Pricing FVA and Cost of Collateral

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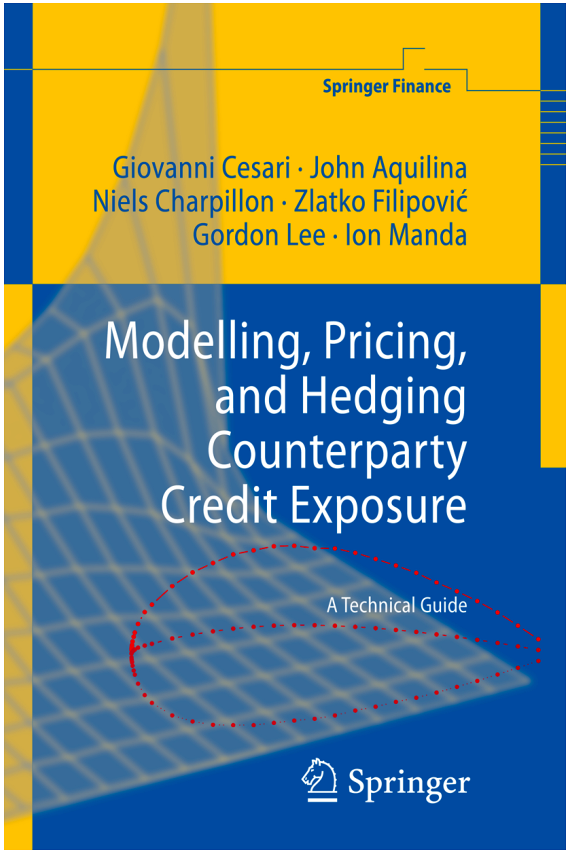
Introduction

- Valuation adjustments now play an essential role in the pricing and risk management of derivatives
- CVA, DVA, FVA, Cost of Collateral (OIS discounting), capital consumption… All of these economic factors are linked and attempt to reflect the value (both from a funding and credit risk perspective) of held/pledged collateral (or of its absence)
- Methodology-wise, all of these values can only be accurately computed in a ***portfolio*** context and require a scenario-consistent estimation of the future values of derivatives and of the corresponding collateral pool values



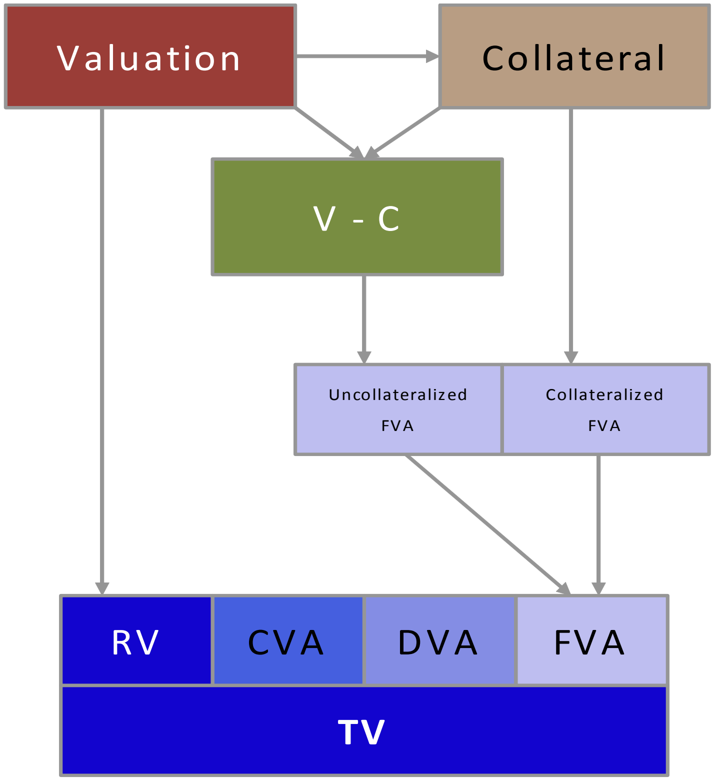
Overview

- Total Valuation Adjustment
- Methodology
- DVA/FVA: double counting?
- Fair Value vs Economic Value
- Cost of Collateral and CSA Pricing vs OIS discounting
- Portfolio valuation adjustments: single trade allocation and impact on option exercise boundaries

Total Valuation Adjustment

- RV: replacement value (e.g. collateralized value using LCH rules)
- CVA: Credit Valuation Adjustment
- DVA: Debit Valuation Adjustment
- FVA (uncollateralized funding valuation adjustment)
- CC (or collateralized FVA): Cost of Collateral
- FVA+CC = Total Funding Valuation Adjustment

Generic framework for both collateralized and uncollateralized portfolios (and anything in between)

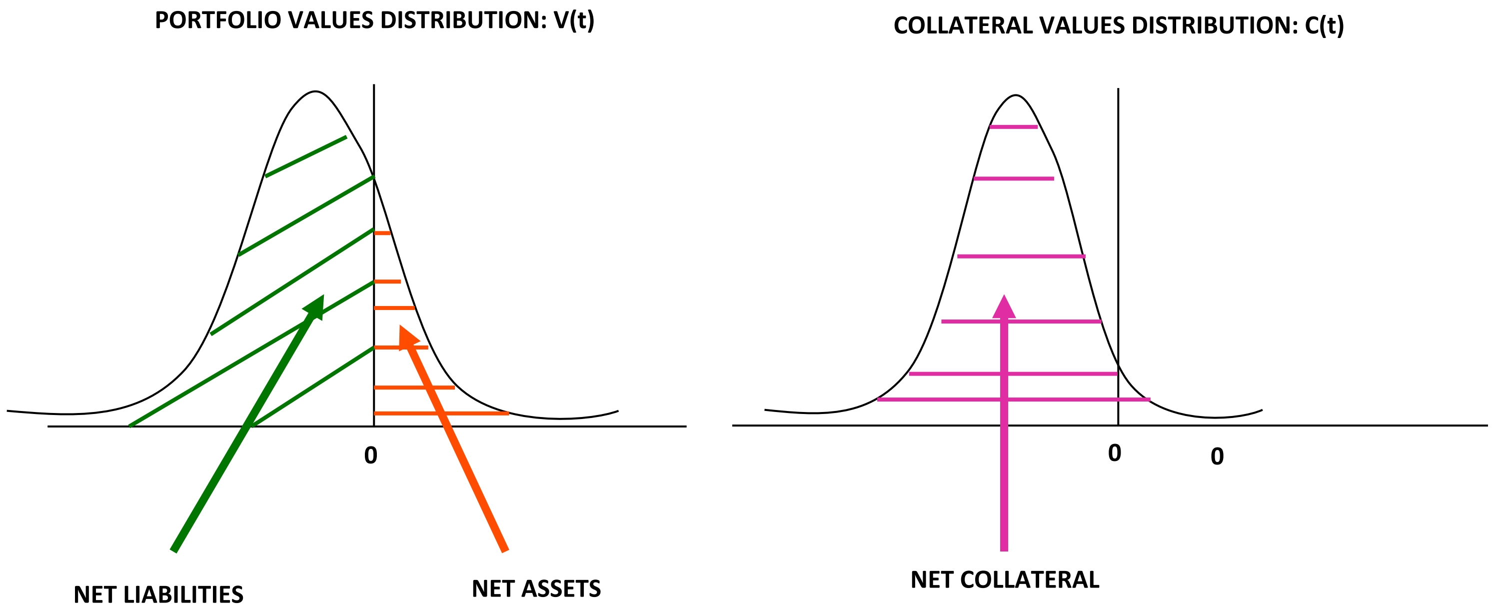


TV = RV – CVA + DVA - FVA

Total Valuation Adjustment

- CVA, DVA, FVA and CC are all valued *relative to the RV*.
- CVA
  - cost to the bank of protecting itself against the default of the counterparty
  - value is price of derivative which pays max(0, V-C) * (1-R) at default time Tc
- DVA
  - cost to the counterparty of protecting itself against default of the bank
  - value is price of a derivative which pays max(0, C-V) * (1-R') at default time Tb
- FVA
  - cost/benefit to the bank of funding the uncollateralized portion of the portfolio (may be different for assets than liabilities)
- CC
  - cost/benefit to the bank of receiving/pledging collateral (i.e. interest earned/ paid on collateral inflows/outflows)

Methodology



- Net uncollateralized assets PV: **EE(t) = N(0) * E[(V(t)-C(t))⁺ / N(t)]**
- Net uncollateralized liabilities PV: **RevEE(t) = N(0) * E[(C(t)-V(t))⁺ / N(t)]**
- Net total uncollateralized PV: **ME(t) = EE(t) – RevEE(t)**
- Net collateral PV: **MEC(t) = N(0) * E[C(t) / N(t)]**

Methodology: CVA & DVA

- CVA = "EE * counterparty default probability (CDS-implied)"

$$CVA = (1 - R_V^C)\int_0^\infty EE(u)\lambda_u^C e^{-\int_0^u \lambda_s^C ds} du$$

- DVA = "RevEE * bank default probability (CDS-implied)"

$$DVA = (1 - R_V^B)\int_0^\infty RevEE(u)\lambda_u^B e^{-\int_0^u \lambda_s^B ds} du$$

- Here, both λ^C and λ^B are usually well defined (or at least if not traded, it is clear that it is the CDS spread / default probability of each counterparty that we want)
- CVA and DVA expressed as unilateral (joint default ignored for simplicity)

Methodology: FVA

- FVA = **FVA_Assets** – **FVA_Liabilities**
- FVA_Assets = "EE * cost of funding"
  - represents is a funding cost

$$FVA_A = \int_0^\infty EE(u)\alpha_A(u)du$$

- FVA_Liabilities = "RevEE * funding benefit"
  - represents a funding benefit

$$FVA_L = \int_0^\infty RevEE(u)\alpha_L(u)du$$

Methodology: FVA

$$FVA_A = \int_0^\infty EE(u)\alpha_A(u)du \qquad FVA_L = \int_0^\infty RevEE(u)\alpha_L(u)du$$

- Depending on pricing methodology and definitions, α_A and α_L are likely to be different
- Asset side: what does it cost the bank to hold uncollateralized assets?
  - Bank senior debt, cash basis, nothing?
- Liability side: what funding benefit does the bank get from uncollateralized liabilities?
  - Bank senior debt (if no DVA)?
- While broad agreement on treatment of liabilities, 2 diverging views (accounting vs "economic"/traditional) still for treatment of assets

Methodology: FVA (Liabilities)

$$FVA_L = \int_0^\infty RevEE(u)\alpha_L(u)du$$

- In the absence of DVA, liabilities attract a funding benefit which should be close to (or equal to) the bank's senior debt
- Same treatment as own credit for deposits
- With DVA, there is a ***risk of double counting***
  - Both DVA and Own Credit reflect the bank's creditworthiness
  - Only difference is DVA is usually calculated from CDS spreads, whereas Own Credit uses Funds Transfer Pricing / Senior Debt curve
- In the presence of DVA, α_L should therefore be a function of the **cash synthetic basis** of the bank (ie bond-cds spread), so that DVA+FVA_L(DVA) = FVA_L(no DVA)
- Note that in this case, the actual calculation of α_L, while simple, is slightly cumbersome

Methodology: FVA (Assets)

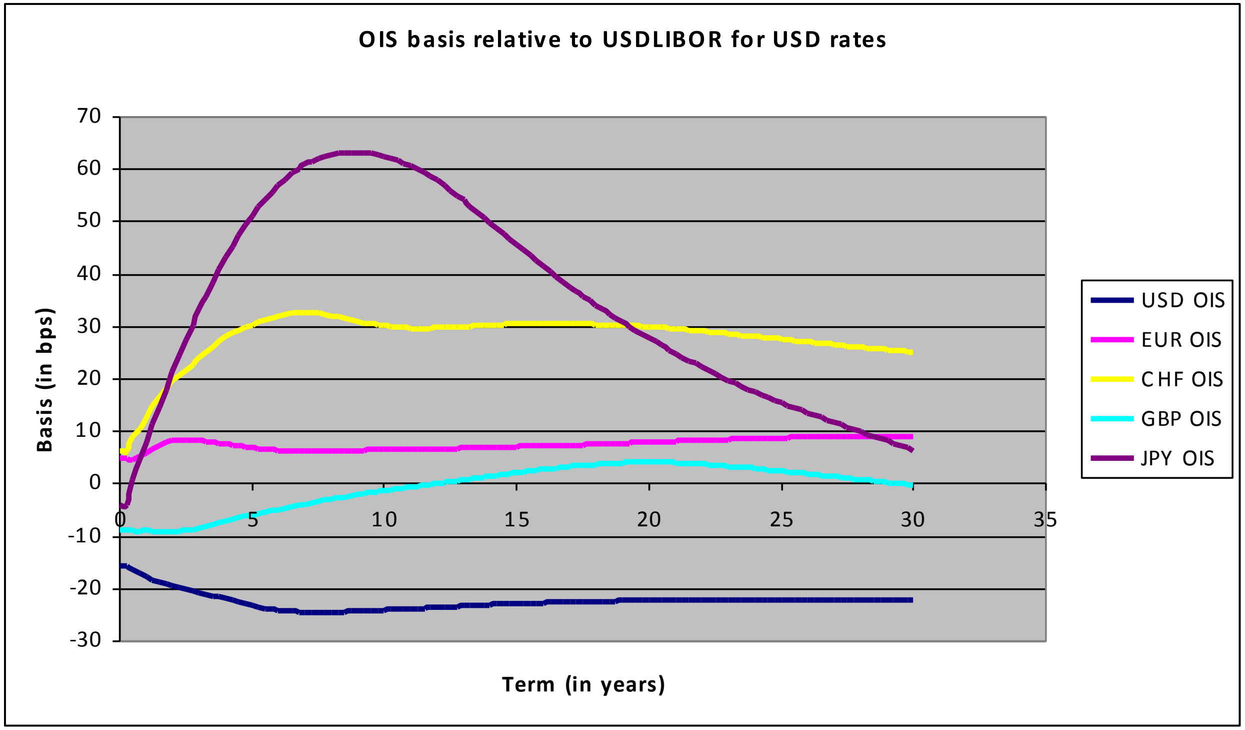
$$FVA_A = \int_0^\infty EE(u)\alpha_A(u)du$$

- The treatment of assets is more controversial
  - H&W vs the world: "*It turns out that the quants arguing about FVA were shouting so loudly at each other that they woke up John Hull and Alan White — the work of whom many a young banker is meant to absorb (mostly by osmosis). The duo spent 1,742 words in the August edition of Risk articulating an intricate argument that we feel can (very) crudely be summarised as: "FVA? Are you smoking crack?"*" (FT Alphaville 29/10/12)
  - Accounting view versus DVA/OCA time decay
- All discussions can be reduced to 2 different values for FVA_A
  - ***"Economic Value"***: α_A is related to the bank's senior debt curve (same treatment as liabilities) → <u>asymmetric pricing</u>
    - Any asset purchase results in issuing debt
    - No recognition of own credit benefit on corresponding debt issuance
  - ***"Fair Value"***: α_A is either zero or related to the cash synthetic basis of the counterparty (or an average across peers) → <u>symmetric pricing</u>

Methodology: CC

- CC = "MEC * collateral rate above RV rate"

$$CC = \int_0^\infty MEC(u)\beta(u)du$$



Methodology: CC

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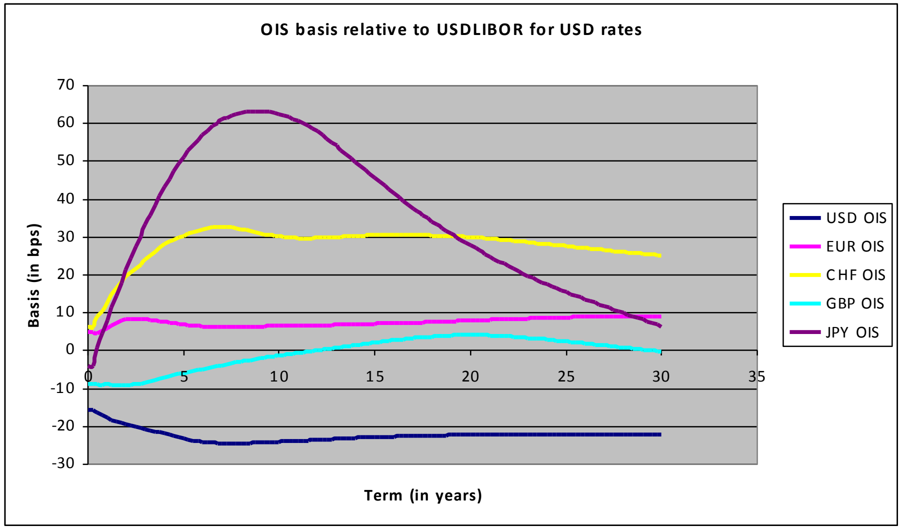


$$CC = \int_0^\infty MEC(u)\beta(u)du$$

- Basis β represents rate of remuneration of collateral above base “risk-free” rate
- Can be used to value CSA optionality (intrinsic value via *blended curves*)
- Depending on jurisdiction, may or not be possible to switch the entire collateral pool (***different substitution rights***), buy only current collateral calls → can still value this by knowing the collateral value distribution (however CC formula becomes more complicated)

CC and OIS discounting

- First industry-wide implementation of a simplified version of CC is OIS discounting
  - Assumes full collateralization (zero thresholds, daily calls, no MTA, etc…) and **full substitution rights**
  - In this case, TV (OIS discouting) = RV – CC
    - Indeed, full collateralization means EE=RevEE=0 (no CVA, no DVA, no FVA)
- Only CC methodology can account for full CSA characteristics, and in particular for ***asymmetric CSAs***
- Main advantage of using CC methodology (via MEC profile) instead of OIS discounting in the case of full collateralization is *computational speed*
  - Once MEC profile is obtained for a given portfolio, CC can be valued on any OIS (or blended) curve at virtually no computational cost → OIS discounting would require revaluing the whole portfolio
  - Only shortfall of MEC shortcut is for the special case of callable trades, where accuracy may not be as good as OIS discounting (frozen exercise boundary)

Optimal Exercise Revisited

- Usually option exercise is *decided at trade level*, and optimal exercise for European and Bermudan / American options **ignores the impact of exercise on the portfolio as a whole** (impact on CVA/FVA/Capital)
  - Can be very wrong!
  - Example 1: uncollateralized physical settled swaption which if exercised would offset a swap we have with the counterparty
  - Example 2: same as above, collateralized – exercise would reduce close out risk → reduction in Basel III capital (lower close out risk)
- Exercise strategy implicit in traditional pricing, can lead to wrong price/ hedge
  - European physical settled swaption payoff: $P = A*\max(0, S-K)$
  - Rewrite $P = A*(S-K) * (S>K)$: the exercise strategy is to exercise if $S>K$

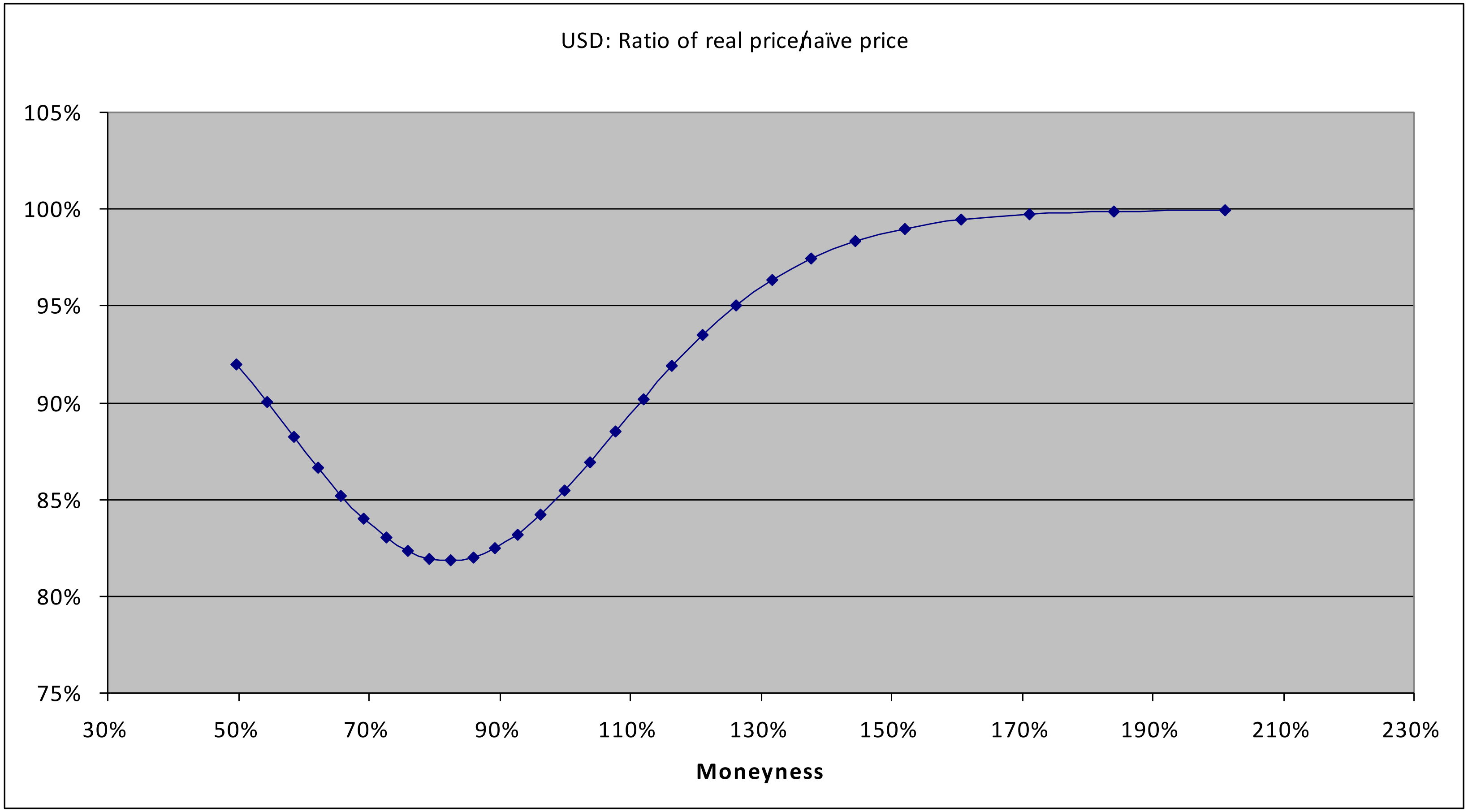
Optimal Exercise Revisited

- Real optimal strategy requires knowledge of impact of exercise on portfolio
  - However, allocation of total CVA/FVA/Capital etc to individual trades is very challenging
  - Portfolio risks are not the sum of individual risks!
- "Corrected" payoff (for European physical swaption) would be P = A*(S-K) * (S-K-PC >0), where PC is the portfolio cost associated with exercise
  - Note that P' < P, ***whatever the sign of PC!***
  - **Ignoring portfolio impact overprices options**

Optimal Exercise Revisited

- European swaption, with maturity t and underlying tenor T, paying fixed
- Payoff at maturity is DV01(t,T) * (s(t,T) – c) * (exercise)
  - Indicator of exercise is determined by swapvalue > portfolio costs given exercise
  - Translates into exercise = s(t,T) > c + PC(t,T) / DV01(t,T)
- Example: 100mm USD 1y into 20y (coupon 2.5%), physically settled and uncollateralized with a new counterparty whose CDS trades at 500 bps
  - Naïve swaption value is roughly 9m USD (underlying swap value around 6.5m USD).
  - Naïve (risk-neutral) exercise probability is about 70%.
  - However, CVA for this swaption using simplified exercise decision is 4mUSD, corresponding roughly to a 25 bps change
  - A better approximation of the exercise decision (assuming vol / cds spreads do not change…) would be that the par rate should be greater than 250+25 = 275 bps.
  - This gives an exercise probability of 55%, ie **20% less likely to exercise once CVA is taken into account**

Optimal Exercise Revisited



Optimal Exercise Revisited

- In practice, finding the right price for the option (i.e. including portfolio effects) is extremely challenging
  - PC is stochastic and portfolio contingent
  - All exercise decisions on a common date should be made simultaneously
  - High dimensional recursive problem
- However, relatively simple to take operational steps to avoid uneconomical exercise decisions
  - Attempt simple allocation of portfolio costs, or do a more accurate estimation of the marginal impact of removing the trade
  - Adjust exercise probabilities (and deltas) by taking static current portfolio charge allocated to the trade

Thank you for your attention!

Q&A