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Stock Price Movements in Response to Stock Issues under Asymmetric Information

WILLIAM S. KRASKER*

ABSTRACT

This paper characterizes the function relating the number of new shares issued by a firm to the resulting change in the firm's stock price, when insiders are asymmetrically informed. We show that, in equilibrium, the stock price will be a decreasing function of the issue size; moreover, the rate of decrease can be so rapid to cause "equity rationing." We also show that there will be underinvestment relative to the symmetric information case.

RECENT EMPIRICAL WORK HAS shown that the announcement of a stock issue is associated with a drop in the corresponding share price[1], and Myers and Majluf [9] have explained why one would expect this result under asymmetric information. If management is acting in the interests of the current shareholders, it will be reluctant to issue new stock when it knows the value of the firm's existing assets is high. A stock issue, therefore, signals to the market that the firm's current assets are overvalued and drives down the share price. In this paper, we generalize the Myers-Majluf model by eliminating the assumption that the firm has a single all-or-nothing investment opportunity whose cash requirements are fixed and known by all investors, and by allowing the firm to choose not merely whether to issue stock, but also how much stock to issue[2]. This generalization allows us to analyze questions about the relationship between the stock price and the issue size, which do not arise in the Myers-Majluf model.

Our principal results are, first, that the stock price following the announcement

* Associate Professor, Graduate School of Business Administration, Harvard University. The author wishes to thank Paul Asquith and a referee for numerous suggestions that improved the exposition.

[1] See, e.g., Masulis and Korwar [7], Asquith and Mullins [1], Hess and Bhagat [3], and Korwar [5]. Dann and Mikkelson [2] have found the same effect for convertible debt.

[2] The size of the stock issue is also treated as a continuous choice variable by Leland and Pyle [6], who examine the financing decision faced by an entrepreneur with superior information about the quality of his own project. Because the entrepreneur is risk-averse and would prefer, other things being equal, to diversify his portfolio completely, the fraction of his firm's equity that he *retains* serves as a signal to the market. The most important difference between their model and ours lies in the nature of the "cost" of false signalling. In Leland and Pyle's paper, to signal falsely, the entrepreneur must increase her exposure to the risk of her own firm. In our paper, specific risk is of no consequence. Management eschews false signalling simply because it is the value of the existing shares conditional on *their* information, rather than the lesser information available to the market, that management wants to maximize. To signal falsely by issuing too few shares would merely deprive the firm of funds that could be used to increase the wealth (relative to the larger information set) of the old stockholders. Other related work is discussed by Myers and Majluf [9, p. 196] and Masulis and Korwar [7].

of a stock issue should be inversely related to the issue size. This prediction is consistent with the empirical findings of Masulis and Korwar [7]. Second, the rate of decrease in the stock price as the issue size increases can be so rapid that the product of the two—the total proceeds of the issue—is bounded. Under these conditions—called "equity rationing"—there is an upper limit to the amount of money that the firm can raise by a stock issue, irrespective of how many shares management issues. Intuition suggests that equity rationing is most likely to occur when the firm's investment prospects are poor, but paradoxically, the opposite is true. We will show that equity rationing *must* prevail if the firm's investment opportunities are known to be "sufficiently good" in a sense we will make precise. Finally, we show that there will be underinvestment in an asymmetric-information equilibrium, relative to the case of complete information.[3] If the information asymmetry is restricted to the value of the firm's assets in place, then the greater is investors' uncertainty about the value of those assets, the smaller will be the expected underinvestment, and the higher will be the stock price prior to the issue announcement. This prediction is also testable using stock prices and other characteristics of firms.

Our analysis is based on a two-period model of a firm that has both assets in place and investment opportunities. The investment requirements of the new opportunities, the potential cashflows resulting from those investments, and the value of the existing assets are all uncertain in the first period, although management has superior information regarding those uncertainties. Management must decide in the first period how much (if any) new stock to issue to finance new investments. In making this decision, they must consider the amount of money that would be raised from a stock issue of any given number of shares. Specifically, in an equilibrium there will be a function, called the "proceeds function," that relates the number of new shares issued to the total amount of money raised by that issue. Management uses this function to determine the issue size that is best for the existing shareholders. (In particular, management has the option of issuing more stock than is necessary for the investments they intend to undertake and paying out the excess proceeds as dividends.) In equilibrium, the market *knows* that management is optimizing relative to the proceeds function, but is nevertheless willing to pay the amount indicated by that function in exchange for management's chosen number of shares.

The model presented in the next section ignores the possibility of raising funds through a debt issue, in addition to or in place of new equity. Actually, debt would complicate the theory only if the firm could issue debt and equity simultaneously. In that case, an equilibrium would have to be described by a more complicated mapping, which maps the number of shares in the stock issue and the promised payments on the debt into the proceeds from the stock issue and the size of the loan. It would be difficult to derive properties of the equilibrium in this more general framework, although conceptually there are no new problems. If, on the other hand, debt and equity issues must be sequential, then the theory described here still applies: our model postulates only that at the moment new equity is issued, investors know how to price that stock. In doing so, investors

[3] Underinvestment also occurs in the model of Miller and Rock [8].

would certainly take into account all the firm's possible future actions (including real investment and securities issues), and also the fact that the manager found it optimal to issue equity rather than raise funds by some other means (including selling assets) or not at all.

In Section I, we will describe our model formally, define an equilibrium proceeds function, and work through an example in which the equilibrium proceeds function is characterized as the solution to a first-order differential equation. In Section II, we establish our results on equity rationing (boundedness of the proceeds function). Section III examines the efficiency of aggregate real investment in an asymmetric-information equilibrium, both in the general case and in the particular case in which the information asymmetry is restricted to the value of the assets in place. The existence of interesting equilibria in the latter case contrasts with the situation in Myers and Majluf [9], in which the authors conclude (p. 203) that you need asymmetric information about *both* assets in place and investment opportunities to get interesting solutions. The results of Section III draw on properties of "completely revealing" equilibria, which are of interest in themselves and are assembled in the Appendix.

I. Proceeds Functions and Equilibria

We will study a two-period model in whose first period the firm raises new equity and invests. The uncertainty about the value of the firm's investment opportunities and existing assets is represented by an n-dimensional random variable, θ, which the manager observes in the first period, but which investors observe only in period 2. Let $V(P, \theta)$ denote the value of the firm in period 2 if management raises an amount, P, in period 1, and uses those proceeds optimally.[4] Since neither undiversifiable risk nor the time value of money plays any role in our results, we will assume that the market value of the firm equals its expected value, conditional on the market's information.

Assume that one perfectly divisible share is outstanding prior to a new stock issue, and suppose s additional shares are issued. The total proceeds from this issue will be denoted by $p(s)$ [so that the stock price is $p(s)/s$]. The "proceeds function" $p(\cdot)$, which describes the amount of money that will be raised by a stock issue of any given size, would become common knowledge over time as a consequence of the issue experience of many firms. [Strictly speaking, the proceeds function is of the form $p(s; \psi)$, where ψ represents the other relevant observable variables that investors use to distinguish among firms. These variables would have to be explicitly incorporated into empirical tests of the model.]

Faced with the proceeds function $p(\cdot)$, and having observed θ, the manager will choose s to maximize the value of the old shares, $[1/(s + 1)]V(p(s), \theta)$. The first-order condition is

$$V(p(s), \theta) = (s + 1)V_1(p(s), \theta)p'(s), \tag{1}$$

[4] Most of our results hold even if θ is infinite dimensional. Moreover, it is not necessary to assume that the manager has complete information. All of our results hold if θ represents not the entire uncertainty, but only the information asymmetry. We need only reinterpret $V(P, \theta)$ to be not the actual period-two value of the firm, but rather the *expected* value conditional on θ.

where the subscript 1 denotes the partial derivative with respect to the first argument. Equation (1) implicitly determines s as a function of θ, given $p(\cdot)$. To emphasize this dependence, we will sometimes denote the issue size by $s(\theta)$.

Let $J(s, p)$ denote the set of all θ satisfying (1). This is the set of possible values for θ, given that the manager found it optimal to issue s shares when faced with the proceeds function $p(\cdot)$, and so represents the market's information. In an equilibrium, the proceeds function must, therefore, satisfy

$$p(s) = \frac{s}{s+1} E[V(p(s), \theta) \mid J(s, p)]. \tag{2}$$

The left side is the total amount paid by investors for the s shares, while the right side is the value of those shares conditional on all the information available to them.[5] Equation (2) must hold for every issue size that might be chosen by management; i.e., for every s that is the solution to the first-order condition (1) for some θ. However, in an equilibrium there may be issue sizes that could never be chosen, and for those, the set $J(s, p)$ is empty, and the equilibrium condition (2) does not determine $p(s)$ uniquely. Therefore, when we speak of an equilibrium proceeds function, we really mean an equilibrium class of equivalent proceeds functions, whereby two proceeds functions are equivalent if they have identical consequences for every θ. We will exploit this distinction in the next section.

Using (1), we can substitute for V in (2) and rearrange terms to obtain an equivalent equilibrium condition:

$$p'(s) = \frac{p(s)}{sE[V_1(p(s), \theta) \mid J(s, p)]}. \tag{3}$$

This suggests that an equilibrium proceeds function will be the solution to a first-order differential equation. However, (3) does *not* express $p'(s)$ explicitly as a function of $p(s)$ and s, since the conditional expectation in the denominator will depend on $p'(s)$ except in special cases.

An extra dollar of proceeds must raise the value of the firm by at least one dollar, because the manager always has the option of retaining that dollar as cash. Formally, $V_1(P, \theta) \geq 1$ for all P and θ. Equation (3), therefore, implies that $p'(s) \leq p(s)/s$, which is equivalent to $d/ds[p(s)/s] \leq 0$, which in turn has the important implication that in equilibrium, the share price $p(s)/s$ is a nonincreasing function of the issue size. [Intuitively, if $p'(s)$ *exceeded* $p(s)/s$, investors could use (1) and $V_1 \geq 1$ to conclude that $(s/(s+1))\ V(p(s), \theta) > p(s)$, so that the price of the shares would be bid up.] This prediction can be tested using data on public firms that issue new stock, by regressing the announcement-period excess return on the relative issue size and other variables that investors use to distinguish among firms (the variables ψ mentioned earlier). Masulis and Korwar [7, section 4] performed an analysis of this form and found that, for industrial

[5] In the trivial case in which there is symmetric information and the firm has only zero-NPV opportunities, so that $V(P, \theta) \equiv V_0 + P$ where V_0 is the value of the existing assets, it is easy to check that $p(s) = V_0 s$ is the equilibrium proceeds function. Thus, the firm can raise as much money as it wants to at price $p(s)/s = V_0$.

firms, the coefficient of the issue size is negative, as our model predicts, although not statistically significant.[6]

It may be helpful at this stage to work through an example, in which θ is two-dimensional. Let $\theta = (\delta, \gamma)$ where δ and γ are non-negative random variables observed by the manager but not by the market, and assume

$$V(P, \delta, \gamma) = \delta P + \gamma. \tag{4}$$

Let

$$u = \log \delta$$

$$v = \log \gamma$$

and suppose that u and v are distributed bivariate normal. The additive component γ can be interpreted as the value of the assets in place, while δ scales the value of extra investment dollars.[7] We will show that the equilibrium condition (3) reduces to a first-order differential equation.

Given the firm's proceeds function, $p(\bullet)$, and having observed δ and γ, the manager chooses s to satisfy the firm's first-order condition

$$\delta p(s) + \gamma = (s + 1)\delta p'(s). \tag{5}$$

This first-order condition can be rewritten succinctly as

$$\gamma/\delta = c(s), \tag{6}$$

where

$$c(s) = (s + 1)p'(s) - p(s). \tag{7}$$

Equation (6) shows that in an equilibrium, the issue size, s, reveals to the market neither γ or δ, but only their ration γ/δ.

Since $V_1 = \delta = e^u$, and the market's only information is that γ and δ satisfy (6), the conditional expectation in the denominator of (3) can be rewritten as $E[e^u \mid \gamma/\delta = c(s)]$. Let $w = v - u$; then (u, v, w) are jointly normal, and the first-order condition (6) is equivalent to $w = \log c(s)$. The regression of u on w has slope $\sigma_{uw}/\sigma_w^2 \equiv \beta$ and intercept $\bar{u} - \beta\bar{w}$, so that $E(u \mid w = \log c(s)) = \bar{u} + \beta(\log c(s) - \bar{w})$. Moreover, the conditional variance of u given w is $\sigma_u^2(1 - \rho^2)$, where ρ is the correlation between u and w. Finally, since $Ee^u = \exp\{\mu + \frac{1}{2}\sigma^2\}$ if $u \sim$

[6] Instead of including as a regressor the change in management's percentage ownership, Masulis and Korwar include only a dummy variable indicating whether the issue was accompanied by an insider secondary offering. This makes it harder to determine whether the negative sign of the issue-size coefficient derives from the mechanism described in this paper, the agency theory of Jensen and Meckling [4], or even the signalling model of Leland and Pyle [6]. In related work, Asquith and Mullins [1] also find that the announcement-period excess return is negatively correlated with the issue size, whereas Hess and Bhagat [3] find a positive but statistically insignificant correlation. However, Hess and Bhagat include in the regression none of the additional variables that investors use to distinguish among firms, and Asquith and Mullins include only one.

[7] Strictly speaking, δ cannot be lognormal because $\delta = V_1(P, \delta, \gamma)$ can never be less than one. This problem can be mitigated by assuming that $\bar{u}$ is positive and σ_u is small. We have been unable to find an analytically tractable two-dimensional example that satisfies $V_1(P, \theta) \geq 1$ for all P and θ.

$N(\mu, \sigma^2)$, we conclude that

$$E[e^u \mid w = \log c(s)] = \exp\{\bar{u} - \beta\bar{w} + \tfrac{1}{2}\sigma_u^2(1 - \rho^2)\}c(s)^\beta. \tag{8}$$

Using the definition of $c(s)$ and (8), we can obtain a special case of (3):

$$p'(s) = \frac{p(s)}{s \exp\{\bar{u} - \beta\bar{w} + \tfrac{1}{2}\sigma_u^2(1 - \rho^2)\}[(s + 1)p'(s) - p(s)]^\beta}. \tag{9}$$

Equation (9) is (implicitly) a first-order differential equation which the equilibrium proceeds function must satisfy. One can express $p'(s)$ explicitly as a function of $p(s)$ and s (and the parameters) if β equals -2, -1, $-\tfrac{1}{2}$, 0, or 1. For example, suppose $\beta = 1$, and let A denote the exponential factor in (9). Then, (9) becomes a quadratic equation $(s + 1)p'(s)^2 - p(s)p'(s) - p(s)/sA = 0$ with solution

$$p'(s) = \frac{p(s) \pm [p(s)^2 + 4(s + 1)p(s)/sA]^{1/2}}{2(s + 1)}, \tag{10}$$

which could be integrated numerically. If $\beta = 0$ (so that u is uncorrelated with $w = v - u$), then Equation (9) becomes the much simpler equation $p'(s) = p(s)/sA$, which will arise again in the context of completely revealing equilibria in Section III and in the Appendix. However, we should emphasize that the equilibrium of the model described here, with proceeds function (9), is not fully revealing. The issue size s reveals to the market only the ratio of the random variables, γ/δ, and not their levels.

This example illustrates how the proceeds function that is observed in the market represents a rational-expectations equilibrium. Investors understand that management is optimizing the old shareholders' wealth. This induces a conditional distribution for $\theta = (\gamma, \delta)$ given s. With respect to this distribution, $p(s)$ is the correct expected value for the s shares.

II. Equity Rationing

Equity rationing is the situation in which the equilibrium proceeds function [or an equivalent version, if the equilibrium does not determine $p(s)$ for every s] is bounded. In this section, we will develop an alternative characterization of equity rationing and establish conditions under which equity rationing must exist.

If there is an upper bound to the number of shares that the manager could optimally choose to issue in equilibrium, say $s(\theta) \le \bar{s}$ for all θ, then we can redefine $p(s)$ for $s > \bar{s}$ so as to make $p(\cdot)$ bounded, without affecting its equilibrium character. Therefore, if $s(\theta)$ is bounded in θ, the equilibrium proceeds function has a bounded equivalent version. A partial converse of this proposition also holds, provided $V(P, \theta)$ is concave in P for each θ ("nonincreasing returns"): if *every* s is optimal for some θ, then the equilibrium proceeds function must be unbounded. To see this, note that concavity implies

$$V(P, \theta) \ge V_1(P, \theta)P \quad \text{for all} \quad P, \theta. \tag{11}$$

Combined with the first-order condition (1), (11) shows that

$$(s + 1)p'(s) \geq p(s) \tag{12}$$

for every s. This implies that, for all s,

$$\begin{aligned} p(s) &= \int_0^s p'(t)\,dt \geq \int_0^s \frac{p(t)}{t+1}\,dt \\ &= \int_0^s \frac{p(1)}{t+1}\,dt - \int_0^s \frac{p(1)-p(t)}{t+1}\,dt \\ &\geq p(1)\log(s+1) - \int_0^1 \frac{p(1)-p(t)}{t+1}\,dt, \end{aligned} \tag{13}$$

so that $p(\cdot)$ is unbounded in s. In short, under nonincreasing returns, equity rationing is essentially equivalent to the existence of an upper bound to the number of shares that the manager could optimally choose to issue.

Intuition suggests that equity rationing is most likely to occur when the firm's investment opportunities are perceived by the market to be poor. However, this intuition does not take into account the implications of the equilibrium condition (3). Under nonincreasing returns, it turns out that equity rationing occurs if the firm's prospects are known to be "sufficiently good." We will begin with a simple result and then show how to extend it.

PROPOSITION 1. *Suppose V is concave in P for each θ. If there exists $\varepsilon > 0$ such that $V_1(P, \theta) \geq 1 + \varepsilon$ for all P and θ, then there will be equity rationing.*

Proof: Since V is concave in P, (12) must hold if the manager chooses s. On the other hand, (3) and the hypothesis of the proposition imply

$$p(s)/s = p'(s)E[V_1 \mid J] \geq p'(s)(1 + \varepsilon). \tag{14}$$

Combining (12) and (14) yields

$$\frac{p(s)}{s(1+\varepsilon)} \geq p'(s) \geq \frac{p(s)}{s+1} \tag{15}$$

which cannot hold unless $s \leq 1/\varepsilon$. Hence, in equilibrium, no issue size larger than $1/\varepsilon$ could be chosen by the manager. Q.E.D.

One can modify proposition 1 to cover the case in which V_1 is *not* bounded away from 1, by defining $\phi(P) = \min_\theta V_1(P, \theta)$ and proceeding as before. Inequality (14) becomes

$$p(s)/s \geq p'(s)\phi(p(s)) \tag{16}$$

and (15) becomes

$$\frac{p(s)}{s\phi(p(s))} \geq p'(s) \geq \frac{p(s)}{s+1}. \tag{17}$$

Comparing the outer expressions in this chain of inequalities yields

$$s[\phi(p(s)) - 1] \leq 1. \tag{18}$$

However, there exists K such that $p(s) \leq Ks$ for sufficiently large s, so that (18) cannot hold for *all* s unless $s[\phi(Ks) - 1] \leq 1$, or equivalently

$$Ks[\phi(Ks) - 1] \leq K, \tag{19}$$

when s is large. This proves that equity rationing must prevail unless $\phi(P) = \min_\theta V_1(P, \theta)$ declines to 1 at least as fast as $1 + P^{-1}$. Formally,

PROPOSITION 2. *Suppose V is concave in P for each θ. Then in equilibrium there will be equity rationing unless $P[\phi(P) - 1]$ is bounded in P.* Q.E.D.

It is possible to get some insight into these paradoxical results by thinking about the form of a solution to (3). Consider the graph of the proceeds function and a point $(s, p(s))$ on that graph. If V_1 is known to be large, then the proceeds, $p(s)$, will certainly generate substantial *added* value for the firm. Hence, the exchange of $p(s)$ for s shares will be a fair gamble for the market (as required by equilibrium) only if the market treats the firm's *existing* assets as being of low value. This will be the case if $p'(s)$ is small, according to the first-order condition (1). (Intuitively, if the manager's marginal rate of substitution between proceeds and shares is very small, then the firm's existing assets must have a low value.) In short, if V_1 is large, the equilibrium condition (3) forces $p'(s)$ to be small, at any given $(s, p(s))$. Along a solution for $p(\cdot)$, there will be some $\bar{s}$ at which inequality (12) becomes an equality. Under nonincreasing returns, only issue sizes smaller than $\bar{s}$ could possibly be chosen by the manager.

III. Inefficiency of Aggregate Real Investment

In an equilibrium, can the level of real investment equal the level that would be achieved under complete information? To be "efficient" in this sense, the level P of investment must satisfy

$$V_1(P, \theta) = 1; \tag{20}$$

i.e., the firm raises money until an additional dollar of proceeds would increase the value of the firm by exactly one dollar (relative to management's information). If instead $V_1(P, \theta) > 1$, valuable investment opportunities are being passed up.

It is easy to show that in equilibrium, investment cannot be efficient for all θ except in the very special case in which the issue size reveals no information about the value of the shares. To establish this result, note that if $V_1(p(s(\theta)), \theta) = 1$ for all θ as efficiency requires, the equilibrium condition (3) reduces to a first-order differential equation $p'(s) = p(s)/s$, so that $p(\cdot)$ is linear in the issue size, say $p(s) = ks$. The post-announcement stock price $p(s)/s$, therefore, necessarily equals k, which is known in advance by investors. This result shows that, in general, real investment will be too low under asymmetric information.[8]

[8] This result need not hold if the investment opportunities are discrete so that the manager's optimization problem is nondifferentiable. As an example, consider a firm with no assets in place,

We can derive some additional implications of equilibrium in the important special case in which the information asymmetry concerns only the value of the firm's existing assets and not the investment opportunities. In this case, we can write $V(P, \theta) = f(P) + \theta$ for some function, f; the univariate random variable, θ, represents the value of the firm's assets in place, while f describes the investment opportunities. If the marginal value of an additional investment dollar is nearly constant over the relevant range, we can approximate f by $f(P) = P/\alpha$ for some constant $\alpha < 1$. In the Appendix, we show that under these conditions, there is an equilibrium with proceeds function $p(s) = s^{\alpha}$. This equilibrium is "fully revealing" in the sense that if the manager announces s, investors can infer that $\theta = (s + 1)s^{\alpha-1} - s^{\alpha}/\alpha$. The larger is θ, the value of the assets in place, the smaller will be the issue size and investment level selected by management. This implies that the *expected* level of investment declines if the probability distribution for θ is shifted to higher values in the sense of stochastic dominance.

More interesting is the fact (derived in the Appendix) that the net value of the firm's investment, $f(p(s)) - p(s)$, is a convex function of θ. This implies that the expected net value of the firm's investment is higher if there is more uncertainty about the value θ of the firm's existing assets (in the sense of a mean-preserving spread; see Rothschild and Stiglitz [10]). Moreover, the ex ante value of the old shares is $E(1/(s + 1))[\theta + f(p(s))] \equiv E\theta + E\{f(p(s)) - (s/(s + 1))[\theta + f(p(s))]\} = E\theta + E\{f(p(s)) - p(s)\}$ by (2). Therefore, under a change in the distribution for θ that leaves $E\theta$ fixed, the change in the ex ante value of the old shares coincides with the change in the expected net value of the firm's investment. Together, these findings have the important implication that the ex ante value of the old shares rises if there is a mean-preserving increase in the uncertainty about θ. This result might appear to be in conflict with the finding of Myers and Majluf [9, p. 205–6] that firm value decreases when the standard deviation of the assets in place increases. However, in the Myers-Majluf model, the capital needs of the firm's investment opportunity are fixed and known by investors, and management's choice is simply to raise the required funds or pass up the investment altogether. An increase in the standard deviation of the existing assets reduces the probability that the firm will exercise its option to invest. By contrast, in our model management's decision is not *whether* to invest, but *how much*. The higher is the value of the firm's existing assets, the lower will be the amount of new investment. Thus, if the *dispersion* of the distribution of the value of the existing assets increases, while the mean stays fixed, there is increased likelihood of extreme investment outcomes, both high and low. However, since the net value of investment is a convex function of the value of the existing assets, the *expected* amount of investment rises.

This prediction of the model is in principle testable, by examining the stock

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and one investment opportunity whose cost θ is known by the manager but not by the market. If the investment is undertaken, it will have value I, which is common knowledge. Consider the concave proceeds function $p(s) = (s/(s + 1))I$. One can show that facing $p(\cdot)$, it can never make sense for the manager to raise *more* money than θ; moreover, if $\theta < 3I/4$, the manager will always choose to raise θ and undertake the investment rather than raise less than θ. Hence, if $\theta < 3I/4$ with probability one, $p(\cdot)$ is an equilibrium proceeds function entailing no underinvestment.

Table I

Expected Value of the Firm's Net Investment when θ is Lognormally Distributed, for Selected Means and Standard Deviations for θ

		Mean of θ		
		½	2	8
Standard Deviation of θ	½	0.75	0.43	0.12
	2	0.83	0.51	0.13
	8	0.86	0.68	0.21

prices of firms for which the information asymmetry pertains only to the assets in place. After controlling for other factors, firms for which the value of the existing assets is more uncertain to investors should have higher stock prices, due to having higher expected values for the fraction of their available investment opportunities that they will undertake.

Table I shows the expected net value of the firm's investment, $E[f(p(s(\theta))) - p(s(\theta))]$, when $\alpha = ½$ and θ has a lognormal distribution, for selected means and standard deviations of θ. In this numerical example, expected net investment is most sensitive to changes in the mean of θ, although an increase in the standard deviation does raise expected net investment.

IV. Summary and Conclusions

Myers and Majluf [9] have developed an equilibrium model in which a stock issue causes a decline in the corresponding stock price. Their model assumes that the firm's investment requirements are fixed and known by all investors, and that the firm raises either that specific amount of equity or none at all. Consequently, their model does not answer, or even pose, questions about the relationship between the stock price and the issue size. In this paper, we generalized the Myers-Majluf model by making the issue size a continuous choice variable. Our model predicts, in agreement with empirical evidence, that in a sample of firms that issue new stock, the stock price will be negatively correlated with the issue size (after controlling for other observable variables). Our model also predicts that if the information asymmetry is restricted to the value of the existing assets, then other things being equal, greater investor uncertainty about the value of the existing assets will be associated with higher stock prices. Finally, our model permits an analysis of "equity rationing," and shows that (perhaps surprisingly) its likelihood is *increased* by a general improvement in a firm's investment prospects.[9]

The results of our more general model strengthen Myers' and Majluf's conclusion that there is an adverse selection problem associated with the issuance of

[9] One remaining issue is that the equilibrium proceeds function will rarely be unique. In cases in which that function is the solution to a differential equation, one still has to specify a "boundary condition." An appealing choice of equilibrium is the one that maximizes the expected net social gain. However, it is not even clear under what conditions there exists an equilibrium with this property.

risky securities, whether debt or equity. Investors must interpret stock issues unfavorably, and indeed, must interpret larger issues more unfavorably than smaller ones. This phenomenon provides a rationale for the "portfolio approach" adopted by many corporations, in which the cash generated by some divisions finances the investments of other divisions, and the need for external financing is minimized.

Appendix

Completely Revealing Equilibria

The number of shares, s, to be issued is just a single variable, and so, in general, cannot reveal all the manager's information to the market. The exception is the case in which the uncertainty θ is itself one-dimensional. We will analyze that special case in this Appendix.

The first-order condition (1) determines the manager's choice, s, as a function of θ, denoted by $s(\theta)$. If θ is one-dimensional, this function will generally be invertible, so that s will reveal θ completely. Equation (1) can then be regarded as identifying the θ observed by the manager as a function of the s he chooses. There will be a function $\Theta(s)$ defined by the property that

$$V(p(s), \Theta(s)) = (s + 1)V_1(p(s), \Theta(s))p'(s) \tag{A1}$$

for all s. Since the set $J(s, p)$ contains the single element $\Theta(s)$, the equilibrium condition (3) reduces to

$$p'(s) = \frac{p(s)}{sV_1(p(s), \Theta(s))}. \tag{A2}$$

In summary, if there are functions, $p(\cdot)$ and $\Theta(\cdot)$, satisfying (A1) and (A2), then $p(\cdot)$ is a completely revealing equilibrium proceeds function. The manager will optimize relative to $p(\cdot)$ by choosing s if and only if she observes $\Theta(s)$; moreover, given that the manager has observed $\Theta(s)$ and wants to issue s shares, $p(s)$ is the shares' fair price.[10]

An important special case, discussed in Section III, is that in which

$$V(P, \theta) = f(P) + \theta \tag{A3}$$

for some function, f, satisfying $f' \geq 1$. One can interpret the random variable, θ, as the value of the firm's assets in place, and f as describing the (nonstochastic) investment opportunities. Substituting (A3) into (A1), we find

$$f(p(s)) + \Theta(s) = (s + 1)f'(p(s))p'(s), \tag{A4}$$

[10] Completely revealing equilibria are anomalous in that the prior probability distribution for θ is irrelevant for the equilibrium proceeds function. This is because the market's information set contains s, but given s, θ is known exactly. (Another way of looking at this is that in a regression of the stock price against the issue size, the distribution of θ would affect the marginal distribution for the independent variable, but would have no effect on the regression itself.) Leland and Pyle's [6] fully revealing model also has this property. Although they state that potential shareholders would have a subjective distribution for the unknown μ in their model, that distribution turns out to play no role.

so that

$$\Theta(s) = (s + 1)f'(p(s))p'(s) - f(p(s)). \tag{A5}$$

Moreover, (A2) becomes the differential equation

$$p'(s) = \frac{p(s)}{sf'(p(s))}. \tag{A6}$$

This equation has a simple closed-form solution if f is linear over the relevant range: let $\alpha < 1$ be a positive constant, and suppose

$$f(P) = P/\alpha \tag{A7}$$

for all $P \leq [\alpha/(1 - \alpha)]^{\alpha}$. (The function, f, can be arbitrarily specified[11] for larger P.) Equation (A6) reduces to

$$p'(s) = \frac{\alpha p(s)}{s} \tag{A8}$$

over the range in which $f'(p(s)) = 1/\alpha$, and this differential equation has the solution

$$p(s) = s^{\alpha}. \tag{A9}$$

(This is shown in Figure A1 for $\alpha = 0.7$.) Substituting the expressions $f'(p(s)) = 1/\alpha$, $p'(s) = \alpha s^{\alpha-1}$, and $f(p(s)) = p(s)/\alpha = s^{\alpha}/\alpha$ into (A5) yields

$$\Theta(s) = [(s + 1)/s^{1-\alpha}] - s^{\alpha}/\alpha. \tag{A10}$$

Equations (A9) and (A10) need be defined only for $s \leq \alpha/(1 - \alpha)$, because as s ranges from $\alpha/(1 - \alpha)$ to zero, $\Theta(s)$ covers the full range from zero to infinity. This example, therefore, exhibits equity rationing of the form described in Section II: by suitably defining $p(s)$ for $s > \alpha/(1 - \alpha)$, we can take $p(\cdot)$ to be bounded.

The net value of the firm's investment, as a function of θ, is $f(p(s(\theta))) - p(s(\theta)) = ((1 - \alpha)/\alpha)p(s(\theta)) = ((1 - \alpha)/\alpha)s(\theta)^{\alpha}$. This is a decreasing function of θ, because $\Theta(s)$ is decreasing in s. A more significant property, whose consequences are explored in Section III, is that $((1 - \alpha)/\alpha)s(\theta)^{\alpha}$ is a *convex* function of θ. The easiest way to establish this is to notice that since $s(\theta)^{\alpha}$ is decreasing in θ, it is convex if and only if its inverse function $\Theta(s^{1/\alpha})$ is convex as a function of s. From (A10), we find that $\Theta(s^{1/\alpha}) = [(s^{1/\alpha} + 1)/s^{(1-\alpha)/\alpha}] - s/\alpha$, and it is easy to establish that $d^2/ds^2\Theta(s^{1/\alpha}) > 0$.

If $\alpha = 1/2$, we can invert formula (A10) explicitly, solving for s as a function of θ:[12]

$$s(\theta)^{1/2} = \frac{-\theta + (\theta^2 + 4)^{1/2}}{2}. \tag{A11}$$

Expression (A11) is used to compute $Es(\theta)^{1/2}$, shown in Table I.

[11] The form of f for large P is irrelevant because, as we will see, in equilibrium the firm could never raise more than $[\alpha/(1 - \alpha)]^{\alpha}$. However, for f to be reasonable, f' should at some point begin to decline to 1.

[12] From $\theta = (s + 1)/s^{1/2} - 2s^{1/2}$, set $g = s^{1/2}$; this gives the quadratic $g^2 + \theta g - 1 = 0$, whose solution for g is (A11).

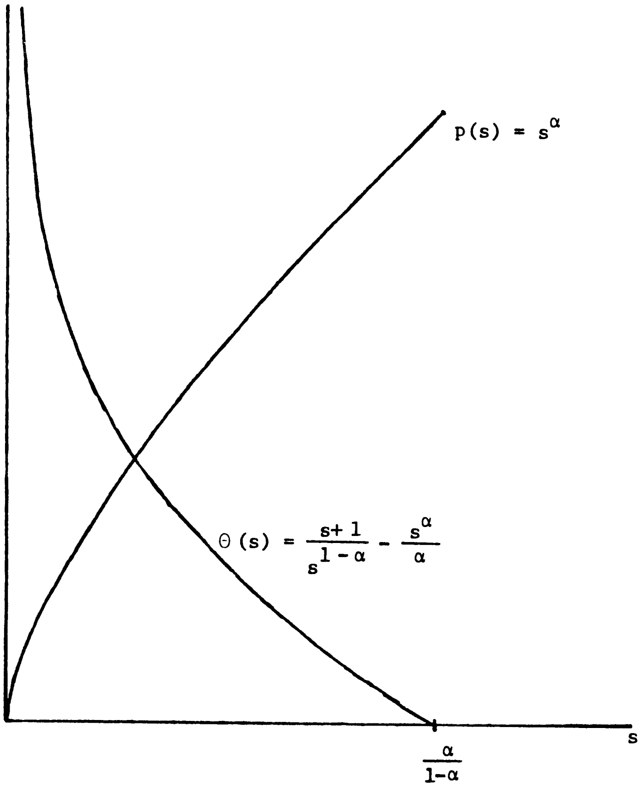

Figure A1. Example of Completely Revealing Equilibrium

REFERENCES

1. P. Asquith and D. W. Mullins. "Equity Issues and Offering Dilution." *Journal of Financial Economics* 15. Forthcoming, 1986.
2. L. Y. Dann and W. H. Mikkelson. "Convertible Debt Issuance, Capital Structure Change and Financing-Related Information: Some New Evidence." *Journal of Financial Economics* 13 (June 1984), 157–86.
3. A. C. Hess and S. Bhagat. "A Test of the Price Pressure Hypothesis Using Announcement Data." Working Paper, University of Utah, 1985.
4. M. Jensen and W. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* 3 (October 1976) 305–60.
5. A. N. Korwar. "The Effect of New Issues of Equity: An Empirical Investigation." Manuscript, 1983.
6. H. E. Leland and D. H. Pyle. "Informational Asymmetries, Financial Structure, and Financial Intermediation." *Journal of Finance* 32 (May 1977), 371–87.
7. R. W. Masulis and A. N. Korwar. "Seasoned Equity Offerings: An Empirical Investigation." *Journal of Financial Economics*, forthcoming, 1986.
8. M. Miller and K. Rock. "Dividend Policy Under Asymmetric Information." *Journal of Finance* 40 (September 1985), 1031–51.
9. S. C. Myers and N. J. Majluf. "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have." *Journal of Financial Economics* 13 (June 1984), 187–222.
10. M. Rothschild and J. E. Stiglitz. "Increasing Risk: I. A Definition." *Journal of Economic Theory* 2 (September 1970), 225–43.