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# OPTION ADJUSTED SPREAD

## Abstract

All readily priced instruments should be valued daily, less-readily priced instruments at least weekly and non-readily priced instruments as often as feasible and whenever a material event occurs. The pricing mechanism and methodologies must be known, understood, follow written policies and be applied consistently.

Interest Rate, Term Structure, and Valuation Modelling is a valuable practitioner-oriented text that thoroughly reviews the interest rate models and term structure models used today by market professionals and vendors of analytical services.

This paper revisits the question of the Option Adjusted Spread Analysis such a tool used by market professionals to analyze and compare bonds with embedded options. Provides a working knowledge of why yield-based analysis breaks down for non-bullet bonds, modelling put and call provisions as embedded options, intrinsic and time components of option values, and other aspects of bond trading.

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# 1. Introduction

There are different methods for valuing bonds with embedded options. Two traditional methods are the Yield-to-workout methodology and the the Spot rate methodology. In this paper we put the emphasis on the Option adjusted spread (OAS) analysis, a third method, which is an efficient stepwise technic used to find the price of the underlying option, embedded option, that is hidden in a mortgage backed security. A bond containing an embedded option has a nature of extra risk to it where, for callables, the issuer has a right but not obligation to call it back before it matures. To find the price of the additional risk that this embedded option causes there are some actions that can be taken. The eight steps of the OAS gives a good valuation to the price of this option.

1. Derive the set of risk-free spot zero rates (via bootstrapping) from benchmark yield curve.
2. Derive the risk-free forward short rates from the benchmark spot zero rates by exploiting the expectations theory of term structure.
3. Build a binomial tree depicting possible future values of the short rates and determine their probabilities
4. Apply the binomial tree to value the cash flows on a benchmark bullet bond
5. Calibrate the model by adjusting the binomial tree values until they matches the actual market price of the bond
6. Apply the same set of calibrated rates to value a callable bond by adding the same number of basis points (the spread factor) to all short rates in the tree. Adjust this spread factor until the model's predicted price equals the actual market price. The result is the bond's OAS.
7. Apply the same OAS to value a bullet bond with terms identical to the callable bond (except that the bullet bond is not callable).
8. Take the difference between the value obtained for the callable bond and the value obtained for the non-callable bullet bond. This difference is the value of the embedded call option.

We will also give an explanation to the concepts of duration and convexity which is the measure of a bonds price sensitivity to a small change in its yield. The convexity is used for larger changes in yield to take into account the curvature movements in the price/yield relationship, because which is shown, the yield curve is a convex function to price.

## **2. Types of bonds**

A bond is nothing more than a loan of which you are the lender. The organization that sells a bond is known as the issuer. You can think of it as an IOU given by a borrower (the issuer) to a lender (the investor).

Of course, nobody would loan his or her hard-earned money for nothing. The issuer of a bond must pay the investor something extra for the privilege of using his or her money. This "extra" comes in the form of interest payments, which are made at a predetermined rate and schedule.

The interest rate is often referred to as the coupon. The date on which the issuer has to repay the amount borrowed, known as face value, is called the maturity date. Bonds are known as fixed-income securities because you know the exact amount of cash you'll get back, provided you hold the security until maturity.

### **2.1 Bullet bond**

A bond that is not able to be redeemed prior to maturity. A bullet bond is usually more expensive than a callable bond (in that the interest rate is lower), since the investor is protected against the possibility of the bond being called when market interest rates fall.

### **2.2 Benchmark bullet bond**

A bullet bond issued by the government and therefore without risk, i.e. treasury bond.

### **2.3 Callable bond**

A callable bond is a bond, which allows the issuer to repurchase the bond for a specified price on certain dates prior to the bond's maturity.

Effectively, the issuer of a callable bond retains a call option on the bond. Correspondingly, the holder of a callable bond is short a call option.

### **2.4 Zero coupon bonds**

A bond in which the principal and the interest are paid at the maturity. No coupons are paid.

## **3. Embedded Options**

An embedded option is an option that is an inseparable part of another instrument. Most embedded options are conversion features granted to the buyer or early termination options reserved by the issuer of a security.

A common embedded option is the call provision in most corporate bonds, which permits the issuer to repay the borrower earlier than the nominal maturity of the bond.

## 4. Duration.

Duration by the dictionary is –A common gauge of the price sensitivity of a fixed income asset or portfolio to a change in interest rates.

When there is a change in the yield, the change in price for a bond with time to maturity =  $t$  will be higher than for a bond with time to maturity =  $t - 1$ . The duration is the aggregated efficient lifetime of a series of cash flows. The duration is found through calculation of the present value for the sum of all cash flows for the bond. One could say that the duration tells the lifespan that a zero coupon bond should have at a specific interest rate to match a coupon paying bond. Duration is also used as risk measure for a bond, and to see why, let  $B_c$  denote the current value of a coupon bond with time to maturity  $y$ . Let the bond pay an annual coupon of  $C$  at dates  $t = 1, \dots, T$ , and have a face value  $F$  dollars. We can write the bonds value as

$$B_c(y) = \sum_{t=1}^T \frac{c}{(1+y)^t} + \frac{F}{(1+y)^T}$$

If the YTM changes by a small amount,  $\Delta y$ , by how much does  $B_c$  change.  $\Delta B_c$  denotes the change in bonds price. So for a small change in YTM, the change in the bonds price can be expressed as

$$\Delta B_c \approx -1 \frac{1}{(1+y)} \left[ \sum_{t=1}^T \frac{tc}{(1+y)^t} + \frac{TF}{(1+y)^T} \right] \Delta y \quad (1)$$

This expression is obtained by using the Taylor series expansion of the bonds price around the bonds yield.

$$B_c(y + \Delta y) - B_c(y) = \frac{\partial B_c}{\partial y} (\Delta y) + \frac{1}{2} \frac{\partial^2 B_c}{\partial y^2} (\Delta y)^2 + \dots$$

The equation shows the sensitivity of the bonds price to changes in its yield. The expression within the brackets is a measure of this sensitivity, i.e the risk. Taking this a little bit further we can see how this expression relates to duration though the classical definition for duration is

$$D_c \equiv \left[ \sum_{t=1}^T \frac{tc}{(1+y)^t} + \frac{TF}{(1+y)^T} \right] / B_c \quad (2)$$

Which is referred to as Macauleys duration. Substituting expression 2 into 1 gives

$$\Delta B_c \approx -B_c D_c \frac{\Delta y}{1+y} \quad (3)$$

The proportional change in the bonds price is related to the duration multiplied by the change

in the YTM divided by one plus the initial YTM. Lets change the definition for duration to obtain a simpler relation. The second definition is referred to as the modified duration

$$D_m = \frac{D_c}{(1 + y)} \quad (4)$$

This expression is familiar for those who have read economics as the price elasticity. When the expression for the modified duration is substituted into 3, we find that the bonds return is directly proportional to the modified duration, i.e

$$\Delta B_c \approx -B_c \times D_m \times \Delta y \quad (5)$$

For a given change in the YTM, the change in the bonds value is the multiplicative product of these three terms on the right side of 5. The minus sign gives the inverse relationship between yield and the price. It measures the sensitivity of the bonds return to changes in the bonds yield.

## 5. Convexity

The use of the modified duration to describe changes in the bonds price to changes in its YTM is accurate only for small changes in the YTM.

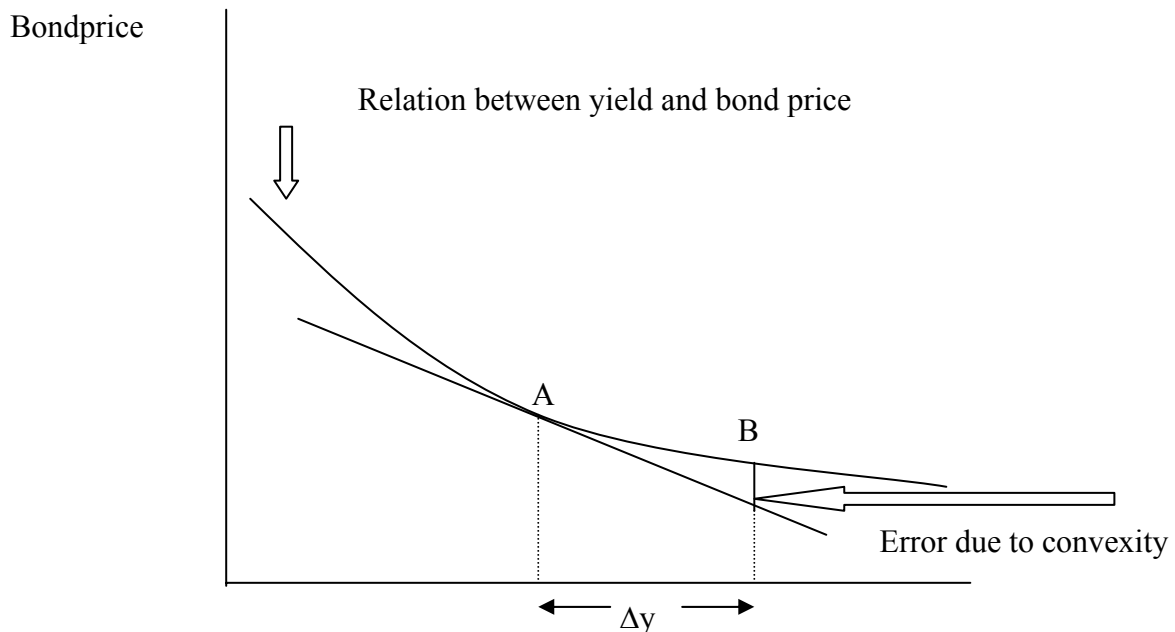


Figure 1

Figure 1 shows the convexity given the relationship between the bonds price and the yield. The modified duration only considers linear changes in the bonds price since its an linear approximation. This is represented by the straight line in the picture. There we can see that in fact the relation between the price of a bond and its yield is a nonlinear function. In fact the

relationship is convex. For small changes this nonlinearity is unimportant, but for larger movements the approximation becomes weaker and the nonlinearity has to be considered. The change in the bond's price, correcting for the convex relation between price and yield is then

$$\Delta B_c = -B_c \times D_m \times (\Delta y) + (1/2) \times B_c \times (\text{Convexity}) \times (\Delta y)^2$$

and the expression for convexity is

$$\equiv \frac{1}{(1+y)^2} \left[ \sum_{t=1}^T \frac{t(t+1)c}{(1+y)^t} + \frac{T(T+1)F}{(1+y)^T} \right] / B_c$$

## 6. Methodology for valuating bonds

### 6.1 Yield-to-workout methodology

In the yield-to-workout methodology all cash flows are discounted at the same rate – the yield-to-workout rate. The price of the bond is simply the present value of all cash flows. This is a traditional method for valuating bonds.

Under the assumption that the bond is paying a coupon today and that this coupon is semiannual, we have the following formula for calculating the present value:

$$PV_{bond} = \frac{1/2 * CR * Par}{(1+y/2)^1} + \frac{1/2 * CR * Par}{(1+y/2)^2} + \dots + \frac{1/2 * CR * Par}{(1+y/2)^{2T}} + \frac{Redemption}{(1+y/2)^{2T}}$$

where y is the yield, CR the annual coupon rate, Par the par value, Redemption is the bond's par amount if held to maturity, or the relevant call price if the bond is called and T is the number of years to workout date. If the bond will not be called, T is time to maturity.

Since it is not known in advance if the bond will be called or not, and if so, on which of the call dates it will happen, there exists a common practice for how to value the callable bonds. Using the current market price of the bond, we calculate all of its yields-to-workout and then make the assumption that the lowest yield (the yield-to-worst or promised yield) is the bond's return.

One big disadvantage in using the yield-to-workout method is how to decide the right yield to determine the credit spread. The credit spread is the additional return to the investor when accepting a higher risk. Usually the yield-to-worst is used, and the bond is treated as if it has maturity at the call date associated with this yield. The problem with this approach is that even small changes in the bond's price can change the yield-to-worst.

Additional weaknesses with the method is that it assumes that all cash flows should be discounted at the same rate, it is unstable when bonds are callable and it has difficulties addressing the volatility of interest rates.

### 6.2 The spot rate methodology



In the spot rate methodology, each coupon payment and the final redemption of a bond may be viewed as zero coupon bonds. This means that we can see the bond as a portfolio of zero coupon bonds, all discounted with the corresponding spot zero rate. The present value of the bond is the present value of the individual present values. Under the assumption that the bond is paying a coupon today and that this coupon is semiannual, we have the following formula for calculating the present value:

$$PV_{bond} = \frac{1/2 * CR * Par}{(1 + {}_0z_1 / 2)^1} + \frac{1/2 * CR * Par}{(1 + {}_0z_2 / 2)^2} + \dots + \frac{1/2 * CR * Par}{(1 + {}_0z_{2T} / 2)^{2T}} + \frac{Redemption}{(1 + {}_0z_{2T} / 2)^{2T}}$$

where  ${}_0z_t$  denote the spot zero rates for discounting zero coupon bonds with maturity  $t$  periods from now. In this case we assume that a period is six months and that the spot rate is annual with semiannual compounding.

The spot rate method, unlike the yield-to-workout method, uses different discount rates for each cash flow. This makes the spot rate method superior to the yield-to-workout method. Still it does not take into consideration the problems with a misleading change in the credit spread and the interest rate volatility. This is where the OAS analysis becomes important.

## 7. Option Adjusted Spread Analysis

Option-adjusted spread (OAS) analysis is recognized as an invaluable tool for analyzing and comparing bonds with different structures.

Option-adjusted spread analysis uses simulated interest rate paths as part of its calculation of bond yield and convexity.

The OAS refers to the yield spread between a callable or mortgage-backed bond and a government benchmark bond. The government bond chosen ideally will have similar coupon and duration values. Thus the OAS is an indication of the value of the option element of the bond, as well as the premium required by investors in return for accepting the default risk of the corporate bond.

When OAS is measured as a spread between two bonds of similar default risk, the yield difference between the bonds reflects the value of the option element only. This is rare, and the market convention is to measure OAS over the equivalent benchmark government bond.

OAS is used in the analysis of corporate bonds that incorporate call or put provisions, as well as mortgage-backed securities with prepayment risk. For both applications the spread is calculated as the number of basis points over the yield of the government bond that would equate the price of both bonds.

As with any methodology, OAS has both strengths and weaknesses; however, it provides more realistic analysis than the traditional yield-to-maturity approach. It has been widely adopted by investors since its introduction in the late 1980s.

### 7.1 Steps of the OAS analysis

There are eight steps involved in the OAS analysis.

1. Derive the set of risk-free spot zero rates (via bootstrapping) from benchmark yield curve.
2. Derive the risk-free forward short rates from the benchmark spot zero rates by exploiting the expectations theory of term structure.
3. Build a binomial tree depicting possible future values of the short rates and determine their probabilities
4. Apply the binomial tree to value the cash flows on a benchmark bullet bond
5. Calibrate the model by adjusting the binomial tree values until they matches the actual market price of the bond
6. Apply the same set of calibrated rates to value a callable bond by adding the same number of basis points (the spread factor) to all short rates in the tree. Adjust this spread factor until the model's predicted price equals the actual market price. The result is the bond's OAS.
7. Apply the same OAS to value a bullet bond with terms identical to the callable bond (except that the bullet bond is not callable).
8. Take the difference between the value obtained for the callable bond and the value obtained for the non-callable bullet bond. This difference is the value of the embedded call option.

## 7.2 Bootstrapping

The first step in the OAS analysis is to derive spot zero rates from a benchmark yield curve. The spot rates can be observed from Treasury zero coupon bonds or calculated via bootstrapping. Suppose that we have the following data for four Treasury securities:

Maturity (years)	Price (\$)	Yield-to-maturity	Coupon rate
0.5	96.15	0.0800	0
1.0	92.19	0.0830	0
1.5	99.45	0.0890	0.0850
2.0	99.64	0.0920	0.0900

We make the assumption that the coupons are paid at exactly the same date as the other securities expire.

The coupon rates are semi-annual compounded and par is \$ 100.00. The first security is a zero coupon bond with a YTM of 8%, which is the actual spot rate for the first period of 0.5 years. The security with maturity of one year is also a zero coupon bond, giving a spot rate of 8.3% the second period (from 6-12 months).

$$s_1=8\%; \quad s_2=8.3\%$$

The third bond however, pays coupons of \$ 4.25 every sixth month. In order to find the spot rate for the third period, we set up the following equation, where the present value of all cash flows will be equal to the price of the bond:

$$\frac{4.25}{1 + \frac{s_1}{2}} + \frac{4.25}{(1 + \frac{s_2}{2})^2} + \frac{104.25}{(1 + \frac{s_3}{2})^3} = 99.45$$

The only unknown factor in this equation is the spot rate for the third period,  $s_3$ . Solving this gives the value of  $s_3=8.24\%$ . The bond with two years to maturity, paying coupons of \$ 4.50, gives the spot rate for the fourth period,  $s_4$ .

$$\frac{4.50}{1 + \frac{s_1}{2}} + \frac{4.50}{(1 + \frac{s_2}{2})^2} + \frac{4.50}{(1 + \frac{s_3}{2})^3} + \frac{104.50}{(1 + \frac{s_4}{2})^4} = 99.64$$

$$s_4=8.18\%$$

We have now calculated the spot-rate-yield-curve:

Maturity (years)	Yield-to-maturity	Spot-rate
0.5	0.0800	0.0800
1.0	0.0830	0.0830
1.5	0.0890	0.0824
2.0	0.0920	0.0818

### 7.3 Derive the risk-free forward short rates

After calculating the spot zero rates, we can derive the risk-free forward short rates (or just forward rates) from the following equation:

$$r_{t_2-t_1}^{forward} = \left[ \frac{(1 + s_2)^{t_2}}{(1 + s_1)^{t_1}} \right]^{\frac{1}{t_2-t_1}} - 1$$

A forward short rate is an estimation of the future interest rate for a given time period, calculated from the spot rate or yield curve.

This gives us

Maturity (years)	Yield-to-maturity	Spot-rate	Forward rate
0.5	0.0800	0.0800	0.0800
1.0	0.0830	0.0830	0.0860
1.5	0.0890	0.0824	0.0812
2.0	0.0920	0.0818	0.0760

The above calculations of the spot rates and forward rates are made under the assumption that all coupon payments are made at the same time as the maturity for a zero coupon bond. This is a simplification of the real world, where we need to be able to calculate the forward rates at the actual time for payment even when it does not coincide with the maturity of a zero coupon

bond. In this case interpolation is used in the model. We will, however, not include this in our report.

### 7.4 Building a Binomial Tree of Short rates

The third step in the analysis is to build a binomial tree of short rates. We assume:

- ➡ That the annual volatility of forward short rates is the same for all forward short rates and that this annual volatility is constant.
- ➡ That forward rates evolve via random walk process.
- ➡ That in order to have a set of rates with which to illustrate the process we have the following four forward short rates all expressed with semi-annual compounding:

$$\begin{aligned}
 {}_0Z_1 &= 6.000 \% \\
 {}_1Z_1 &= 7.200 \% \\
 {}_2Z_1 &= 8.150 \% \\
 {}_3Z_1 &= 8.836 \%
 \end{aligned}$$

- ➡ That annual volatility of the forward short rate is 15%.

We need to describe how, over time, the estimates of future short rates might evolve from the current estimates of those future short rates, which are the forward rates.

Under a standard binomial approach, we assume that with each passing time period, the estimate of a future short rate can go up to one and only one higher level or down to one and only one lower level.

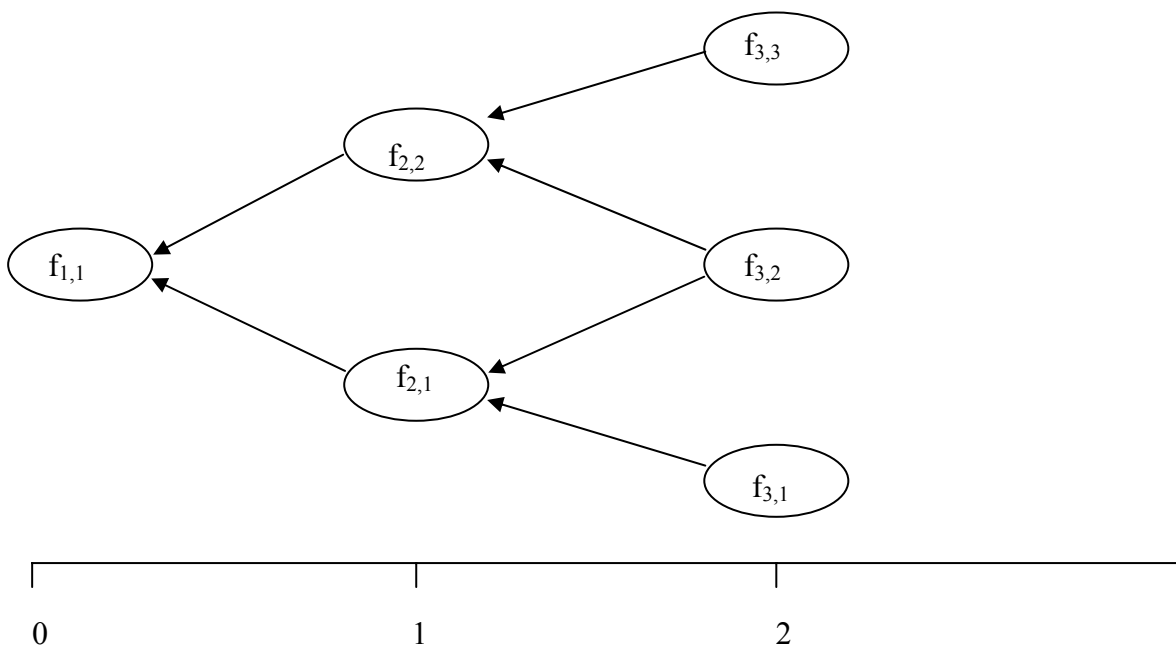


Figure 2 – Binomial tree

Looking at the binomial tree, we start at the period 0, we suppose that it's now and denote it  $f_{1,1}$ , moving forward inside the tree at the period 1 having only two alternatives  $f_{2,1}$  that denotes the lower node, and  $f_{2,2}$  denoting the upper node. We continue with our next step inside the tree at the period 2 looking at our tree alternatives denote by  $f_{3,1}$  for the lower node,  $f_{3,2}$  for the middle node and  $f_{3,3}$  for the upper nodes.

We are denoting the t-period forward short rate by  $f_{i,j}$ , but we are interested in where this forward rate will be j periods from now and we denote this by  $f_{i,j} = Z_i^{j-1} * f_{i,1}$ .

Thus, the calculation of the first node is:

$$f_{i,j} = Z_i^{j-1} * f_{i,1} \Rightarrow f_{1,1} = f_1 \Rightarrow f\left(0, \frac{1}{2}\right) \text{ and this is our forward rate in the period from 0 to half a year.}$$

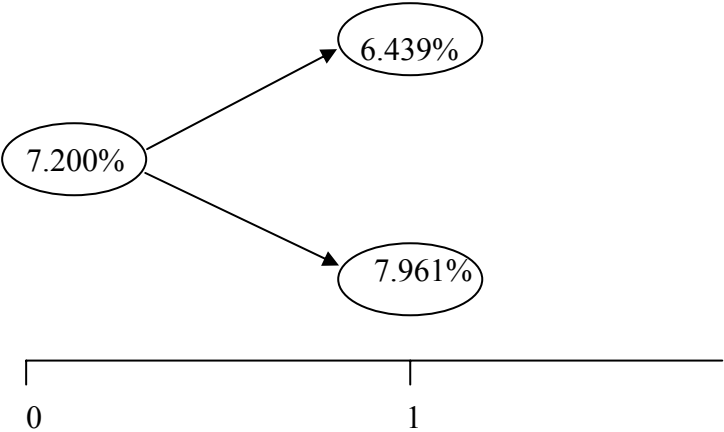


Figure 3. Shows the result of a forward rate in terms of its value in the prior period.

Note that when  $j = t$  the forward short rate has become the realized short rate. We need to know all possible short rates that might be realized for each period.

We will assume that the probability of the forward short rate rising for each period and the probability of the forward short rate declining each period are the same.

Building the Binomial tree, we must to keep in mind different notation to make it easy our calculations.

The first notation is  $Z_i$  that's mean the volatility element, and it's calculating according to the formulae  $Z_i = e^{2\sigma\sqrt{t_i - t_{i-1}}}$ ; where  $t_i - t_{i-1} = \frac{1}{2} \text{ year}$ .

The second notation is the relation between the nodes  $f_{i,j} = Z_i^{j-1} * f_{i,1}$ .

The third notation is the probability element that is building inside the binomial tree, taking different values according to our movements in the different periods.

Keeping in mind the mentioned above we can calculate the forward rate for the second period.

$$\left\{ \begin{array}{l} f_{2,2} = Z_2 * f_{2,1} \\ f_{2,1} = \frac{2 * f_2}{1 + Z_2} \end{array} \right. \Rightarrow f_2 = \frac{1}{2} f_{2,1} + \frac{1}{2} f_{2,2} \Rightarrow f_{2,2}$$

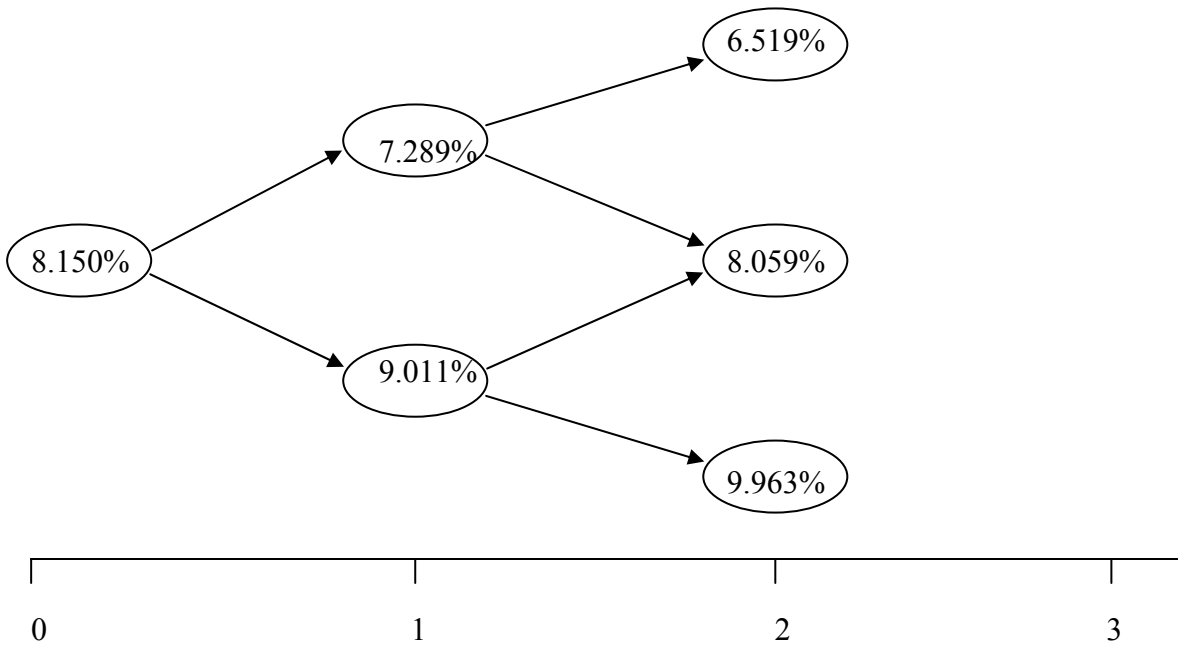


Figure 4. Shows the realized values of the short rate two periods from now  $f_{3,3}$ .

The forward rate for the third period is calculated by:

$$\left\{ \begin{array}{l} f_{3,3} = Z_3^2 \times f_{3,1} \\ f_{3,2} = Z_3 \times f_{3,1} \\ \frac{1}{4} f_{3,1} + \frac{1}{2} f_{3,2} + \frac{1}{4} f_{3,3} = f_3 \end{array} \right. \Rightarrow f_{3,1} = \frac{4 \times f_3}{1 + 2 \times Z_3 + Z_3^2} \Rightarrow f_{3,2} \Rightarrow f_{3,3}$$

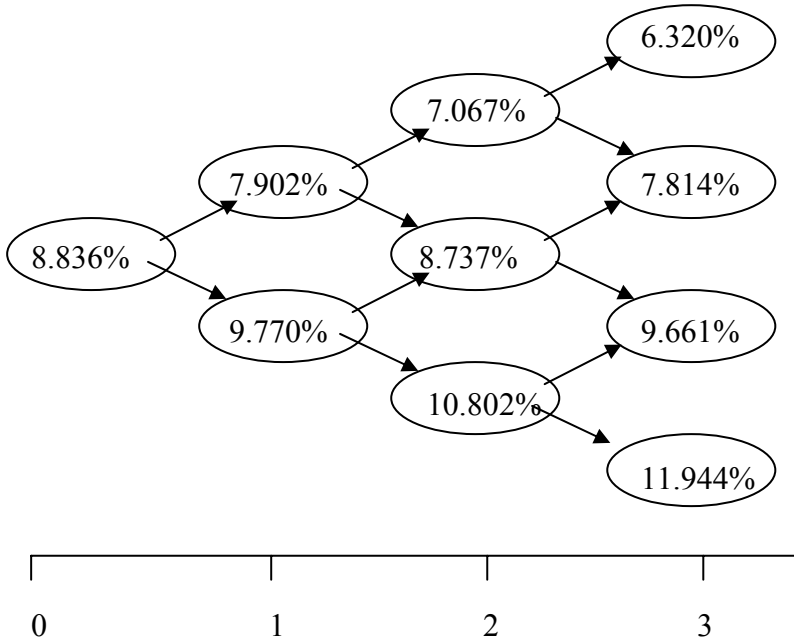


Figure 5. Shows the realized values of the short rate three periods from now

In this way, we can continue for the fourth, and so on. The general formula is

$$f_{n,1} * \sum \frac{n-1}{i} * Z_n^i = 2^{n-1} * f_n \Rightarrow f_{n,1} \Rightarrow f_{n,2}, \dots, f_{n,n}$$

We are not interested in the intermediate values associated with arriving at the set of possible realizable values of each short rate.

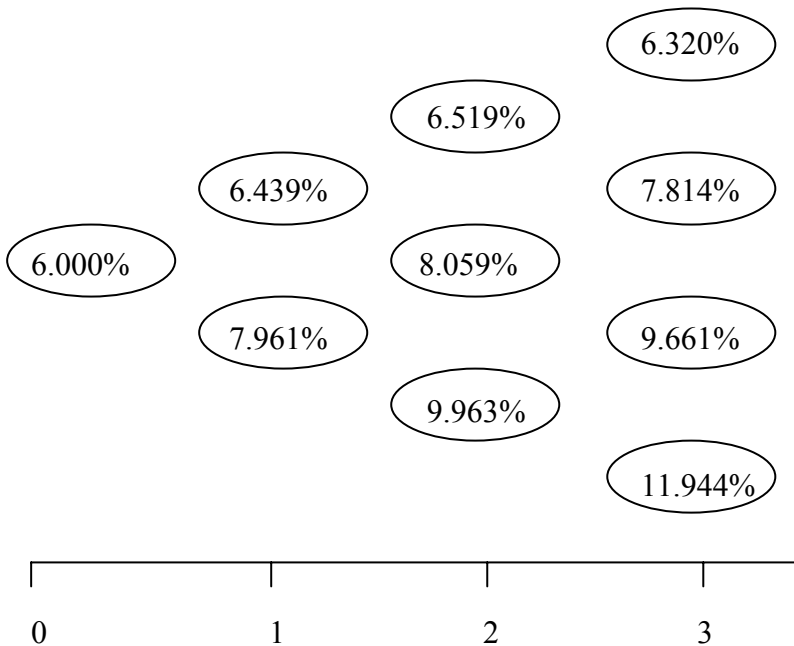


Figure 6. Show the binomial tree of risk-free short rates.

We are now ready to employ the range of possible values the various short rates might take on to value a bond.

We begin with a benchmark bullet bond. Suppose that the bond pays a 7.50% annual coupon in two semi-annual instalments of \$3.75 (per \$100 of par) and that the bond currently has 24 months (4 periods) to maturity.

Given the forward short rate we have assumed, this bond should be priced exactly at par (1000,000), which we will assume it is.

The first coupon of \$3.75 will be received exactly 1 period i.e six months. What is the value now (present value) of this future payment? We obtain this by building a price tree for this payment, because this payment is only one period away and the short rate associated with this one period is known with certainty.

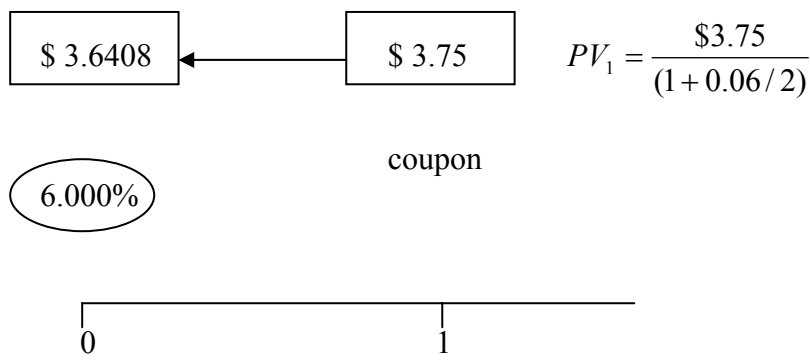


Figure 7. Shows the present value of the future payment.

Next, we use the short rates to determine the value of the \$3.75 coupon payment that will be received two periods (12 months) from now by building a price tree for the second cash flow. This requires that we discount sequentially backward through every possible path potentially realizable short rates.

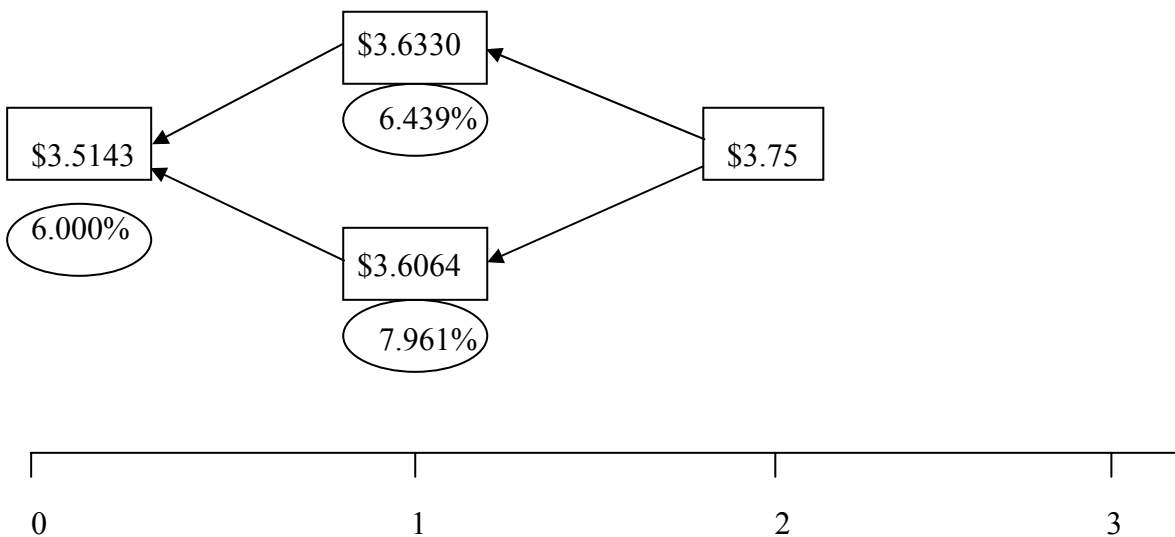


Figure 8. Shows the coupon instalments, received 2 period from now.



Now, we use the short rates to determine the value of the \$3.75 coupon payment that will be received 3 periods from now by building by building a price tree for the third cash flow. Again this requires that we discount sequentially backward through every possible path of potentially realizable short rates.

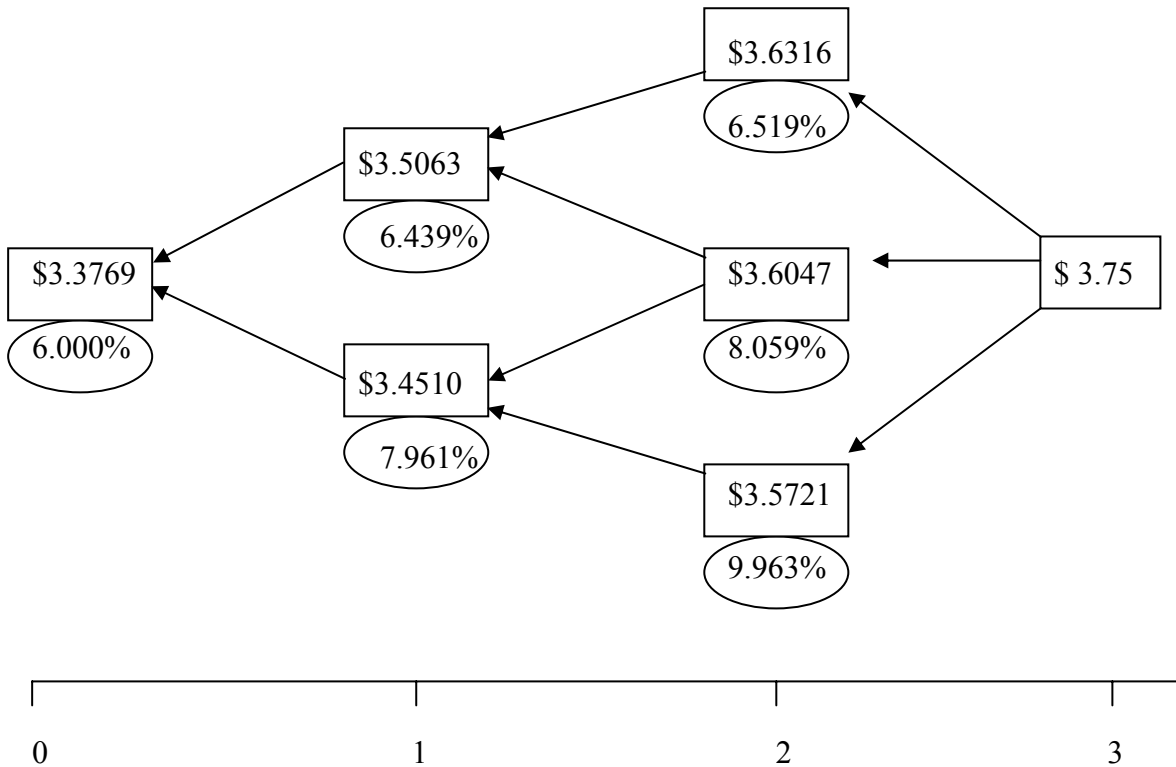


Figure 9. Shows the coupon instalments, received 3 period from now.

Finally, we use the binomial tree of short rates in the same manner to calculate the expected present value of the final cash flow. This final cash flow, which occurs two years from today, includes the final coupon payment of \$3.75 and the bond's par value of \$100. Thus, the final cash flow is \$103.75.

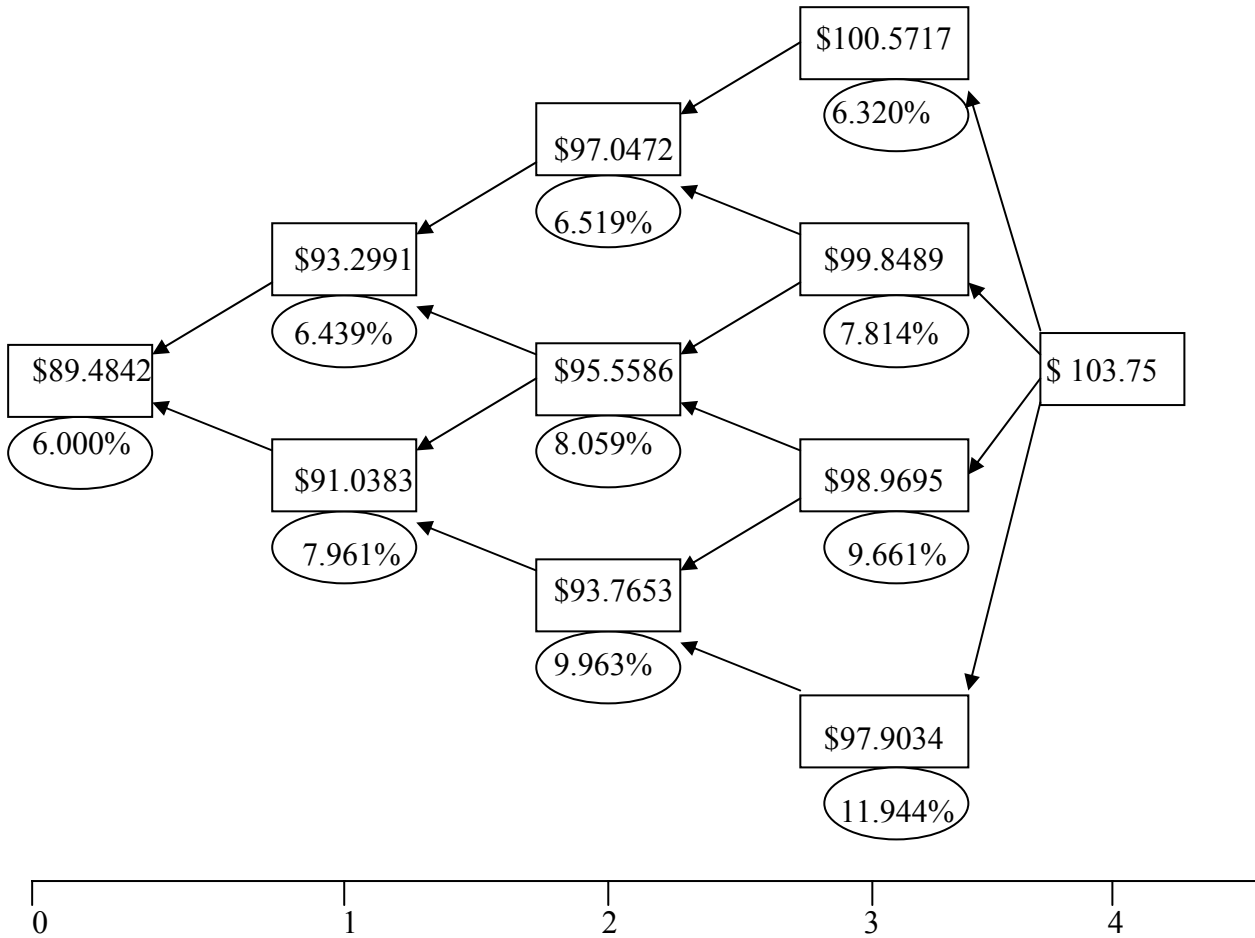


Figure 10.

### 7.5 Value the bond from its cash flows

If we sum the present values of the cash flows from the binomial tree, we get the following value of the bond:

Period	Cash flow	Present value
1	\$ 3.75	\$ 3.6408
2	\$ 3.75	\$ 3.5143
3	\$ 3.75	\$ 3.3769
4	\$ 103.75	\$ 89.4842
		\$ 100.0161

The market price of the bond is actually \$ 100.0000. What has caused this error in our calculations? The answer is a combination of two factors – the convexity of present value functions and the volatility of forward rates.

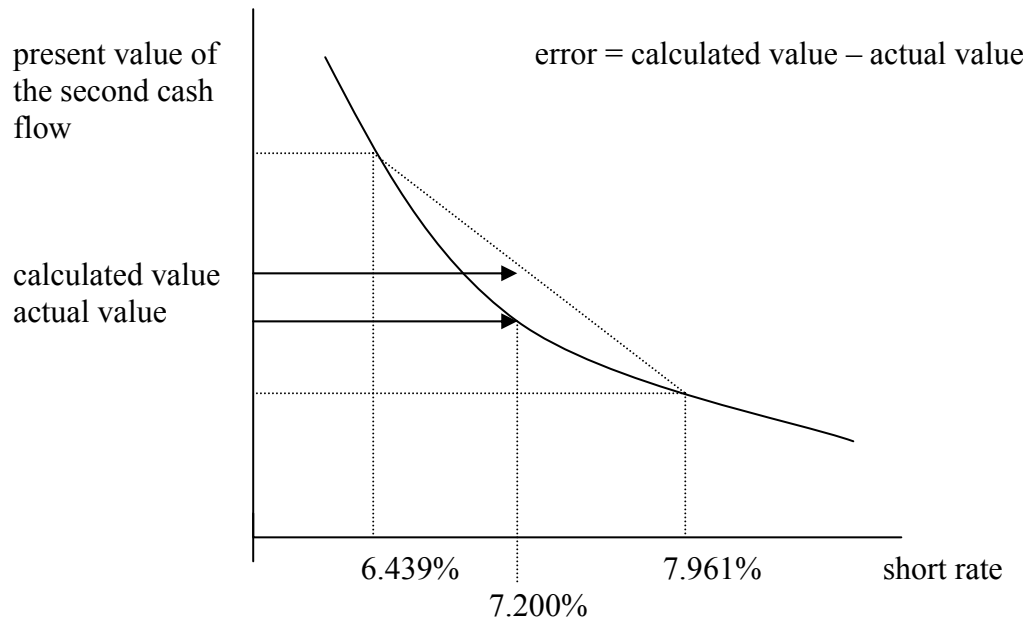


Figure 11.

The present value curve is, as we see in the figure above, curved and not linear. This convexity, together with the volatility of interest rates, is the source of the error. If we discount the second cash flow of \$ 3.75 first with the one period forward short rate (7.200%) and then again with the spot short rate (6.000%) we get the actual value. If the cash flow is discounted by two different short rates (6.439% and 7.961%), the calculated average value will lie on the straight line between these values and, therefore, above the actual value.

### 7.6 Calibrate the binomial tree of short rates

How can we correct the calculated error? This is done by so called calibrating of the binomial tree of short rates. We need to change the short rates so that the tree will give us the same prices as the forward rates.

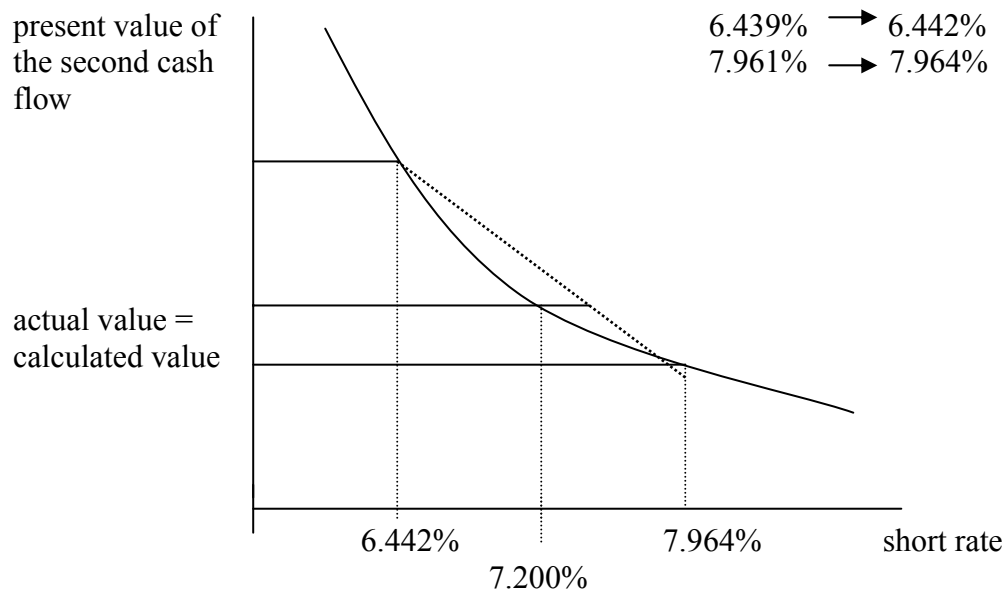


Figure 12.

Choose any cash flow (cf) for each period of the tree, starting at time 1, that we will estimate. The discounted value of this cash flow should give the same result if we use the forward rate or use the binomial tree. At time 2, we will get the following equation

$$\frac{1}{2} \left[ \frac{cf}{1 + f_{2,1} * (t_2 - t_1)} + \frac{cf}{1 + Z_2 * f_{2,1} * (t_2 - t_1)} \right] * \frac{1}{1 + f_{1,1} * (t_1 - t_0)} = \frac{cf}{(1 + f_1 * (t_1 - t_0)) * (1 + f_2 * (t_2 - t_1))}$$

The left hand side is the discounted cash flow using the tree (first at time 2, then at time 1). The right hand side is the discounted cash flow by the current forward rate. This equation can be solved analytical, but for higher nodes numerical methods must be used.

Observe! During the calibration the equality  $f_{i,j} = Z_i^{j-1} * f_{i,1}$  must hold.

After calibrating all nodes, we see that the rates have increased slightly in order to correct the error.

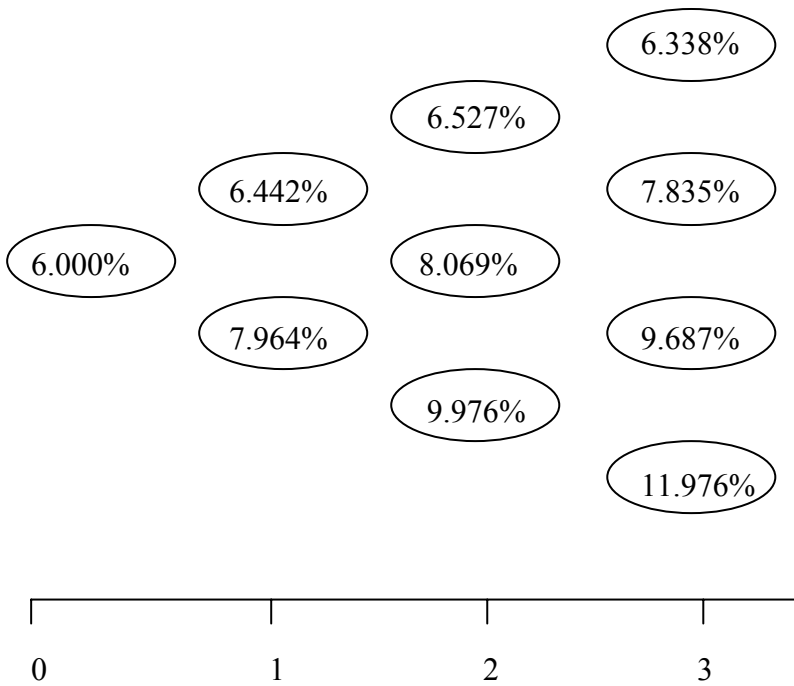


Figure 13.

When we sum up the present values of the calibrated tree, we get:

Period	Cash flow	Present value
1	\$ 3.75	\$ 3.6408
2	\$ 3.75	\$ 3.5143
3	\$ 3.75	\$ 3.3767
4	\$ 103.75	\$ 89.4683
		\$ 100.0000

The error is corrected. This tree can be used in valuating a treasury bond.

### 7.7 Adding the spread factor

In order to value a non-benchmark callable bond, like a callable corporate bond, the tree must be calibrated once again. Let us assume that there are no transaction costs, and that the issuer always call the bond when this is rational, just to simplify the calculations.

Suppose that we have a callable corporate bond with maturity in 24-months with an annual coupon of 10.50% (semi annual payments). The bond can be called in 18 months at a price of \$ 101.00, and the price today is \$ 103.75.

To begin with we calculate the values of the price tree for the first three cash flows, just like we did before. The first cash flow,

$$PV_1 = \frac{5.25}{(1 + (0.06000/2))} = 5.0971$$

the second,

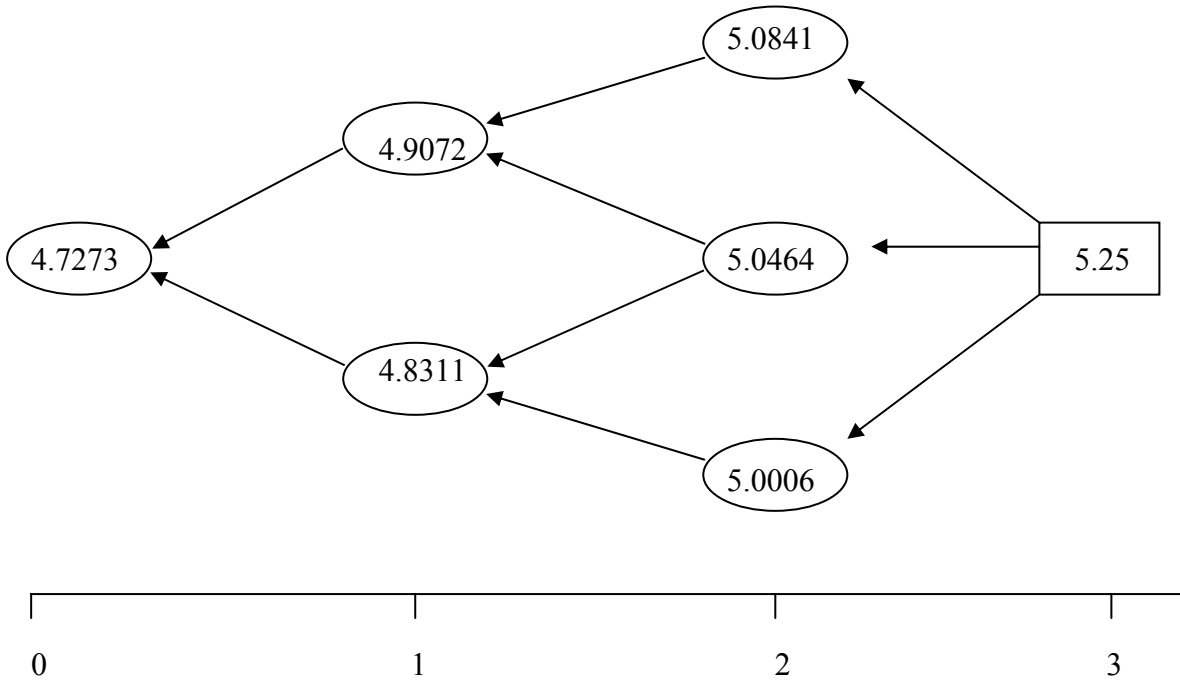


Figure 14.

The value of the fourth cash flow looks like this:

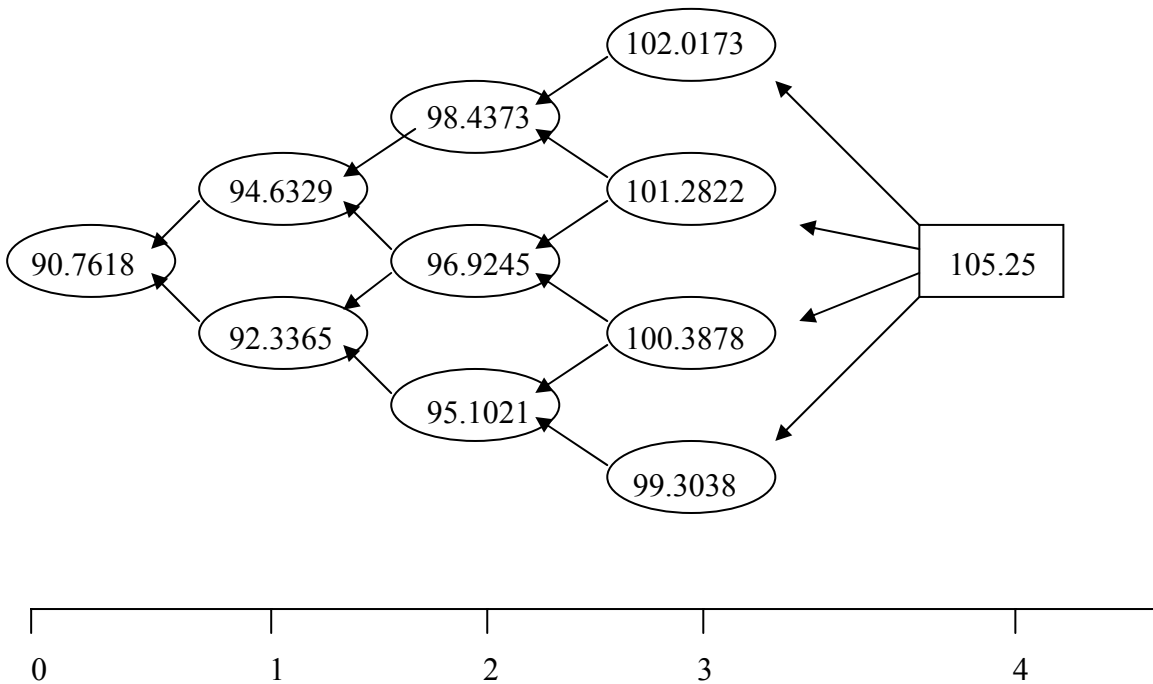


Figure 15.

The fourth cash flow can take one four values, \$102.0173, \$ 101.2822, \$100.3878 or \$ 99.3038. The bond will be called by the issuer only if the value exceeds the call price of \$ 101.00. This means that the maximum value of the bond at call date will be \$ 101.00, and we need to adjust the tree like this:

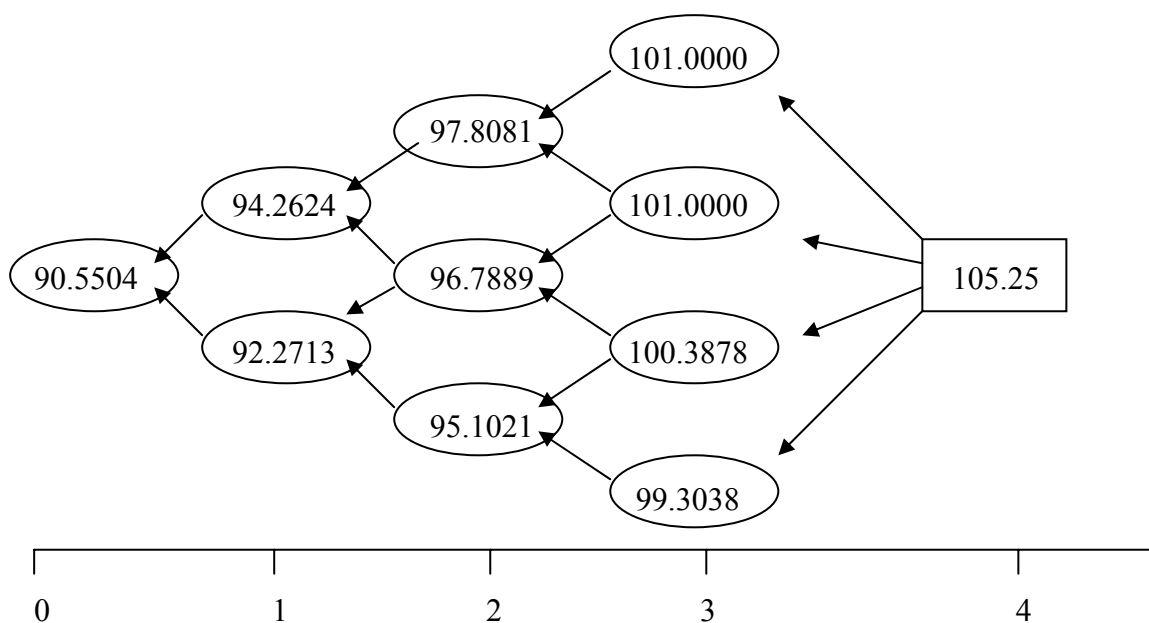


Figure 16.

If we now sum the present values for the different cash flows, we get the model's predicted price (MPP):

<u>Period</u>	<u>Cash flow</u>	<u>Present value</u>
1	\$ 5.25	\$ 5.0971
2	\$ 5.25	\$ 4.9200
3	\$ 5.25	\$ 4.7273
4	\$ 105.25	<u>\$ 90.5504</u>
		\$ 105.2947

In this case the MPP is higher than the market price, and though the bond is overvalued. The reason for this is that we have used risk-free short rates when valuating the bond. A corporate bond is riskier than a Treasury bond, and investors are not prepared to pay as much for the corporate bond. This means that we need to discount the value of the cash flows with a higher interest rate than the risk-free one.

Lets go back to the calibrated binomial tree of risk-free short rates. We should add a constant spread to all the interest rates in the tree. The spread factor is measured in basis points, where 100 basis points equals to 1.0%. In order to find the right interest rates, we use trial and error. Adding 50 basis points, for example, generates a bond value of \$ 104.4664, indicating that we need a higher spread to get the desired value of \$ 103.75.

Whit a spread factor of 90.465 basis points, we get the following:

<u>Period</u>	<u>Cash flow</u>	<u>Present value</u>
1	\$ 5.25	\$ 5.0748
2	\$ 5.25	\$ 4.8772
3	\$ 5.25	\$ 4.6659
4	\$ 105.25	<u>\$ 89.1321</u>
		\$ 103.7500

This spread factor is the bond's option adjusted spread.

## 7.8 Use the OAS to value the embedded option

The bond's option adjusted spread is found, but we would like to know how much the embedded option in a callable bond is actually worth. Lets calculate the value of a non-callable bond with the same OAS. We use the same tree as before, but no bond values will be exchanged, since the bond can not be called. The value of the non-callable bond will be \$ 103.8143, which is higher than the callable bond price.

We might think of the callable bond as a portfolio consisting of a long position in a bullet bond and a short position in a call option on a bullet bond that begins on the option's call date:

$$B_{callable} = B_{bullet} - C_{bullet}$$

$$103.7500 = 103.8143 - C_{bullet}$$

$$C_{bullet} = 0.0643$$

The option is worth \$ 0.0643 for every \$ 100 of par.

## 7.9 Effective duration and convexity

If we have a callable bond with cash flows that are depending on the market interest rate we can't settle with the Macauleys duration for measuring the risk of possessing the bond. In this case we need to work with an efficient duration since there is presence of embedded options in these bond's. The OAS (option adjusted spread) –analysis helps to find a better measure of the interest rate risk.

To calculate this measure we need to find the price of the bond according to the steps in the OAS, second we shift the yield curve one point upwards and one down and calculates the new prices these shifts gives arise to. The spot rate is changed so the forward rate needs to be recalculated and calibration for each new calculation in the binomial tree has to be done. The efficient Macauleys duration and convexity can be found through

$$\text{Duration} = \frac{P_{down} - P_{up}}{2 \times P_{unshifted} \times \Delta y}$$

and

$$\text{Convexity} = \frac{P_{up} + P_{down} - 2 \times P_{unshifted}}{P_{unshifted} \times (\Delta y)^2}$$



## **Conclusion**

The OAS analysis is superior to more traditional methods of valuating bonds with embedded options. It involves bootstrapping, forward rates, building of binomial trees and calibrations of the trees.

To perform the calculations over many periods, numerical methods must be used, and the calculations are made by computers.

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