Exotic option II

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   - Introduction
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Asian Options

Introduction

- The Asian Option (also called **Average Option**) is the option whose payoff depends on the average value of the underlying asset over pre - specified period

- Basic forms:
- **Average price option**
- **Average strike option**

Introduction

- The averaging period
- The sampling frequency
- The averaging method:

- Arithmetic average: $$\frac{s_1 + s_2 + ... + s_n}{n}$$
- Geometric average: $$\sqrt[n]{(s_1 * s_2 * ...... \ s_n)}$$

Introduction

- Weighted arithmetic average:

$$\frac{w_1 s_1 + w_2 s_2 + \ldots + w_n s_n}{w_1 + w_2 + \ldots. + w_n}$$

- Weighted geometric average:

$$\sqrt[w_1 w_2 \ldots.. w_n]{s_1^{w_1} s_2^{w_2} \ldots. s_n^{w_n}}$$

A mean value option

- Payoff of the Average Option at time T_2

$$X = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)du$$

- Price of the Average Option:

$$\Pi[X|F] = e^{-r(T_2-t)} E^Q_{t,s}\left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)du\right] = \frac{e^{-r(T_2-t)}}{T_2 - T_1} \int_{T_1}^{T_2} E^Q_{t,s}[S(u)]du$$

$$\Pi[X|F] = \frac{s \cdot e^{-r(T_2-t)}}{T_2 - T_1} \int_{T_1}^{T_2} e^{r(u-t)} du = \frac{s/r}{T_2 - T_1} \cdot \left(1 - e^{-r(T_2 - T_1)}\right)$$

Pricing geometric averaging options

$$P_{call} = Se^{(b-r)T} N(d_1) - Ke^{-rT} N(d_2)$$

$$P_{put} = Ke^{-rT} N(-d_2) - Se^{(b-r)T} N(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(b + \frac{\sigma^2{}_A}{2}\right)T}{\sigma_A \sqrt{T}} \qquad b = \frac{1}{2}\left(r - \frac{\sigma^2}{6}\right)$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

and the adjusted volatility equals

$$\sigma_A = \sqrt{\frac{\ln(M_2)}{T} - 2b}$$

Pricing arithmetic averaging options

- The analytic approximation

$$P_{call} \approx Se^{(b-r)T} N(d_1) - Ke^{-rT} N(d_2)$$

$$P_{put} \approx Ke^{-rT} N(-d_2) - Se^{(D-r)T} N(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(b + \frac{\sigma^2{}_A}{2}\right)T}{\sigma_A \sqrt{T}}$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

$$b = \frac{\ln(M_1)}{T} \qquad \sigma_A = \sqrt{\frac{\ln(M_2)}{T} - 2b}$$

where M_1 is a first and M_2 second moment function

Monte Carlo simulation

- applying the *control variate method*

$$V_A = V'_B - V'_A + V_B$$

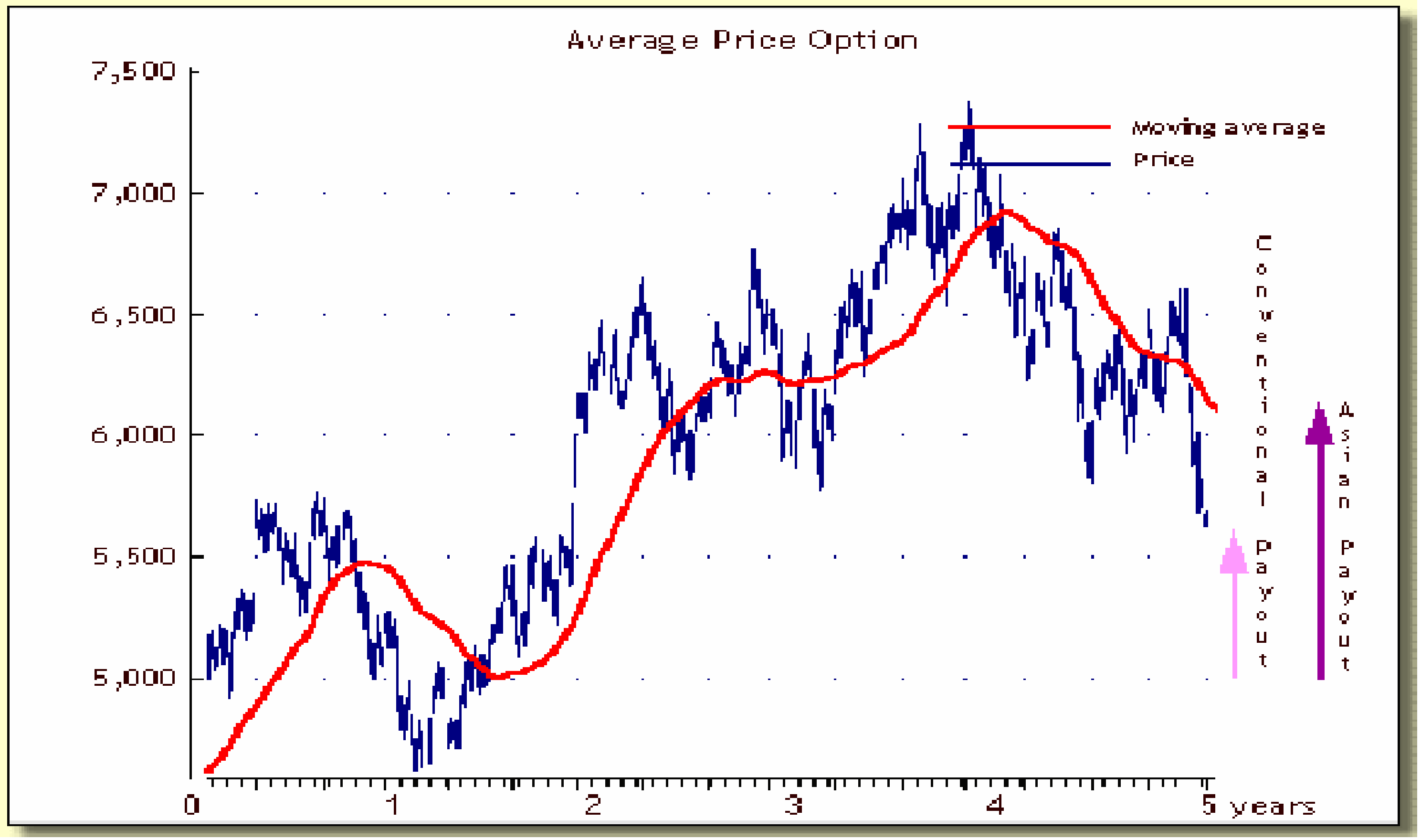
where

V'_A - estimated value of the arithmetic Asian through simulation

V'_B - simulated value of the geometric Asian

V_B - value of the geometric Asian

Comparison of the payoff



Conclusion

- Lower volatility
- Option price's sensitivity is reduced
- Cheaper than conventional one

Forward Options

Outlines

- Introduction
- Pricing
- Conclusion

Introduction

- Start at some future time
- Usually at-the-money at start
- 3 date $t < t_g < T$
- When enter, pay premium at t, grant you at t_g, expire at T
- European option start future time (call/put)
- Main target → pricing these forward options

European Option Pricing

- Standard BS European option formula

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S(t)^2 \frac{\partial^2 C}{\partial S^2} + rS(t)\frac{\partial C}{\partial S} - rC = 0$$

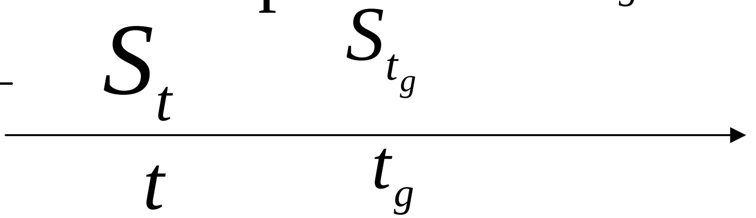
$$C(S(t),t) = e^{-r(T-t)}\,\tilde{E}[C(T,S(T))\,|\,I_t]$$

$$C = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$P = Xe^{-r(T-t)}N(-d_2) - SN(-d_1)$$

Forward Call Options

- Target: Find F(t)
- C(t)=value of ATM option with period $T-t_g$
- ATM option $F(t_g)=C(t)\frac{S_{t_g}}{S_t}$
- Feynman-Kac



$$F(t)=e^{-r(t_g-t)}E^*(F(t_g)\,|\,I_t)$$

$$E^*(S_{t_g})=S_t e^{(r-q)(t_g-t)} \longrightarrow ce^{-q(t_g-t)}$$

- where q is a dividend rate of the stock

Results

- F(t) equal to the value of present ATM option with same life period for no dividend paying stock (q=0)

$$F(t) = c$$

Forward Call Options

- Consider dividend pay over the stock

$$F(t) = ce^{-q(t_g - t)}$$

$$C(t) = e^{-q(t_g - t)}(e^{-q(T - t_g)} S_{t_g} N(d_1) - Xe^{-r(T - t_g)} N(d_2))$$

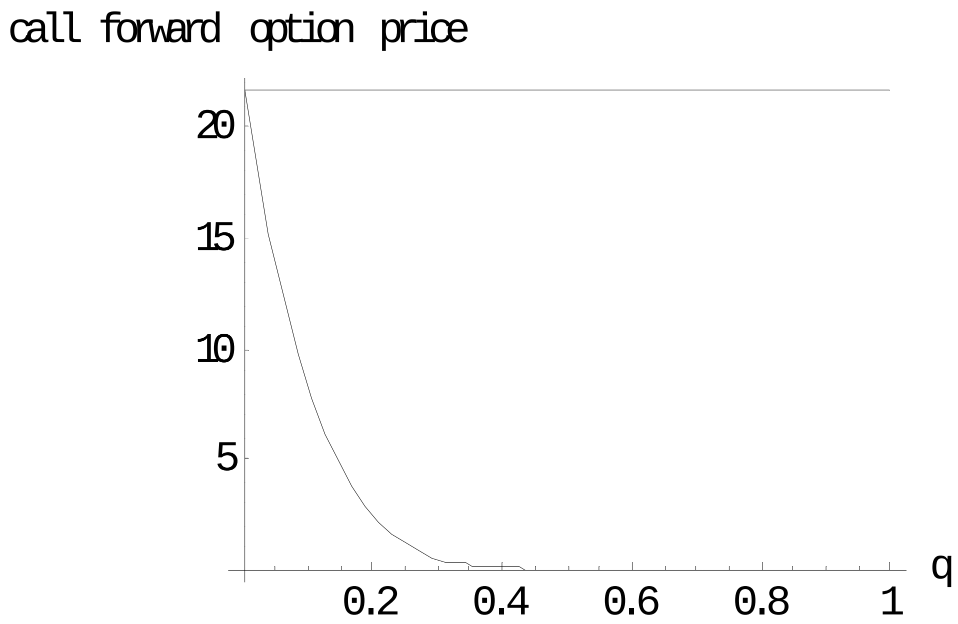
$$P(t) = e^{-q(t_g - t)}(Xe^{-r(T - t_g)} N(-d_2) - e^{-q(T - t_g)} S_{t_g} N(-d_1))$$

- q -> value of stock↓ -> value of option↓

Calculate forward call value

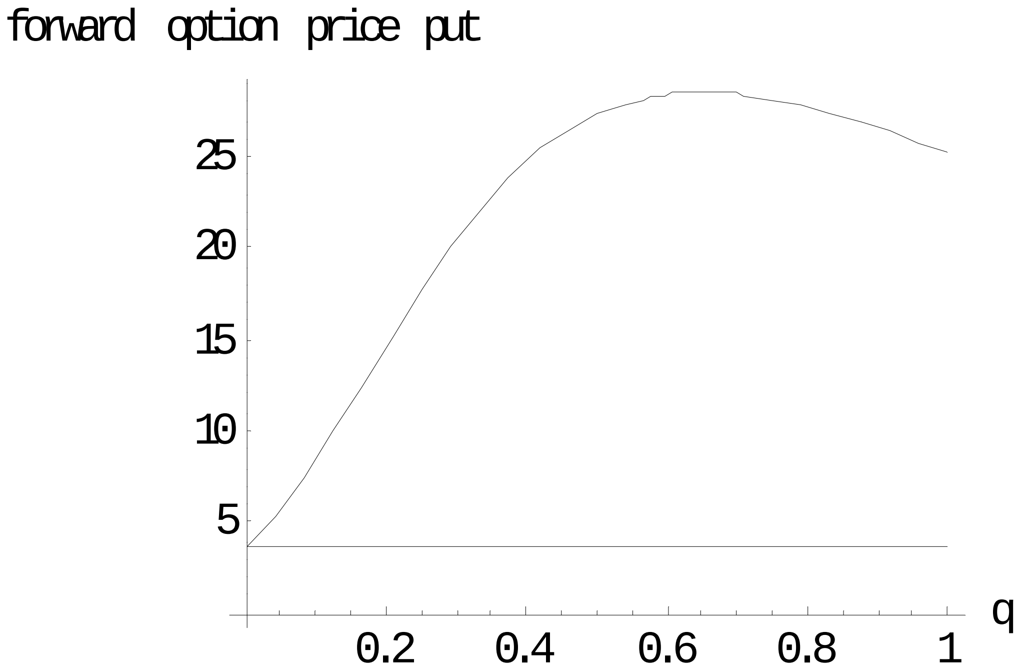
- Take $S_{t_g} = S_t = X = 100, r = 0.1, \sigma = 0.2$

  $t = 0, t_g = 1, T = 3$



Calculate forward put value

- The number same as before



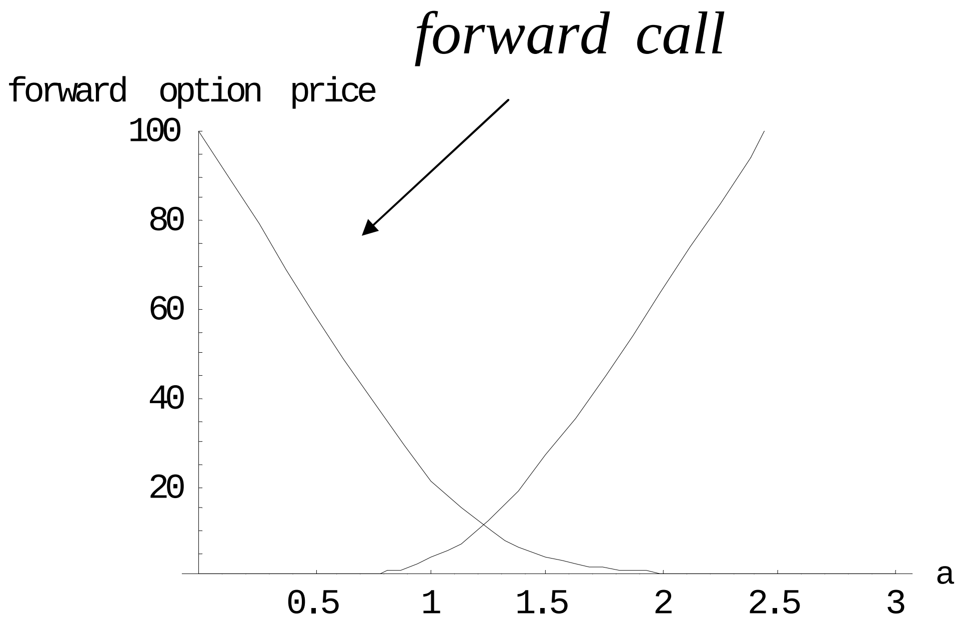
ITM/OTM Forward Options

- Consider forward option not ATM $X = S_{t_g}$ when granted
- But with a ratio $X = \alpha S_{t_g}$
- α is given at time t when you enter the contract
- Call: α <1, ITM ///// α >1, OTM
- The change of α -> change of price too

Calculate ITM/OTM forward call/put

- Take $S_{t_g} = S_t = 100, r = 0.1, \sigma = 0.2$

  $t = 0, t_g = 1, T = 3$



Remarks

- The larger the α , the larger the strike price, the more OTM the call option, thus the lower price
- Similarly, the more ITM the put option, thus the higher price

Conclusion

- Can price the forward option, hedging of these options difficult
- Assume const. volatility of asset over the time period but is stochastic variable
- wrong assumption of const. volatility -> significant risk in hedging these options
- Since dealing with forward option, need to estimate the forward volatility

The end

Thank you