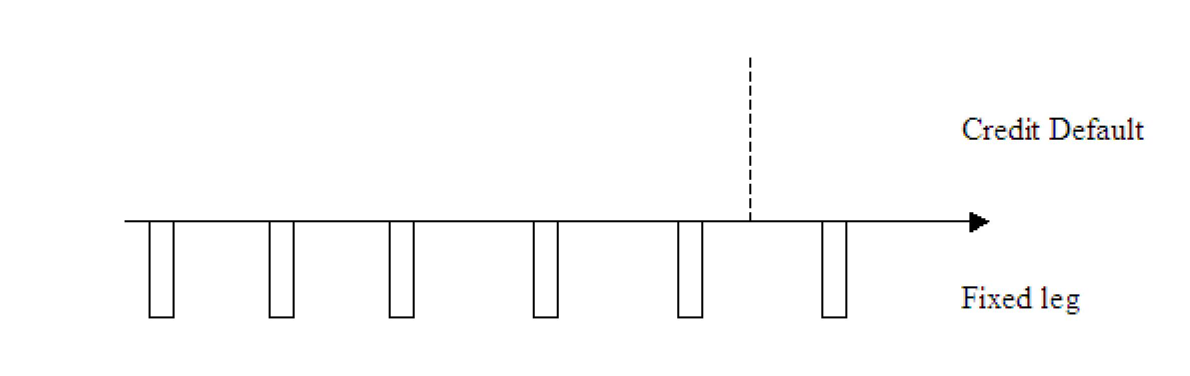
Party A owns a security that pays an annual of 10% but it doesn't want to be exposed to default risk of this security for a given period of time. So the company A buys a credit guarantee from party B. In return, it will pay a regular payment to B. So the A transferred the risk of default.



- A pays B: x basis points
- B pays A: par minus the post-default value

What is interesting, if there are many Credit Default Swaps dependent on the same underlying bond, in the case of default, all companies will be trying to buy the bond in order to resell it for its face value. This will drive up the price of the bond and reduce the payoff from the credit derivative.

The cash flow of a Credit Default Swap:



Credit default will only be paid in case of default and is calculated as:

Nominal * (100- after default price)

And the second leg is fixed payment that only be paid if no credit default has taken place:

Nominal * fixed rate * time period

2. Pricing

A credit default swap (CDS) is an exchange of a periodic payment against a one-off contingent payment if some credit event occurs on a reference asset. The ingredient of the basic structure is specification of maturity T: usually from 1 to 10 years, Underlying: corporate or sovereign, Credit event: default, bankruptcy, downgrade.

Let c(T) is the fixed coupon that the protection buyer pays. The payment continues until either default or maturity. In case of default, assume that the payment from the protection seller to the protection buyer is equal to the difference between the notational amount of the bond and the recovery value δ. The fixed side of the payment is set so that the contract value is zero at the beginning.

Thus since the cash flow at coupon date i for the protection buyer is c(T) and the payment for the protection seller at time of default is 1-δ, we obtain

$$(1-\delta)E^*(e^{-r\tau}\big|_{\tau\leq T})-c(T)\sum_{i=1}^{n}e^{-ri}P*(\big|_{\tau>i})=0$$

$$c(T)=\frac{(1-\delta)E^*(e^{-r\tau}\big|_{\tau\leq T})}{\sum_{i=1}^{n}e^{-ri}P*(\big|_{\tau>i})}$$

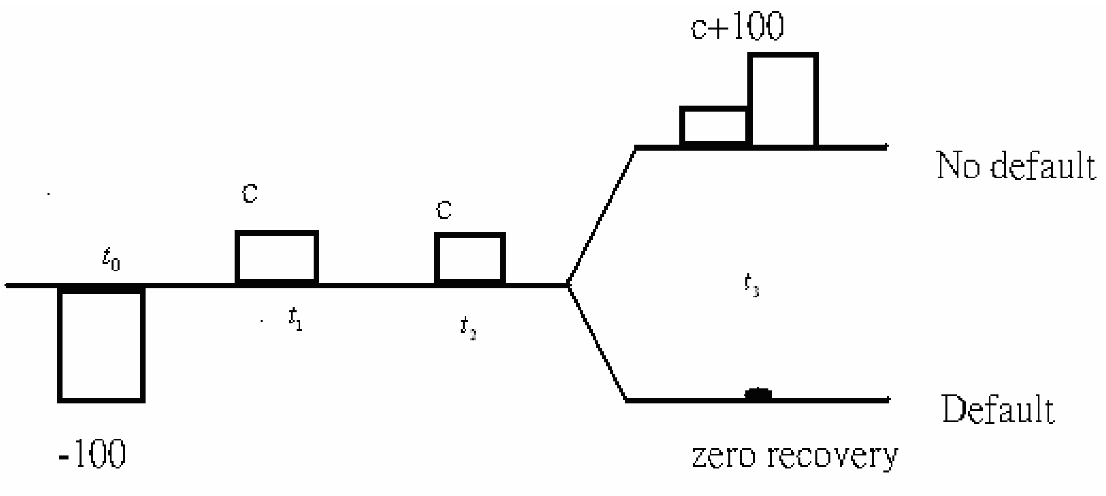
where we assume the interest rate is constant. Both $E^*(e^{-r\tau}\big|_{\tau\leq T})$ and $P*(\big|_{\tau>i})$ are readily available from market data.

3. Hedging

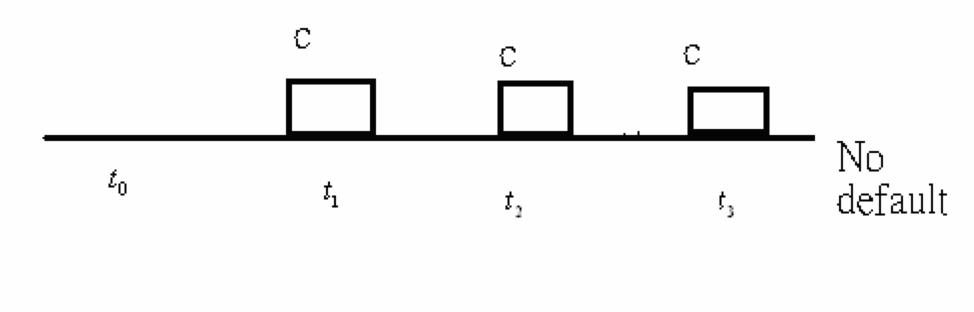
In order to hedge the risk of the market maker who is selling the CDS instrument, we first would like to know how to construct the CDS in the market.

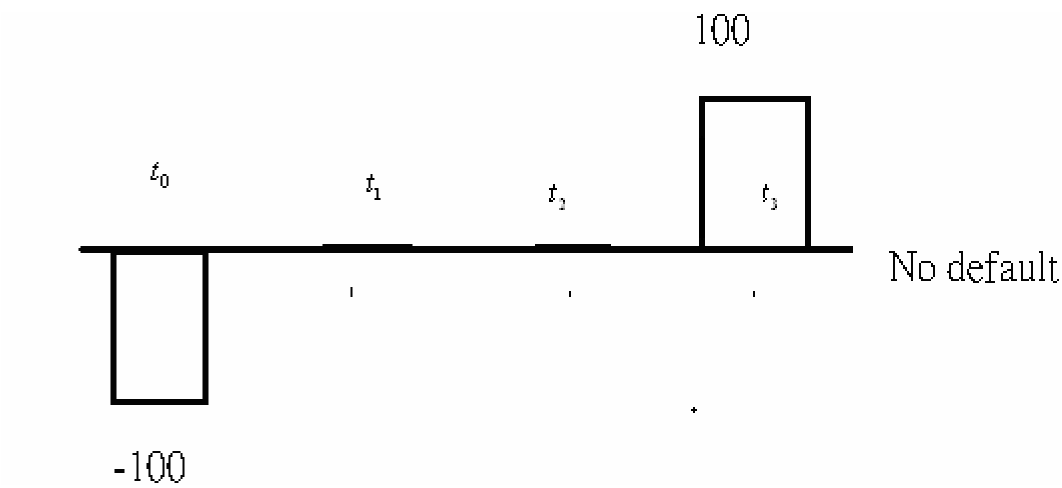
Create a CDS

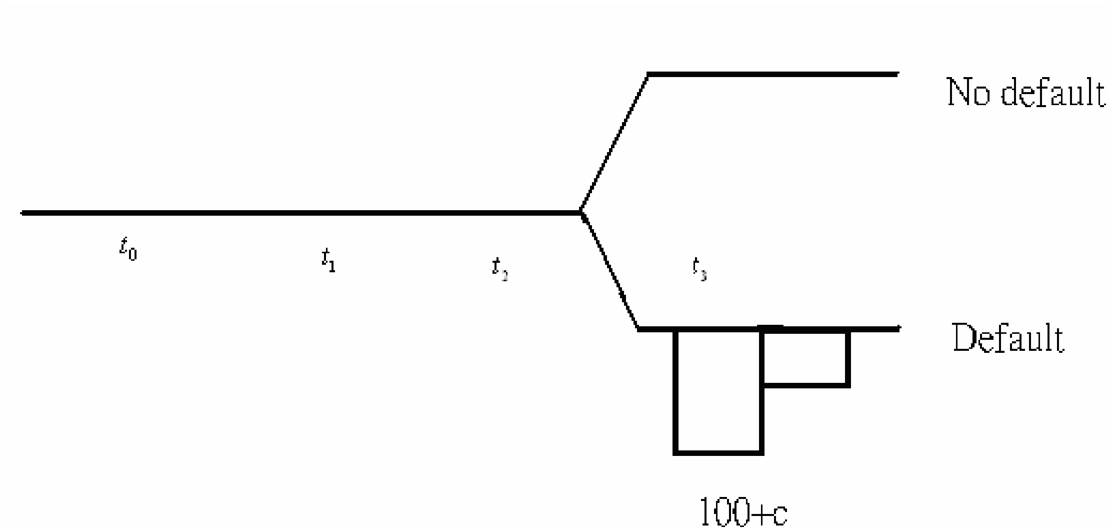
Here we take a risky bond as an example and decompose it into some simpler, liquid constituents. Consider a risky bond, purchased at time t_0, subject to default risk. It pays a coupon c annually over three years. The bond is originally sold at par. First, we assume that, in the case of default, the recovery value is zero. Second, without the loss of generality, we assume that the default occurs only in period t_3. Here show the cash-flow of the bond.



The bond is purchased for 100 in the beginning, three coupon payments are made, and the principle of 100 is returned if there is no default. The bond pays nothing if there is default. At time t_3, there are two possibilities and the claim is contingent on these. Now we are going to decompose the defaultable bond and isolate the underlying default risk as a single instrument, CDS. We decompose the cash-flow of the risky bond as follow:





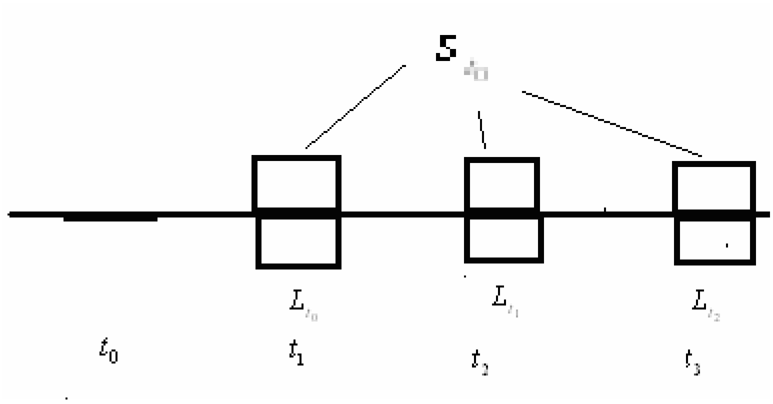


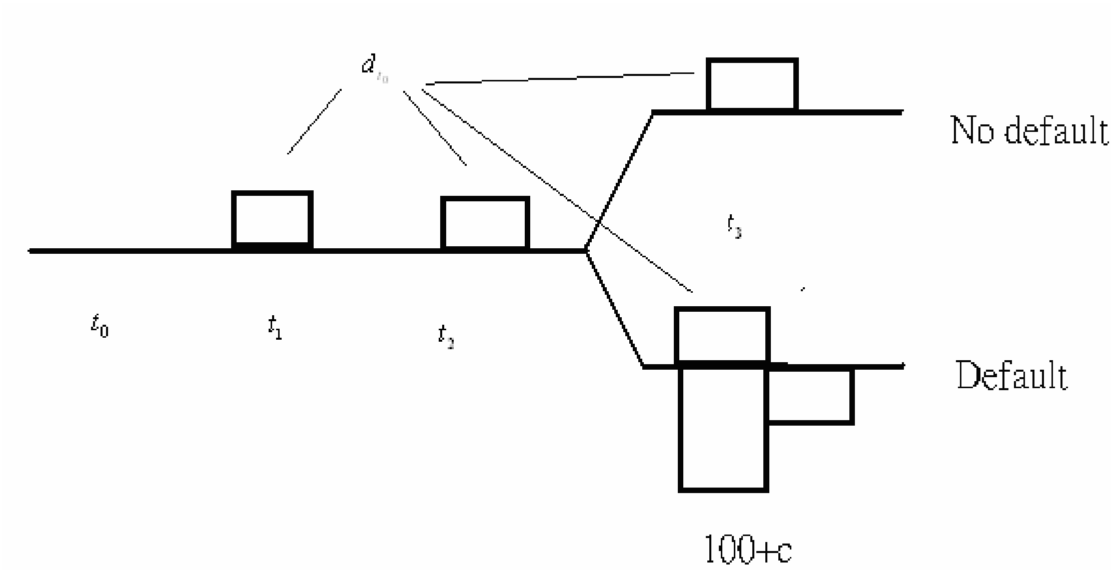
When added back these process together, it should give back the original risky bond cash-flow. Next step is to convert the three cash flow diagrams into recognizable and, preferably liquid contracts in markets.

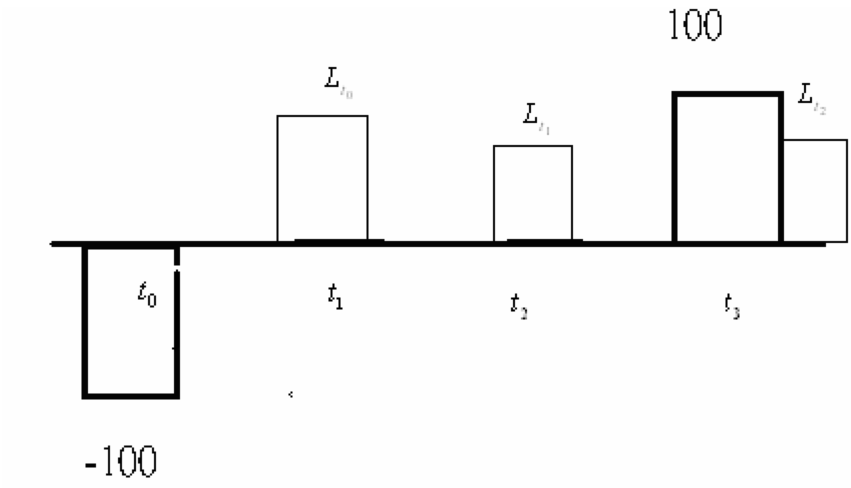
For the first diagram, if we add floating Libor-based payments, L_{t_i} at time t_1, t_2, t_3, the cash-flow will look like a fixed-receiver interest rate swap. This is good because swaps are very liquid instruments. However, one additional change is required. The fixed-receiver swap rate involves a rate that is less than the coupon of a par bond issued at time t_0. Thus, we have

$$d_{t_0} = c - s_{t_0}$$

where d_{t_0} is the credit spread over the swap rate. This is how much a credit has to pay over and above the swap rate due to the default possibility. So we will replace the above three diagrams with the below three.


S_{t_0}
L_{t_0}
L_{t_1}
L_{t_2}
t_0
t_1
t_2
t_3


d_{t_0}
No default
t_3
t_0
t_1
t_2
Default
100+c



The first diagram is a typical cash-flow of a fixed receiver interest rate swap. The second cash-flow is the CDS of selling protection. The third one can be regarded as a default-free money market deposit.

Finally, we come up with a contractual equation:

Defaultable bond on the credit = receiver swap + default-free deposit + CDS on the credit

The application of this equation can show the relation of creating such a CDS on the market. We can, therefore, show how to obtain a hedge for a CDS position by manipulating the contractual equation. Suppose a market maker sells a CDS on a certain name, how would he or she hedge this position? To obtain a hedge for the CDS, we first rearrange the term in the contractual equation:

CDS on the credit = Defaultable bond on the credit - receiver swap -

default-free deposit

Negative sign implies the opposite position in the relevant instrument.

Thus we can rewrite it as

CDS on the credit = Risky bond on the credit + payer swap + default-free loan

The market maker who sold such a CDS and provides protection needs to take the opposite position on the right hand side of this equation. That is, the credit derivatives dealer will first short the risky bond, deposit the received 100 in a default-free deposit account, and contract a receiver swap. This and the long CDS position will then 'cancel' out. The market maker will make money on the bid-ask spread.

4. References

Applied Math for Derivatives John S. Martin

Principles of Financial Engineering Salih N. Neftci

Options, Futures, And Other Derivatives John C. Hull

Lecture Notes in Analytical Finance II Jan R. M. Röman