

Course MT1410 , 5p
Analytical Finance 1 C
Project /Seminar

Analytical Comparison Of 3 Binomial Models by Matlab Simulation

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Part one: Introduction

1.1 Aim

The aim of this paper is to compile codes - in the chosen of us programming language MATLAB - for 3 different binomial models. These are CRR (Cox-Ross-Rubinstein), TIAN and Leisen-Reimer models. The objective is to study the result of binomial pricing for 3 different American put options: one in-the-money, one at-the-money and one out-of-the-money. Finally, analytical approximation comparison of the graphical results is realized.

1.2 Method

The characteristic descriptions of the three binomial models are based mainly on student literature from courses Analytical Finance I. Also, the MATLAB program used to build the graphs part is based on self-reading reference literature supplied by the previous study course Numerical Methods with MATLAB.

Part Two: Theoretical Study of 3 Binomial Models

2.1 The Cox-Ross-Rubenstein model (the most popular model)

$$\begin{cases} u = e^{\sigma\sqrt{\Delta t}} \\ d = 1/u = e^{-\sigma\sqrt{\Delta t}} \end{cases}$$

$$\rho = \frac{e^{r\Delta t} - d}{u - d}$$

2.2 The TIAN model

$$\begin{cases} u = \frac{M \cdot V}{2} \left[V + 1 + \sqrt{V^2 + 2V - 3} \right] \\ d = \frac{M \cdot V}{2} \left[V + 1 - \sqrt{V^2 + 2V - 3} \right] \end{cases}$$

where

$$\begin{cases} M = e^{r\Delta t} \\ V = e^{\sigma^2 \Delta t} \end{cases}$$

$$\rho = \frac{e^{r\Delta t} - d}{u - d}$$

2.3 The Leisen Reimer model

This is a model giving more accurate approximation of the option value compared to the other models. This model has an important advantage against the other models. The model have quadratic converges in the number of time steps, while the other models have a linear convergence. That's why the accuracy is much better.

$$a = e^{r\Delta t}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) \cdot (T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

where we can see that d_1 and d_2 from Black-Scholes equations. Now we introduce:

$$\rho = B(d_2, N)$$

$$\bar{\rho} = B(d_2 + \sigma \cdot \sqrt{T-t}, N)$$

where B is the inverse of the binomial distribution and N - the number of time steps. We use the Peizer-Pratt method to invert the binomial distribution $[j+1/2=n-(j+1/2), n=2j+1]$:

$$\rho = B(z, n) = \frac{1}{2} \mp \left[\frac{1}{4} - \frac{1}{4} \cdot \exp\left\{ -\left(\frac{z}{n+1/3}\right)^2 \cdot \left(n + \frac{1}{6}\right) \right\} \right]^{\frac{1}{2}}$$

and we get

$$\begin{cases} u = a \cdot \frac{\bar{\rho}}{\rho} \\ d = a \cdot \frac{1-\bar{\rho}}{1-\rho} \end{cases}$$

Part Three: Codes Composition and Analytical Comparison of Generated Graphical Results

To compare the models, we shall study American put options with the following parameters:

- Underlying price $S_0 = 40$
- Strike price $K = 45$ [in-the-money]
 $K = 40$ [at-the-money]
 $K = 35$ [out-of-the-money]
- Time to maturity $T = 5/12$
- Risk-free interest rate $r = 0.08$
- Volatility $\sigma = 0.30$
- Number of time steps $[25, 250]$

3.1 “In-the-money” American put option pricing with 3 diverse binomial models

3.1.1 CRR Model

```
% File: crr1.m
% Objective: Binomial pricing of "in-the-money" american put
option using CRR binomial model
% Author: Group 4
So = 40;
K = 45;
r = 0.08;
T = 5/12;
sigma = 0.30;
N = 25:250;
price = zeros(size(N));
for k = 1:length(N);
    delta = T/N(k);
    [StockPrice, OptionPrice] = binprice(So, K, r, T, delta,
sigma, 0);
    price(k) = OptionPrice(1,1);
end
plot(N, price);
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');
```

The above code produces the following graph:

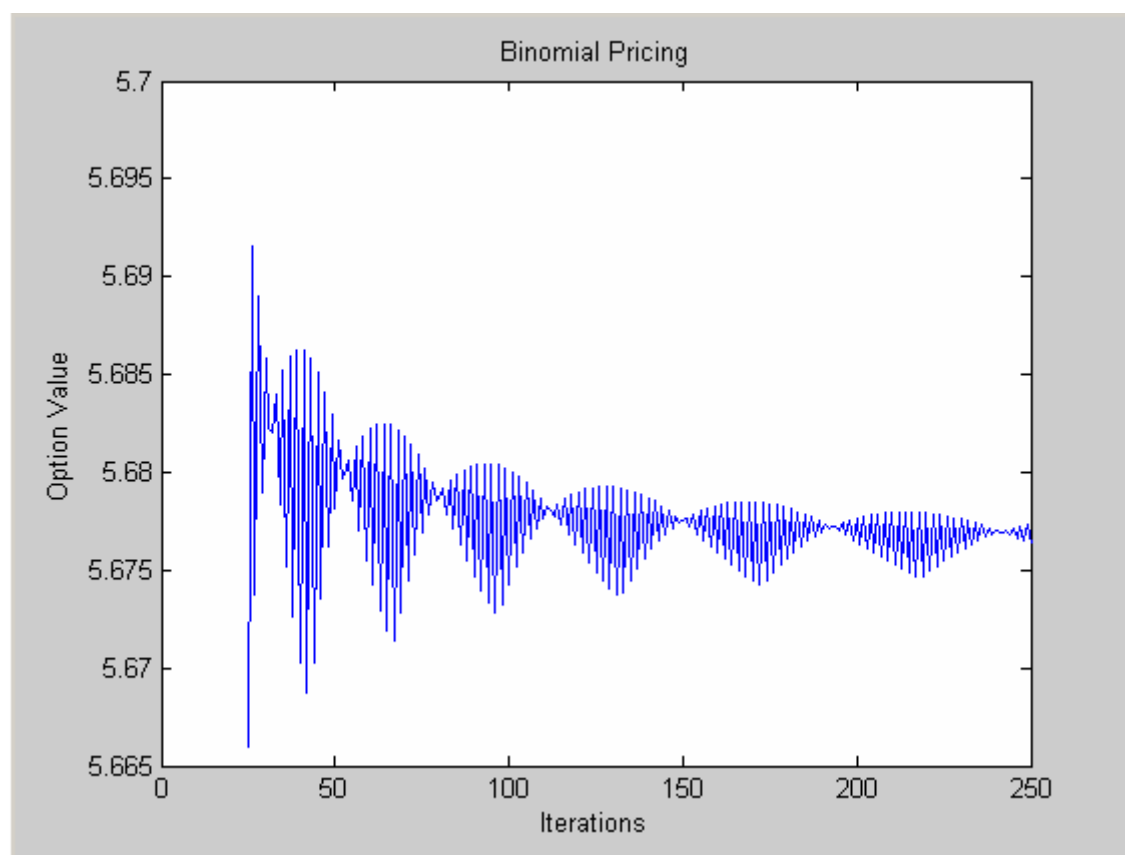


Fig. 1 A graph of American put option price values against the number of time steps.

3.1.2 TIAN Model

```
% File: TIAN1.m
% Objective: Binomial pricing of "in-the-money" american put
option using TIAN binomial model
% Author: Group 4
S0=40;
K=45;
T= 5/12;
r= 0.08;
sigma = 0.30;
price=1:250;
for n=25:250
    deltat=T/n;
    M=exp(r*deltat);
    V=exp(sigma^2*deltat);
    s=sqrt(V^2+2*V-3);
    u=M*V*(V+1+s)/2;
    d=M*V*(V+1-s)/2;
    p=(M-d)/(u-d);
    beta=1/M;
    V=pric(S0,K,u,d,p,beta,n);
    price(n)=V(1,1);
end;
plot(25:250,price(25:250));
xlabel('Iterations');
```

```
ylabel('Option Value');
title('Binomial Pricing');
```

The latter script displays the following graphical result:

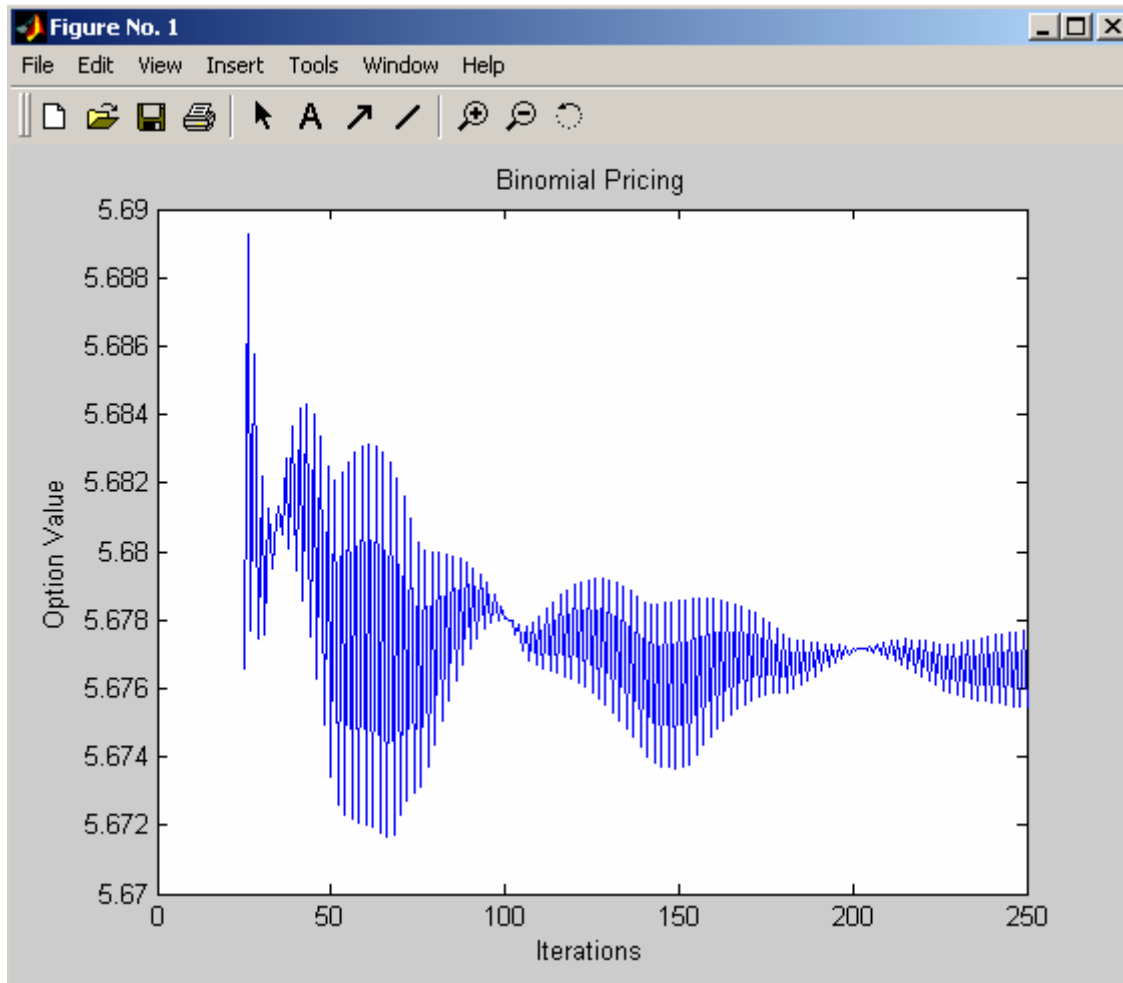


Fig. 2 A graph of American put option price values against the number of time steps.

3.1.3 Leisen Reimer Model

```
% File: LR1.m
% Objective: Binomial pricing of "in-the-money" american put
option using Leisen Reimer binomial model
% Author: Group 4
So = 40;
K = 45;
r = 0.08;
T = 5/12;
sigma = 0.30;
price = 1:250;
for n = 25:250
    deltat = T/n;
    a = exp(r*deltat);
    beta = 1/a;
```



```

d1 = (log(S0/K) + (r + 1/2 * sigma^2) * (T)) / (sigma * sqrt(T));
d2 = d1 - sigma * sqrt(T);
p1 = 1/2 + (1/4 - 0.25 * exp(-(d2/(n+1/3))^2 * (n+1/6)))^0.5;
p2 = 1/2 - (1/4 - (1/4) * exp(-(d2/(n+1/3))^2 * (n+1/6)))^(1/2);
q1 = 1/2 + (1/4 - (1/4) * exp(-(d1/(n+1/3))^2 * (n+1/6)))^(1/2);
q2 = 1/2 + (1/4 - (1/4) * exp(-(d1/(n+1/3))^2 * (n+1/6)))^(1/2);
if 0 < p1 < 1,
    p = p1;
else p = p2;
end
    if 0 < q1 < 1,
        q = q1;
    else q = q2;
    end
u = a * q / p
d = a * (1 - q) / (1 - p)
V = pric(S0, K, u, d, p, beta, n);
price(n) = V(1, 1);
end
plot(25:250, price(25:250));
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');

```

The latter code returns the following graph:

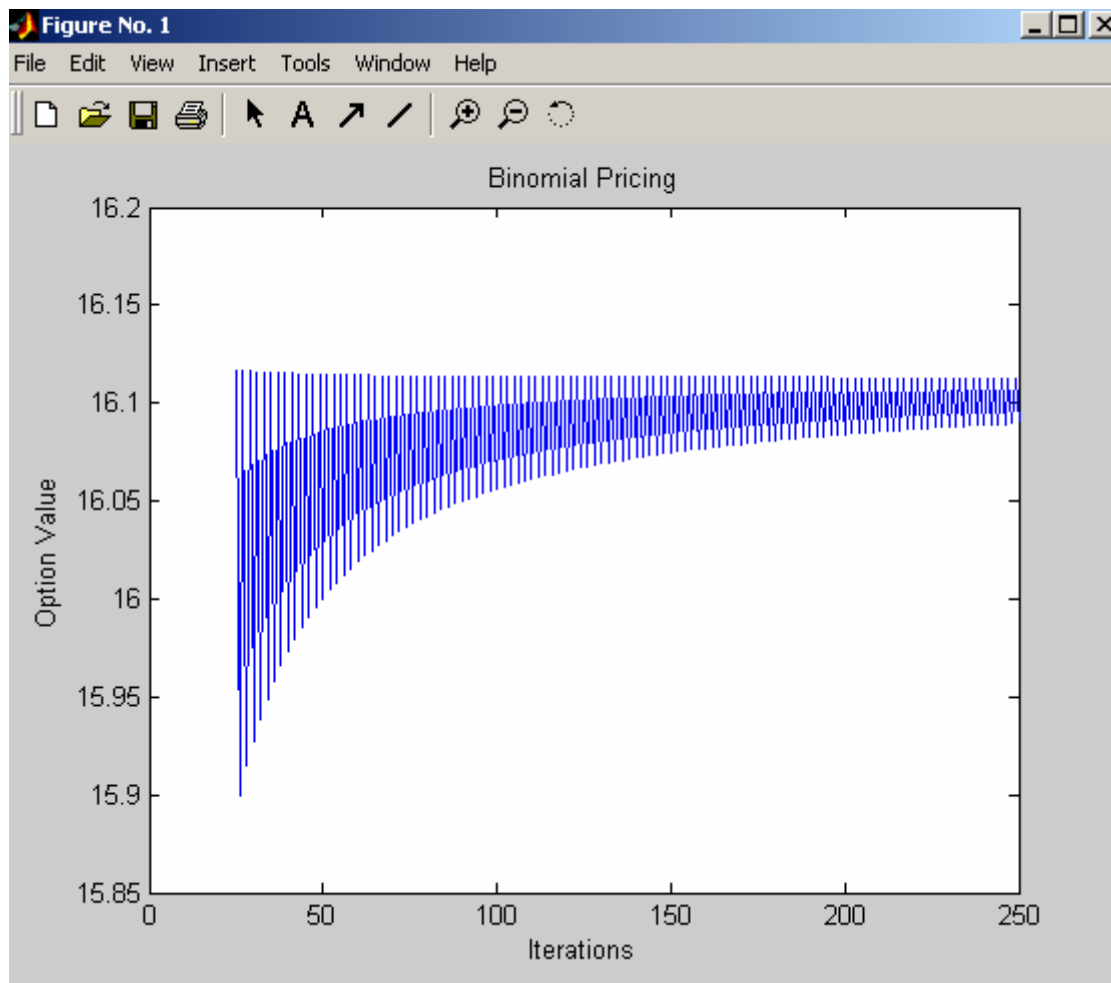


Fig. 3 American put option price values with respect to time iterations.

3.2 “At-the-money” American put option pricing with 3 diverse binomial models ($K = S_0$)

3.2.1 CRR Model

```
% File: CRR2.m
% Objective: Binomial pricing of at-the-money american put
option using CRR model for binomial pricing
So = 40;
K = 40;
r = 0.08;
T = 5/12;
sigma = 0.30;
N = 25:250;
price = zeros(size(N));
for k = 1:length(N);
    delta = T/N(k);
    [StockPrice, OptionPrice] = binprice(So, K, r, T, delta,
sigma, 0);
    price(k) = OptionPrice(1,1);
end
plot(N, price);
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');
```

Running the above M-file in Matlab command window generates the following graph:

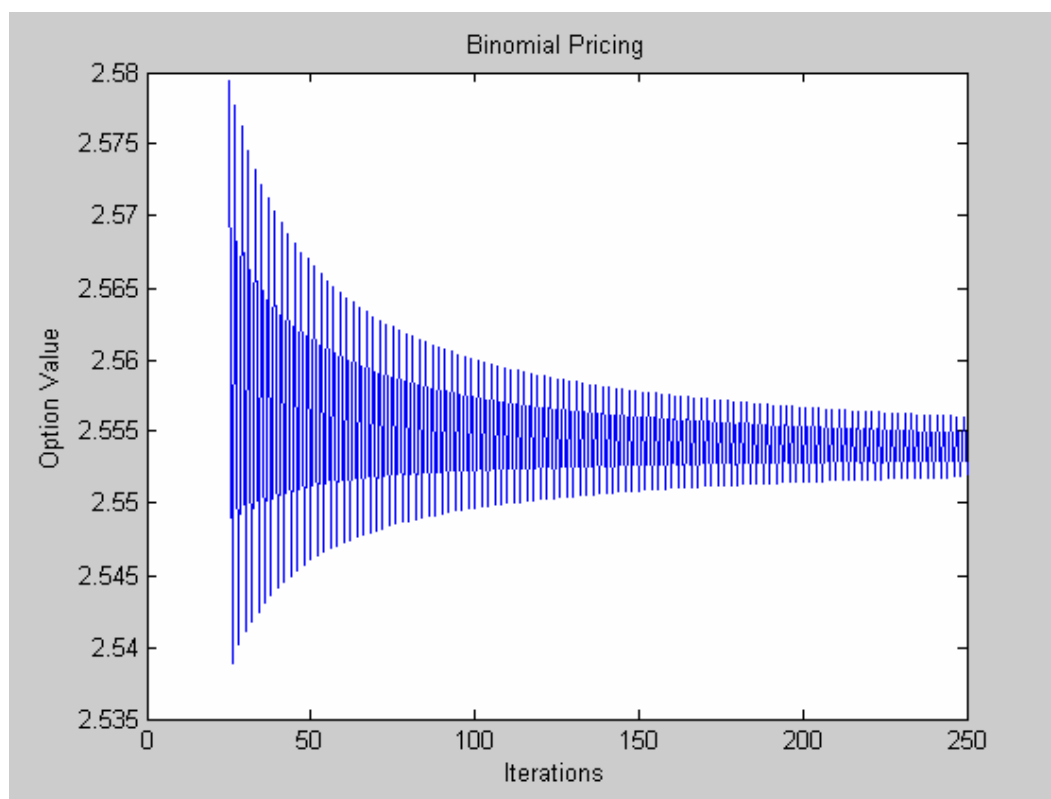


Fig. 4 American put option price values against the number of time steps.

3.2.2 At-the-money TIAN binomial pricing

The code compiled for TIAN model at-the-money American put option pricing is as follows:

```
% File: TIAN2.m
% Objective: Binomial pricing of "at-the-money" american put
option using TIAN binomial model
% Author: Group 4
S0=40;
K=40;
T= 5/12;
r= 0.08;
sigma = 0.30;
price=1:250;
for n=25:250
    deltat=T/n;
    M=exp(r*deltat);
    V=exp(sigma^2*deltat);
    s=sqrt(V^2+2*V-3);
    u=M*V*(V+1+s)/2;
    d=M*V*(V+1-s)/2;
    p=(M-d)/(u-d);
    beta=1/M;
    V=pric(S0,K,u,d,p,beta,n);
    price(n)=V(1,1);
end
plot(25:250,price(25:250));
xlabel('Iterations');
ylabel('Option Value');
```

```
title('Binomial Pricing');
```

This code returns the following graphical output:

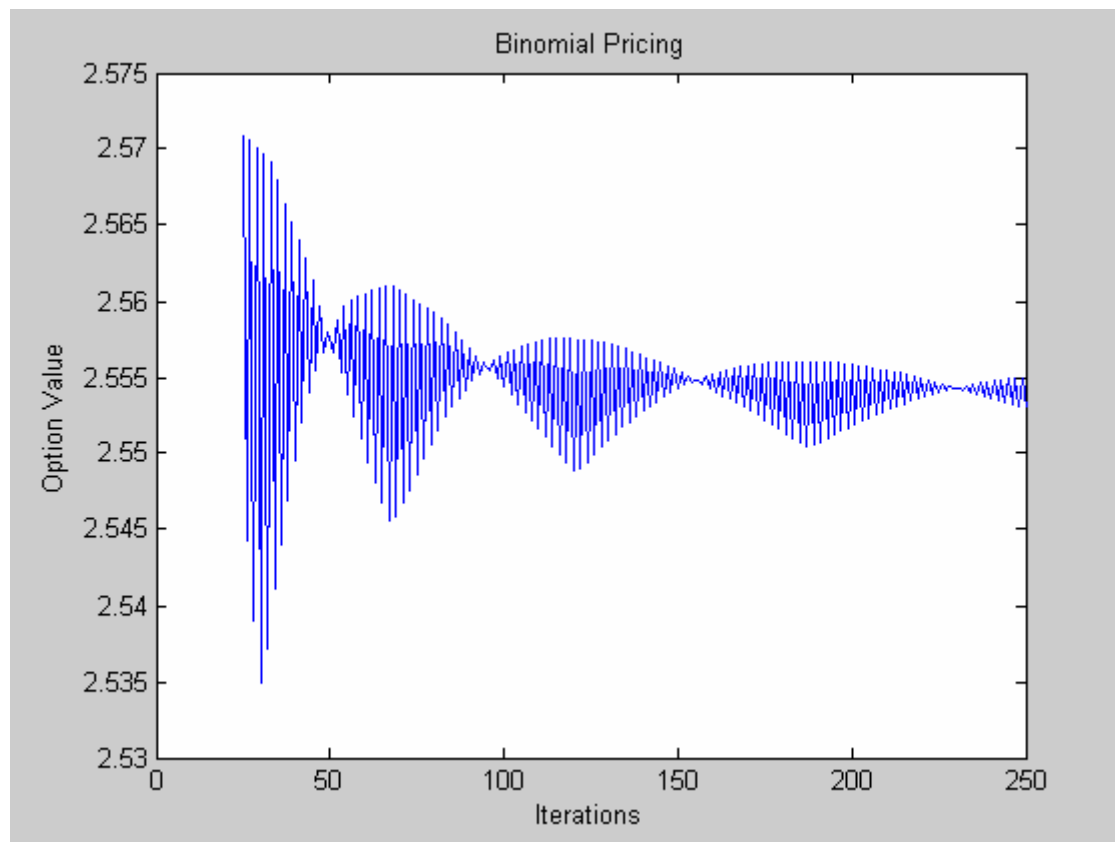


Fig. 5 American put option price values against the number of time steps.

3.2.3 Leisen Reimer model at-the-money American put option pricing

Using Leisen Reimer binomial model theory, we have composed the following codes:

```
% File: LR2.m
% Objective: Binomial pricing of "at-the-money" american put
option using Leisen Reimer binomial model
% Author: Group 4
So = 40;
K = 40;
r = 0.08;
T = 5/12;
sigma = 0.30;
price=1:250;
for n = 25:250
    deltat=T/n;
    a = exp(r*deltat);
    beta = 1/a;
    d1 = (log(S0/K)+(r+1/2*sigma^2)*(T))/(sigma*sqrt(T));
    d2 = d1-sigma*sqrt(T);
    p1 = 1/2 + (1/4-0.25*exp(-(d2/(n+1/3))^2*(n+1/6)))^0.5;
    p2 = 1/2 - (1/4-(1/4)*exp(-(d2/(n+1/3))^2*(n+1/6)))^(1/2);
```

```

q1 = 1/2 + (1/4 - (1/4)*exp(-(d1/(n+1/3))^2*(n+1/6)))^(1/2);
q2 = 1/2 + (1/4 - (1/4)*exp(-(d1/(n+1/3))^2*(n+1/6)))^(1/2);
if 0 < p1 < 1,
    p = p1;
else p = p2;
end
    if 0 < q1 < 1,
        q = q1;
    else q = q2;
    end
u = a*q/p
d = a*(1-q)/(1-p)
V=pric(S0,K,u,d,p,beta,n);
price(n)=V(1,1);
end
plot(25:250,price(25:250));
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');

```

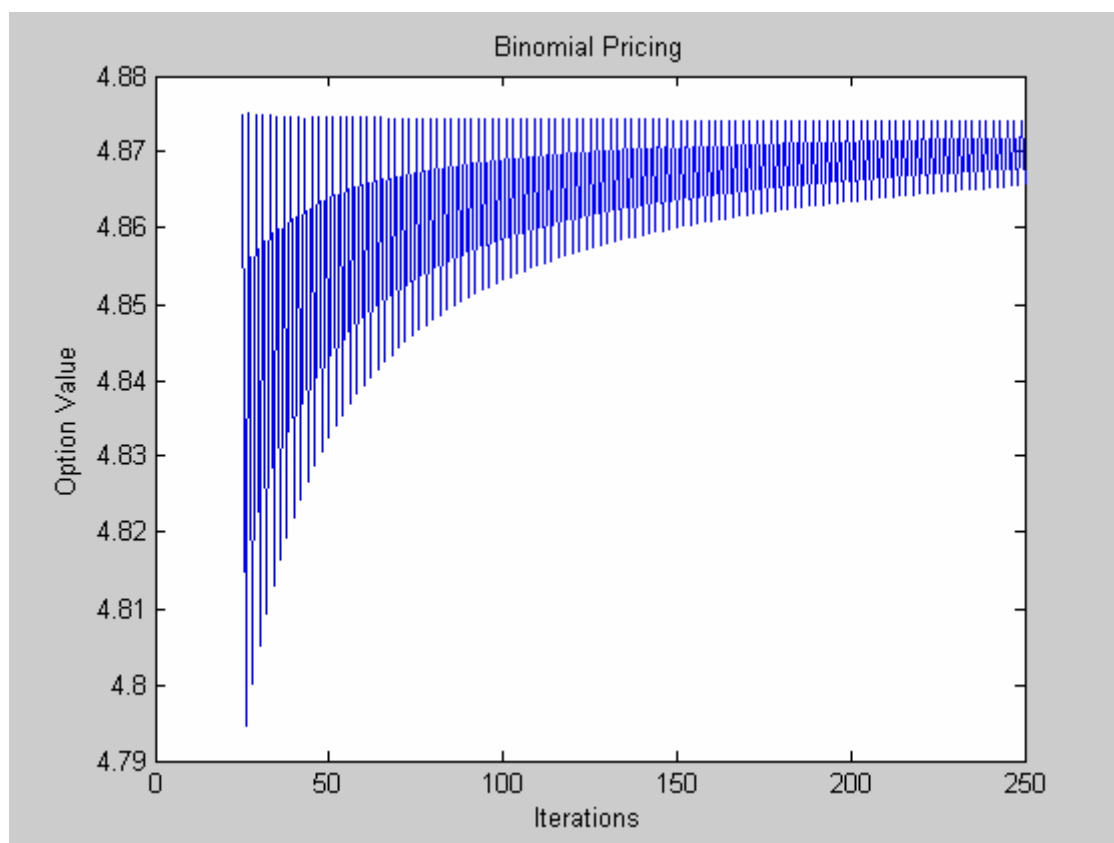


Fig. 6 American put option price values against the number of time iterations.

3.3 “Out-of-the-money” American put option pricing with 3 diverse binomial models

3.3.1 CRR Model

```

% File: crr3.m
% Objective: Binomial pricing of "out-of-the-money" american
put option using CRR binomial model
% Author: Group 4
So = 40;
K = 35;
r = 0.08;
T = 5/12;
sigma = 0.30;
N = 25:250;
price = zeros(size(N));
for k = 1:length(N);
    delta = T/N(k);
    [StockPrice, OptionPrice] = binprice(So, K, r, T, delta,
sigma, 0);
    price(k) = OptionPrice(1,1);
end
plot(N, price);
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');

```

This M-file code called by Matlab returns the following graphical output:

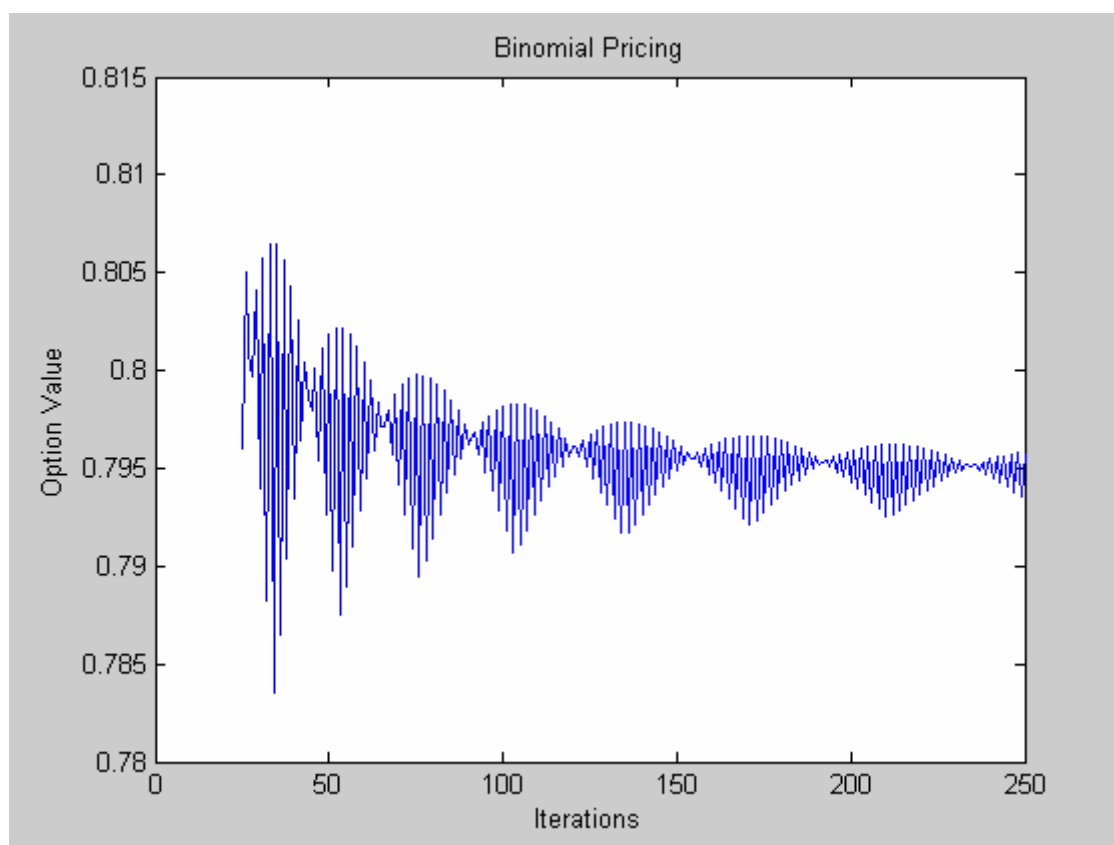


Fig. 7 A graph of American put option price values against the number of time steps.

3.3.2 TIAN Model

In consistency with the TIAN binomial model we create the M-file code as below:

```
% File: TIAN3.m
% Objective: Binomial pricing of "out-of-the-money" american
put option using TIAN binomial model
% Author: Group 4
S0 = 40;
K = 35;
T= 5/12;
r= 0.08;
sigma = 0.30;
price=1:250;
for n=25:250
    deltat=T/n;
    M=exp(r*deltat);
    V=exp(sigma^2*deltat);
    s=sqrt(V^2+2*V-3);
    u=M*V*(V+1+s)/2;
    d=M*V*(V+1-s)/2;
    p=(M-d)/(u-d);
    beta=1/M;
    V=pric(S0,K,u,d,p,beta,n);
    price(n)=V(1,1);
end
plot(25:250,price(25:250));
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');
```

And the graphical result produced looks as follows

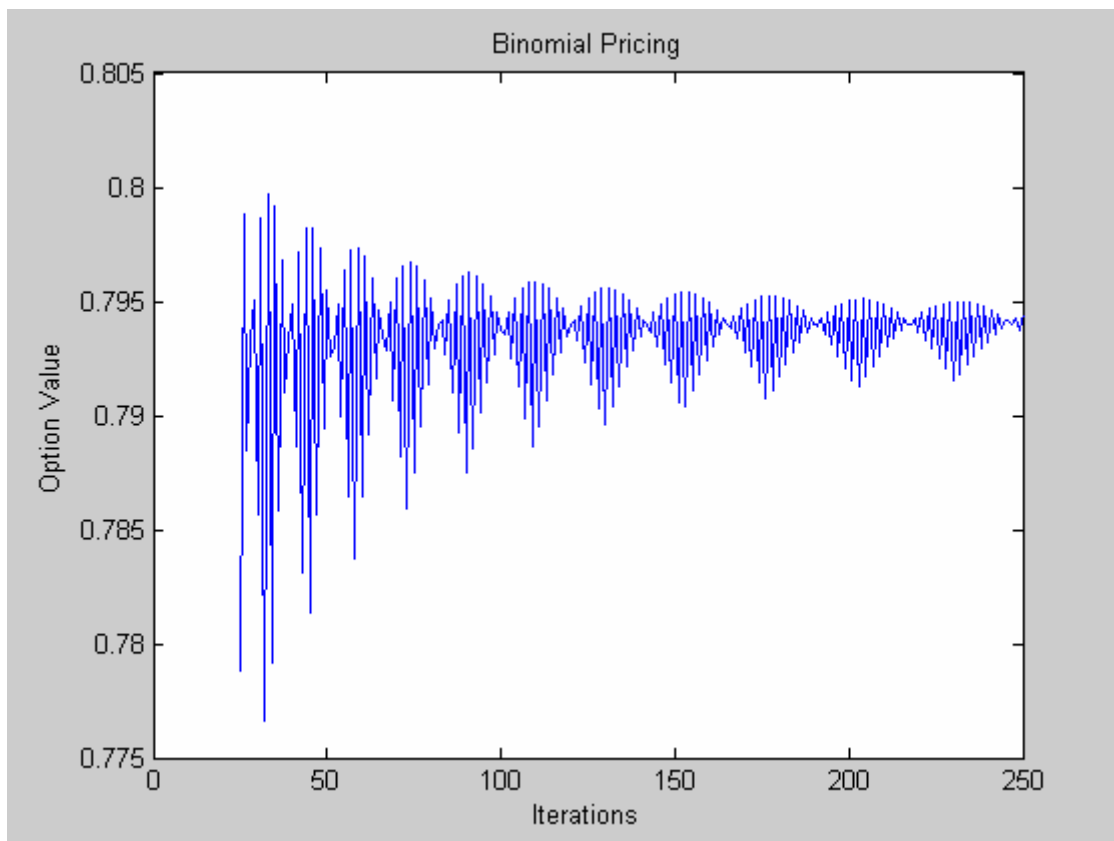


Fig. 8 American put option price values with respect to the number of time steps.

3.3.3 Leisen Reimer Model

Finally, by applying the Leisen Reimer model for binomial pricing of the American put option, we have written the following code:

```
% File: LR3.m
% Objective: Binomial pricing of "out-of-the-money" american
put option using Leisen Reimer binomial model
% Author: Group 4
So = 40;
K = 35;
r = 0.08;
T = 5/12;
sigma = 0.30;
price=1:250;
for n=25:250
    deltat=T/n;
    a = exp(r*deltat);
    beta = 1/a;
    d1 =(log(S0/K)+(r+1/2*sigma^2)*(T))/(sigma*sqrt(T));
    d2 = d1-sigma*sqrt(T);
    p1 = 1/2 +(1/4-0.25*exp(-(d2/(n+1/3))^2*(n+1/6)))^0.5;
    p2 = 1/2 -(1/4-(1/4)*exp(-(d2/(n+1/3))^2*(n+1/6)))^(1/2);
    q1 = 1/2 +(1/4-(1/4)*exp(-(d1/(n+1/3))^2*(n+1/6)))^(1/2);
    q2 = 1/2 +(1/4-(1/4)*exp(-(d1/(n+1/3))^2*(n+1/6)))^(1/2);
    if 0 < p1 < 1,
        p = p1;
    else p = p2;
    end
    if 0 < q1 < 1,
        q = q1;
    else q = q2;
    end
    u = a*q/p
    d = a*(1-q)/(1-p)
    V=pric(S0,K,u,d,p,beta,n);
    price(n)=V(1,1);
end
plot(25:250,price(25:250));
xlabel('Iterations');
ylabel('Option Value');
title('Binomial Pricing');
```

The M-file code above generates a window with the following graphical plot:

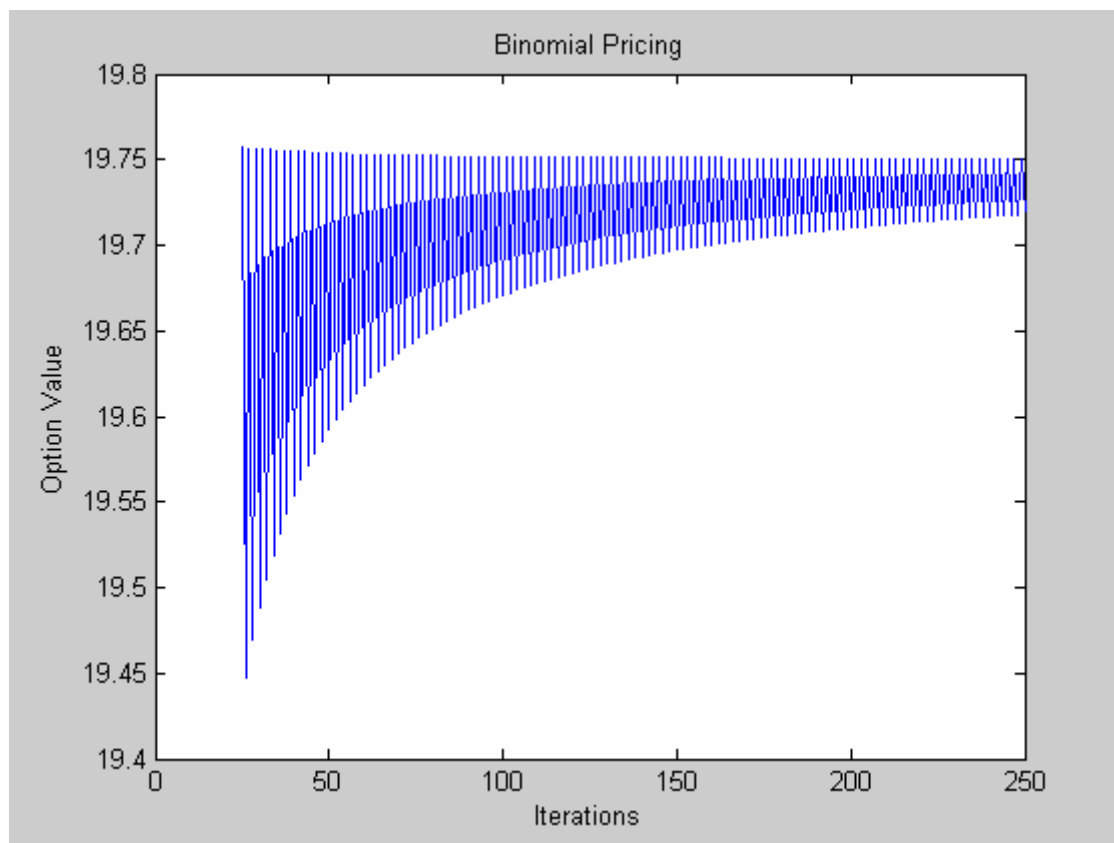


Fig. 9 American put option price values against the number of time steps.

Part Four: Conclusion

The graphical results under each of the models and for each of the cases - in-the money, at-the-money, and out-of-the-money - allow to compare how the option value changes over certain time. We observed it changes differently for each of the models giving diverse option value approximation. Under the CCR, the most popular model, the option value shows more fluctuations at the start. As the number of time steps increases, the amplitude of oscillations reduces and as iterations go to infinity, the option value converges to a certain limit.

Under the last model – Leisen Reimer model – option value oscillates significantly less, which gives more compact graph and clearly a smoother approximation.