Closing out DVA

The choice of a close-out convention applicable on the default of a derivatives counterparty can have a significant effect on the credit and debit valuation adjustments, as can the order of defaults. ***Jon Gregory*** *and* ***Ilya German*** *examine this phenomenon in detail*

Financial institutions often consider their own default in the valuation of liabilities, including a so-called debit valuation adjustment (DVA) opposite the credit valuation adjustment (CVA) accounting for the counterparty's default. DVA is a double-edged sword. On the one hand, it creates a symmetric world where counterparties can readily agree on pricing. On the other hand, its nature creates some potentially unpleasant effects, such as institutions booking profits arising from their own declining credit quality. The controversy over DVA can be seen when comparing accountancy standards and capital rules. While accounting rules such as IFRS 13 and FASB 157 require DVA, the Basel III framework does not allow any DVA relief in capital calculations (Basel Committee on Banking Supervision, 2011).

The debate over DVA use centres on whether or not institutions can monetise their own default. Ways that institutions attempt to do this include selling credit default swap (CDS) protection on highly correlated counterparties, buying back own debt and unwinding trades (see, for example, Gregory, 2009, and Burgard & Kjaer, 2011). While not completely impossible, such techniques are often seen as dubious and only leading to unintended consequences such as the creation of systemic risk. Another possible way to realise DVA is when closing out trades in the event of the default of the counterparty. In such a case, DVA can be incorporated into the so-called risky close-out amount, as opposed to the risk-free close-out, which ignores the adjustments. However, any realised DVA gain would immediately be paid out in a CVA charge on any replacement trade.

An additional theoretical complexity brought about by the use of bilateral CVA (BCVA) is that it implies that the CVA depends on the credit quality of the institution in question alone. This is because the probability of default of the counterparty must be weighted by the probability that the institution has not previously defaulted. This captures the first-to-default nature of a contract and avoids double counting. However, it also means that even a pure asset, such as a bond, appears to bear the credit risk of both parties, which is counter-intuitive. However, Brigo & Morini (2011) have shown that in such a case, the dependence on own default risk disappears if a risky close-out is assumed. This article aims to investigate the more general case.

Bilateral CVA

Extending the classic CVA formula bilaterally leads to the following representation (see, for example, Gregory, 2009, and Brigo, Buescu & Morini, 2011):

$$
\begin{aligned}
BCVA = CVA + DVA = &\int_0^\infty EE(t)\left[1 - F_I(t)\right]dF_C(t) \\
&+ \int_0^\infty NEE(t)\left[1 - F_C(t)\right]dF_I(t)
\end{aligned}
\tag{1}
$$

where $EE(t)$ and $NEE(t)$ represent the discounted expected exposure and negative expected exposure, respectively, and $F_C(t)$ and $F_I(t)$ are the cumulative default probabilities of the counterparty and institution respectively. This assumes that the defaults are independent, although this can be readily relaxed (see, for example, Gregory, 2009). Putting other potential objections to DVA aside, an issue with the above formula is that an institution's own default probability affects its CVA. Furthermore, the assumption of independent defaults is a strong one and some model for this dependency should surely be chosen. However, some institutions calculate both CVA and DVA unconditionally (UBCVA) according to:

$$
\begin{aligned}
UBCVA = UCVA + UDVA = &\int_0^\infty EE(t)\,dF_C(t) \\
&+ \int_0^\infty NEE(t)\,dF_I(t)
\end{aligned}
\tag{2}
$$

This may appear somewhat naive at first glance as it neglects the first-to-default aspect. However, the results of Brigo & Morini (2011) show that in a unilateral case, UCVA (or UDVA) is the correct formula in the case of a risky close-out assumption. This would tend to suggest that equation (2) is indeed the correct representation of bilateral CVA.

However, according to a recent survey by consultancy Ernst & Young (2012), banks are divided on whether to use conditional or unconditional representations (see also Carver, 2011). The survey found six banks using BCVA and seven using UBCVA. The aim of this paper is therefore to extend the Brigo and Morini unilateral case. Unfortunately, this will be far from trivial and not allow an unambiguous answer. However, we will describe assumptions that will make the UBCVA approximately, but not exactly, valid.

Close-out and DVA

In deriving formulas for CVA and DVA, a standard assumption is that, in the event of default, the close-out value of transactions will be based on risk-free valuation. This is an approximation that makes quantification more straightforward, but the actual payout is more complex and subtle. Let us consider the situation when a counterparty defaults on some derivatives contract. Suppose the position's valuation is negative, say –\$900, with a DVA component making it –\$800. A risk-free close-out would require the institution to pay \$900 and also make an immediate loss of \$100. If the DVA can be included in the close-out calculation then the institution pays only \$800 and has no jump in its profit and loss that would otherwise occur (Brigo & Morini, 2011). If instead the institution has a bilateral position with a current net positive value of \$1,000, of which \$900 is risk-free value and \$100 is

DVA, then a risk-free close-out amount is based on \$900, leading to a certain loss of \$100. On the other hand, a risky close-out allows a claim of \$1,000. Documentation tends to support this approach. For example, under the International Swaps and Derivatives Association (2009) protocol, the determination of a close-out amount "may take into account the creditworthiness of the determining party", which suggests that an institution may consider its own DVA in determining the amount to be settled.

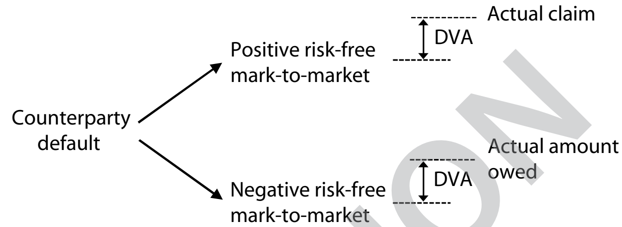
Brigo & Morini (2011) show that the inclusion of DVA in the close-out amount generally leads to a more intuitive theoretical result than a risk-free close-out. These authors illustrate the impact on a zero-coupon bond and discuss the special cases of independence and perfect correlation of default times. The zero-coupon bond alone, with its one-sided payout profile, might be quite a limiting simplification, since it naturally neglects one side of the CVA/DVA pair. There are three potential ways in which to extend such an analysis. The first of these is to consider the impact of default correlation on the results. The second is to look at the recursive nature of this effect – the close-out amount has an impact on the current CVA and DVA and vice versa. The third and very important point of interest is to calculate the impact on bilateral derivatives exposures.

To account for risky close-out in counterparty risk valuation, an institution should quantify the additional gain arising when its counterparty defaults. This comes from two components. The first is an increased claim in the event of a positive future value (of which a recovery will be achieved). The second is a gain resulting from using the DVA to offset any amount owed. The situation we assume under risky close-out is represented in figure 1. A positive value leads to a claim on the amount owed, which includes the cost of DVA that would be incurred on a replacement transaction. A negative value requires a settlement of the amount to the counterparty that is offset by the DVA.

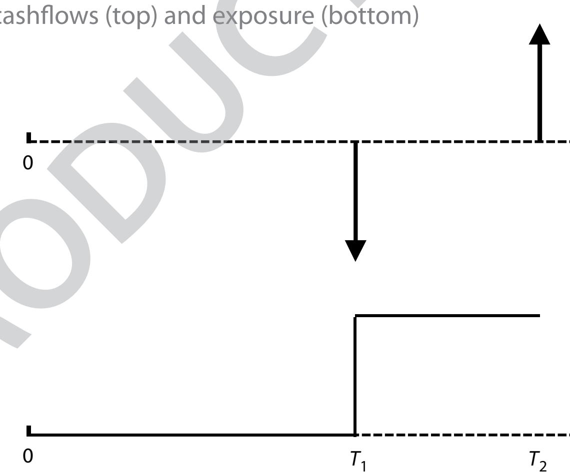
An institution also needs to consider the symmetric case that occurs when it defaults. In this case, the counterparty can increase its valuation in exactly the same way. To the institution, this increase in valuation from DVA appears as a reduction in valuation by CVA. The four resulting cases are shown in table A, based on the bilateral CVA formula in equation (8) in Brigo, Buescu & Morini (2011). Having CVA and DVA appear in its own payout is complex but seemingly unavoidable. Indeed, similar effects occur in cases such as the exercise of physically settled options where the CVA and DVA of the underlying affect the exercise boundary (see, for example, Arvanitis & Gregory, 2001).

We note that there are some potential objections to the above stylised assumptions regarding close-out amounts, which will be discussed at the end of this article. However, we will first show that under the assumptions described above and represented by figure 1, the strong first-to-default effect of bilateral CVA valuation is largely removed when assuming risky close-out. However, in contrast to previous research, we will also show that, even then, risky close-out is not a perfectly clean theoretical solution in that aspects such as default correlation are still important.

1 Illustration of the impact of DVA on the close-out amount when a counterparty defaults



2 Illustration of the simple example showing the cashflows (top) and exposure (bottom)



Simple example

A good intuition of bilateral CVA and close-out interdependence is provided by analysing a simple case of cashflows in opposite directions. The logic would be the same regardless of the sizes of those cashflows, so to simplify the exposition we assume them to be equal. Assume an institution pays a unit cashflow at time T_1 and receives a unit cashflow at a later time T_2 (see figure 2). We assume that both the institution (I) and its counterparty (C) can default – though of course the counterparty cannot be a creditor in this particular example – and have associated fixed hazard rates of h_I and h_C respectively. Percentage recovery rates are given by R_I and R_C and interest rates are assumed to be zero. The exposure based on risk-free close-out is zero until T_1 and one from T_1 to T_2. The fact that the above case represents only positive exposure is not a concern due to the inherent symmetry of the problem (although we deal with the more general case below). The aim now is to calculate the formula for the CVA.

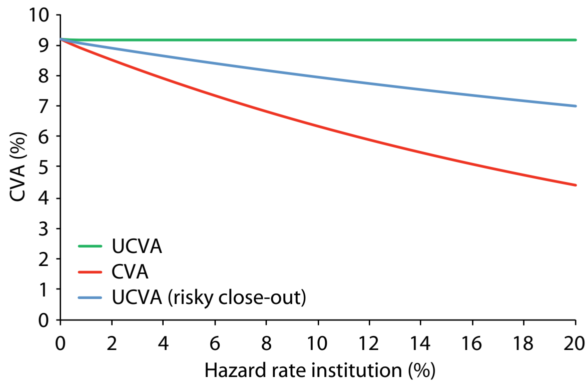
Note that for ease of exposition the representation below assumes independence of defaults but the more general case is an

A. Payouts for risk-free and risky close-out

	Risk-free close-out		Risky close-out	
	Positive exposure	Negative exposure	Positive exposure	Negative exposure
Counterparty defaults	$R_C \times Max(MtM, 0)$	$Min(MtM, 0)$	$R_C \times Max(MtM - DVA, 0)$	$Min(MtM - DVA, 0)$
Institution defaults	$Max(MtM, 0)$	$R_I \times Min(MtM, 0)$	$Max(MtM - CVA, 0)$	$R_I \times Min(MtM - CVA, 0)$

Note: R_C and R_I denote recovery value for counterparty and institution respectively. DVA negative by convention

3 CVA for simple two-cashflow example



Note: CVA for simple two-cashflow example with maturities T_1 = 2.5 and T_2 = 5 years. Calculated with both risk-free and risky close-out as a function of the hazard rate of the institution. The counterparty hazard rate is 8.33% and recovery rates are 40%

easy extension. For example, we can represent the hazard rates under conditional independence as in some factor model. We define $F(T_1, T_2)$ as the default probability between dates T_1 and T_2 and $S(T_1, T_2)$ as the associated survival probability. We denote the first-to-default probability and associated survival functions as $F^1(\cdot)$ and $S^1(\cdot)$ respectively. With a standard close-out based on the risk-free value of the claim, the CVA at time zero, which intuitively should reflect the fact that if the counterparty defaults first in the interval $[T_1, T_2]$ then the institution makes a loss due to not receiving the final cashflow, can be written as:

$$\begin{aligned} CVA(0) &= (1-R_C)\frac{h_C}{h_C+h_I}F^1(T_1,T_2) \\ &= (1-R_C)h_C\int_{T_1}^{T_2}\exp\left(-(h_C+h_I)s\right)ds \end{aligned} \quad (3)$$

The ratio $h_C/(h_C + h_I)$ gives the probability that the counterparty is the first to default. Clearly, as the institution's default probability increases, the CVA tends to zero.

Let us now look at the impact of the case of a risky close-out, including DVA, on the above calculation. If the institution defaults, the counterparty will include DVA (or CVA from the institution's point of view). We therefore have to consider two additional components corresponding to the two different time periods.

■ **Institution defaults first in the period [0, T_1].** Here, the counterparty will claim its DVA benefit (which is the institution's CVA) but will receive only a recovery fraction of it. This requires an addition term of:

$$R_I h_I\int_0^{T_1}CVA_{\tau_I=s}(s)\exp\left(-(h_C+h_I)s\right)ds \quad (4)$$

which evaluates the CVA component at the default time of the institution. Since the institution has defaulted, its hazard rate will drop to zero and the CVA will become $CVA_{\tau_I=s}(s) = (1 - R_C)[\exp(-h_C(T_1 - s)) - \exp(-h_C(T_2 - s))]$. Substituting this into the above and integrating again, we obtain:

$$R_I(1-R_C)F_C(T_1,T_2)F_I(0,T_1) \quad (5)$$

The intuition behind this is that if the institution defaults before T_1 and then the counterparty defaults in the interval $[T_1, T_2]$, then the counterparty will claim its DVA on the remaining cashflows and the institution (because it is in default) will pay only a recovery fraction of this. Another way to look at this is to consider how much it will cost the counterparty to replace the transaction in case of the institution defaulting prior to T_1. A party providing the replacement transaction will have to assess the probability of the counterparty default in the interval $[T_1, T_2]$ and will incorporate this in the price.

■ **Institution defaults first in the period [T_1, T_2].** Here, the counterparty will subtract its own DVA from the unit payment it is obliged to make. Since it owes the institution, there is no recovery value as in the previous case. This gives an additional term of:

$$h_I\int_{T_1}^{T_2}CVA_{\tau_I=s}(s)\exp\left(-(h_C+h_I)s\right)ds$$

The CVA at this point will be $CVA_{\tau_I=s}(s) = (1 - R_C)[1 - \exp(-h_C(T_2 - s))]$. Again evaluating the integral gives:

$$(1-R_C)\left[\frac{h_I}{h_C+h_I}F^1(T_1,T_2)-S_C(0,T_2)F_I(T_1,T_2)\right] \quad (6)$$

The probability in the brackets gives the probability that the institution defaults in the interval $[T_1, T_2]$ and the counterparty defaults second but before T_2. The CVA with risky close-out is found by adding the terms in equations (3), (5) and (6) above, giving:

$$UCVA-(1-R_I)(1-R_C)F_C(T_1,T_2)F_I(0,T_1) \quad (7)$$

where the unilateral CVA is given by $UCVA = (1 - R_C)F_C(T_1, T_2)$. The second term is a correction due to the fact that, in the event of the institution's own default, the counterparty may claim a recovery fraction of their DVA benefit. If $T_1 = 0$, or equivalently, when the institution has no liability, then we obtain the result of Brigo & Morini (2011) corresponding to the UCVA with no sensitivity to the institution's own hazard rate. However, in the bilateral case, neither CVA nor UCVA is the correct solution to the problem and there is an adjustment term. In figure 3, we compare the different close-out assumptions for this simple example showing CVA, UCVA and the true risky close-out result of equation (7). In this example, the actual result is somewhere between CVA and UCVA.

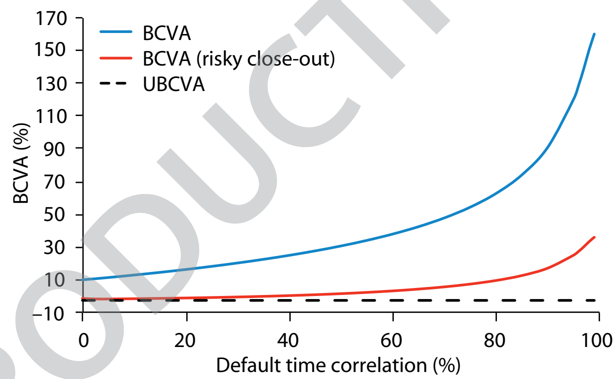
We have seen that in the general bilateral case, risky close-out assumptions do not lead to an obvious simple CVA formula as they do in the unilateral case of Brigo & Morini (2011). However, the above example was rather extreme as only one party had a DVA component. Furthermore, we have not yet considered the impact of other aspects such as default correlation. We will look at the more general case below.

Actual example

We now consider an example of bilateral exposures arising from a typical five-year \$100 million notional swap on a quarterly fixing (see figure 4). We approximate this by assuming the underlying at each date t is normally distributed with mean $-0.25 \times (T - t)\sqrt{t}$ and standard deviation $(T - t)\sqrt{t}$, which results in exposure profiles of $EE(t) = -0.25(T - t)\sqrt{t}\Phi(-0.25) + (T - t)\sqrt{t}\varphi(-0.25)$ and $NEE(t) = -0.25(T - t)\sqrt{t}\Phi(0.25) - (T - t)\sqrt{t}\varphi(0.25)$. The negative

B. CVA, DVA, BCVA and UCVA of swap portfolio assuming independent defaults

Conditional		Unconditional	
CVA	149,800	UCVA	162,407
DVA	−140,213	UDVA	−165,179
BCVA	9,587	UBCVA	−2,772



Note: bilateral CVA of interest rate swap example, as a function of the correlation between the default of the institution and its counterparty. Also shown is the approximation arising from using the unconditional BCVA (UBCVA)

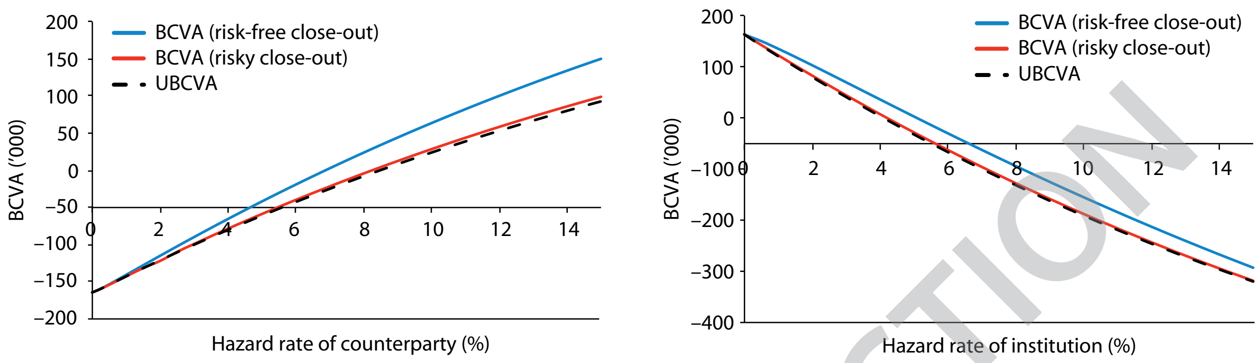
expected exposure (which drives the DVA) is greater in absolute terms than the expected exposure (which drives the CVA). Assuming a 40% recovery value for both parties and hazard rates of h_C = 8.33% and h_I = 4.17% corresponding roughly to spreads of 500 basis points and 250bp for counterparty and institution respectively. This gives that the CVA and DVA are approximately equal and opposite, and the BCVA is close to zero (see table B). We also show the UBCVA results exhibit similar behaviour but give rise to a materially different valuation.

To introduce dependence between the default times of the institution and counterparty, we use a simple and well-known Gaussian copula approach. Another aspect to consider is that the CVA (or equivalently DVA) defined at the time of close-out should naturally include the value of any future close-out adjustments (on the replacement transaction), which leads to a recursive problem. We solve this by simply recalculating the above integrals numerically and iteratively solving until a convergence is reached. More details can be found in Gregory & German (2012).

The impact of correlation on the BCVA (see figure 5) shows a strong effect, with BCVA increasing towards the unilateral value as the correlation increases to 100%. This is due to the aforementioned comonotonic feature where the most risky name is certain to default first and therefore the DVA benefit is lost. The first-to-default impact on BCVA is clearly very significant. On the other hand, the results of the BCVA with a risky close-out (including the impact of DVA and CVA and the recursive effect) show that default correlation now has a much smaller impact on the BCVA. This is due to the fact that the institution can benefit from its DVA even in the event that the counterparty defaults first.

Interestingly, in this more general case, the UBCVA approach gives close agreement with the case of risky close-out, especially for low correlation values. We test this over a wider range of situations and figure 6 shows the same quantities as a function of the hazard rate of the counterparty and institution for a fixed default time correlation of 50%. While we know from the simple result

6 Impact of risky close-out on BCVA for interest rate swap



Note: BCVA for interest rate swap as function of counterparty (left) and institution (right) hazard rates. Also shown is the approximation arising from using the unconditional BCVA (UBCVA). A fixed default time correlation of 50% is used

given in equation (7) that UBCVA will not always give the correct risky close-out valuation, in more realistic cases it appears to be in very close agreement.

Conclusion

We have examined the pricing of bilateral counterparty risk using risky close-out assumptions where a surviving party would be able to include their DVA in the amount paid or claimed from their defaulted counterparty. Risky close-out tends to cancel out some of the complicated features created by the use of DVA, in particular the strong impact of correlation between defaults. It seems unlikely that, given the complexity of CVA calculation, any institution would attempt to properly reflect risky close-out assumptions, especially since doing so requires a recursive calculation. It is therefore partially reassuring that the UBCVA formula gives a very close result to the true risky close-out case in the example considered above. Our results suggest that, in the absence of a more complex calculation, UBCVA should be used rather than BCVA. Since, as mentioned earlier, the market appears rather equally divided between these choices, this is an important conclusion.

However, unfortunately the bilateral exposure case is not as clear cut as the previous unilateral case considered by Brigo & Morini (2011). In extreme cases, UBCVA may not be a particularly good approximation to the actual case (as seen in figure 3), especially when an institution's own default probability is high. In certain cases therefore, it appears important to take into consideration the dependency between default and the recursive nature of the bilateral CVA payout.

An added problem is that the precise assumptions we have made for risky close-out could also be questioned. Indeed, risky close-out has not always been observed in practice. Consider, for example, the Peregrine Fixed Income Limited versus Robinson Department Store PLC case (see, for example, Parker & McGarry, 2009), although we note that the most recent Isda documentation supports risky close-out more than previous versions. An intuitive criticism could be the lack of recognition of the CVA of the replacement transaction, that is, the implicit assumption that the replacement counterparty is risk-free. Or one might consider a dealer market with homogeneous credit quality and symmetric exposures. Here, CVA and DVA are reduced by the use of collateral and should in any case cancel, so that the correct replacement cost (ignoring transaction costs) would simply be the risk-free amount. It remains to be seen what the implication of using other assumptions would be but it is unlikely that they will simplify the complex problem of default dependency and close-out assumptions in the pricing of bilateral counterparty risk. Furthermore, the possible inclusion of potential funding costs in close-out assumptions, not considered here, will make the problem even more complex. ■

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