Pricing Beyond the Curve

Derivatives and the Long Term

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9 February 2001

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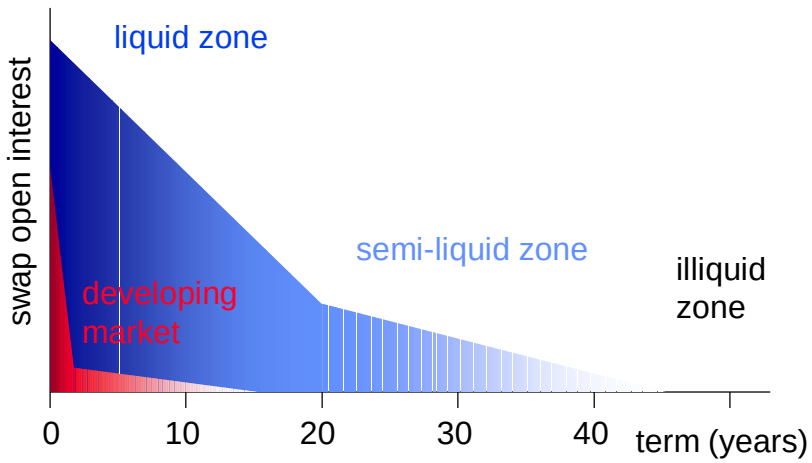
Demand and Liquidity



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Some Commercial Questions

- In the semi-liquid zone
  - still some demand for swaps
  - margins are wider
  - observable prices unreliable or absent
- Can I make money dealing in this area?
  - how do I know when I've made money?
  - how can I manage the risk?
  - what about other derivatives?

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Combine several techniques

- How should the long end behave?
  - theoretical behaviour
  - practical inputs for parameter estimation
- Implications for yield curve calibration
  - fitting observed rates
  - subject to long term constraints
- Medley of different techniques required
  - hence 3 separate papers

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Simplest Approach - Extrapolation

- This is currently dominant standard
- Take current swap curve
  - or spot, or forward
- Fit a functional form
  - eg exponential spline
- Work backwards, to spot & forward rates
- Separate position limits on long spot curves

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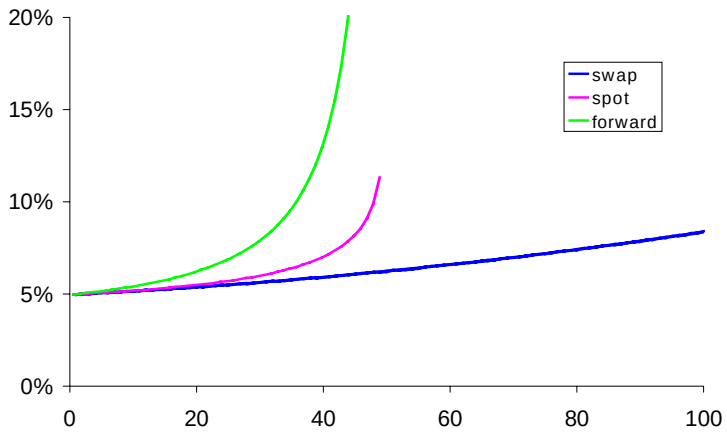
Extrapolation can be Unstable



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What we need is …

- A method for pricing long flows
  - that calibrates short flows exactly
  - and does not become unstable
- To avoid instability
  - calibration constraints necessarily linear in discount bond prices
  - why not constrain discount bond price vector to linear manifold?
  - this forces a hard-coded long forward rate

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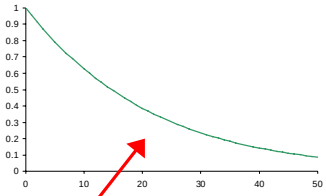
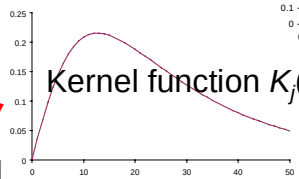
Building Yield Curves

Discount function $P(t)$

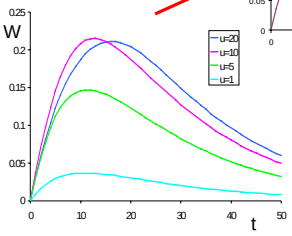


linear combination



Kernel function $K_j(t)$

linear combination



Wilson's function $W(t,u)$

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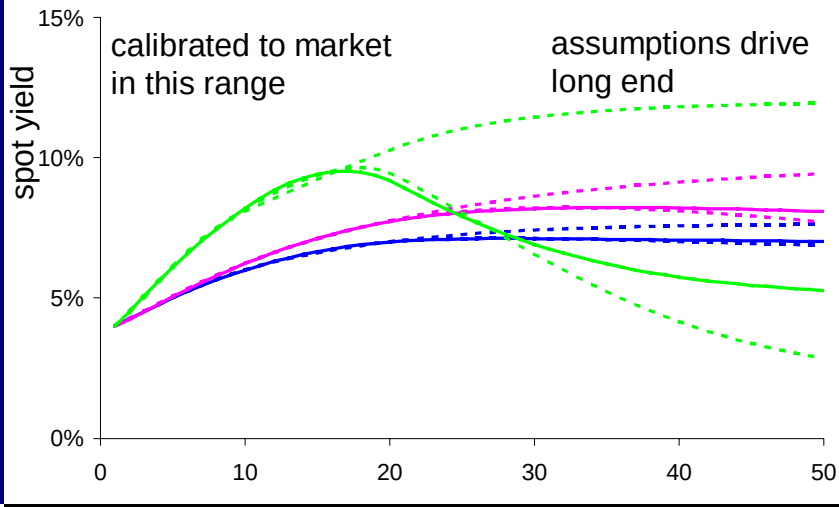
Fixing the Long Forward Rate



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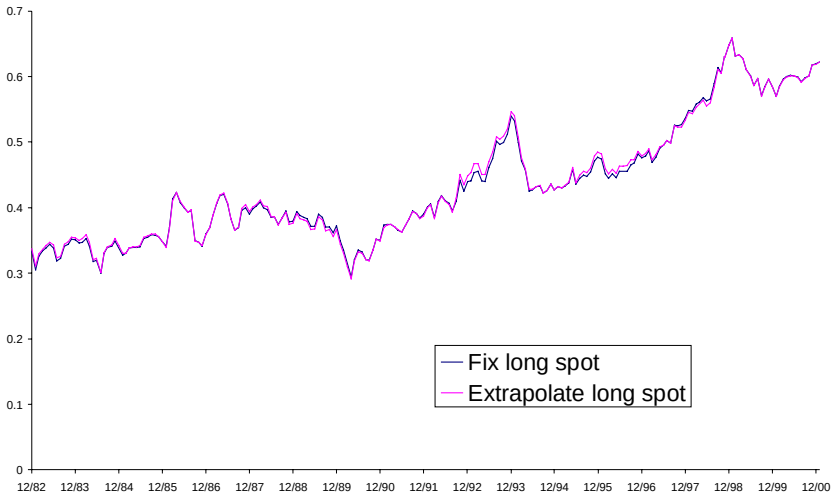
Results – 10 year Zero (based on gilts)



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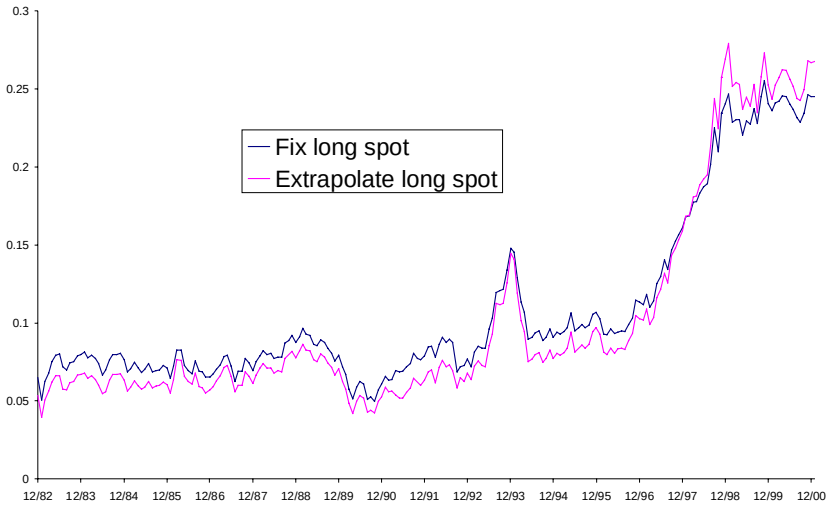
Results – 30 year Zero



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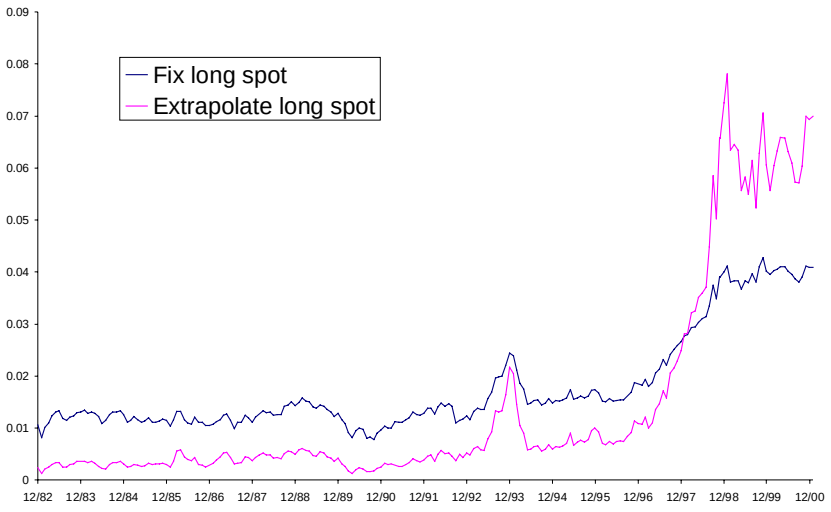
Results – 60 year Zero



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Absence of Arbitrage: D.I.R. Theory

- Convexity effects
  - Suppose 50 year swap rate always = 30 yr swap rate
  - Hedge ratio based on duration
  - Negative convexity if sold fixed
  - Drift on long rate is infinite (HJM)
- Solution – fixed long rate
  - 50 year swap rate = function(30 yr rate, 20 yr rate, …)
  - Careful choice of basis (via Wilson's function) to get stability
  - Differentiation gives hedge ratio – concave in term
  - Theta pays for Gamma position

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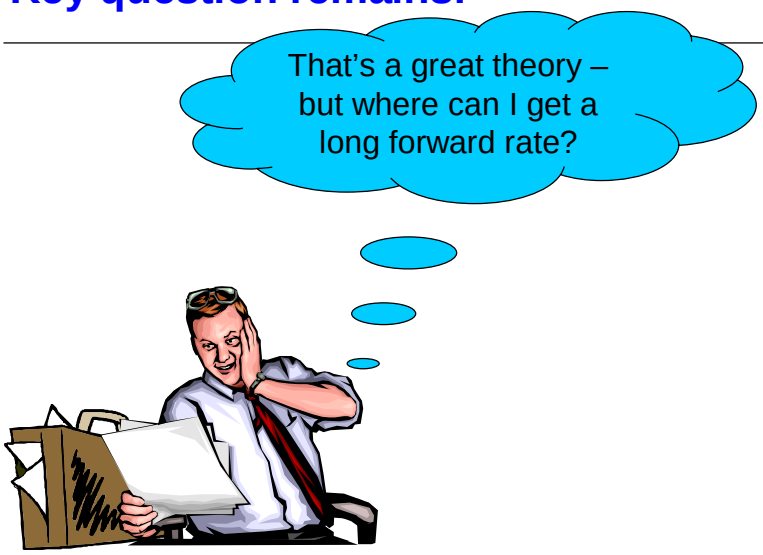
Key question remains:



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Options analogy

- Pricing a call or put on exotic stock
  - look at time series volatility
  - assume future like the past
    - or take deliberate view
  - apply Black-Scholes to get an option price
  - add a contingency margin
  - hedge
  - monitor hedge slippage

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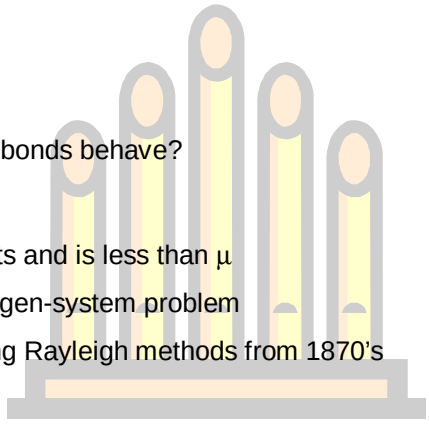
Solving Long Forward Rates is Hard

- Example: suppose
  - $dr = \alpha(\mu - r)dt + \sigma r\, dz$
  - under risk neutral law
  - how do prices of long bonds behave?
- What we can show:
  - long forward rate exists and is less than μ
  - in general this is an eigen-system problem
  - solve numerically using Rayleigh methods from 1870's



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Proposed Modelling Approach

- Work under real-world law
  - so can reconcile to longitudinal data sets
  - stationary Markovian models
  - 3 parameters: level, volatility, reversion
- Risk adjustment for pricing
  - via explicitly generated state price deflator
  - introduces one more risk premium parameter
- Discrete time
  - general error distribution (non-diffusion limit)
  - incomplete market

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Possible Calibration Inputs

- Could express long forward as function of the following four inputs:
  - Level to which short rate mean-reverts
  - Mean reversion factor
  - One-step-ahead short rate volatility
  - Long bond risk premium
- Alternatives
  - long bond volatility
  - instead of mean reversion factor

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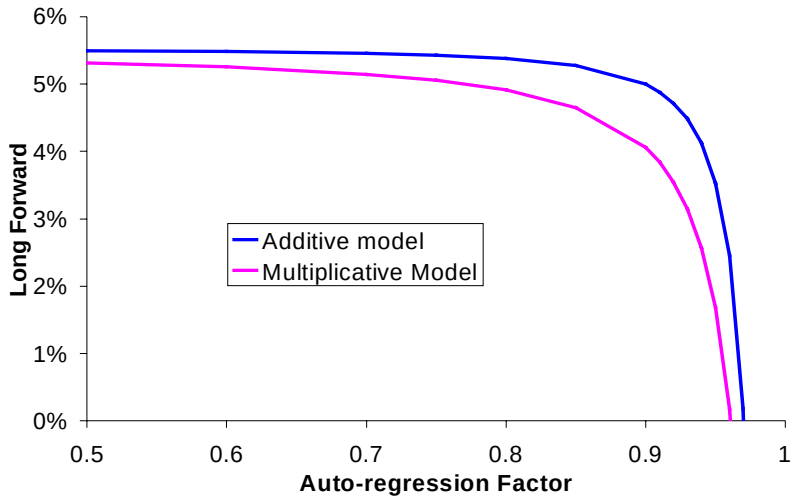
Mean Reversion is Critical



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What does hedging rely on?

- Hedge slips if volatility turns out different from estimate
  - familiar hedging problem from options
- When hedging with bonds;
  - hedge *much* more effective than with volatile long forward
  - possible dislocation when new bond issued
  - good model is one which forecasts new long issue prices
- With swaps, new long swap issued every day
  - hedge performance critical at longest node
  - analysis of long actual / synthetic
  - provides an alternative calibration methodology

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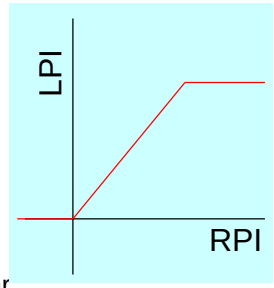
Application: Valuing LPI

- Limited price indexation
  - Popular formula for pensions in payment
- Annual pension increase =
  - rate of price inflation in previous year
  - subject to a maximum of 5%
  - and minimum of zero
- Pension increases compound year-by-year
  - no provision for "catching up" volatile inflation
- We compute long forward rate



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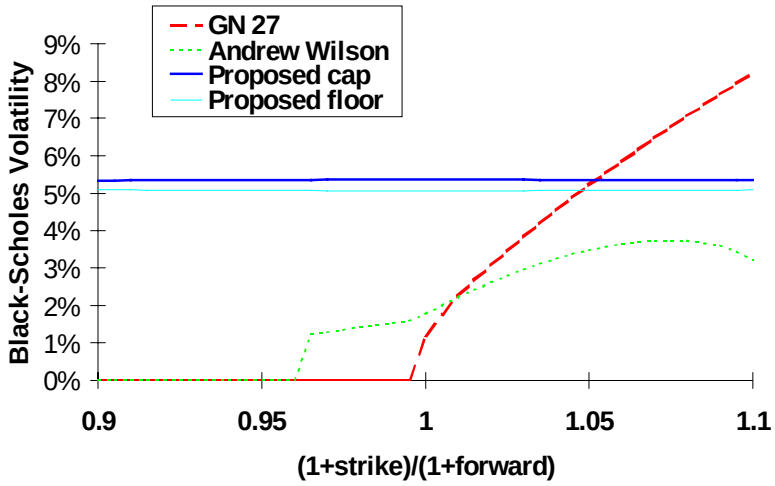
LPI: Implied Volatility



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Discuss: Strengths and Weaknesses

- Stable linear calibration of yield curve
- Daily moves consistent with arbitrage-free models
- Very long bonds less volatile than we thought
- Empirical basis for choosing long parameters
- Subjective input needed
- May be vulnerable to lobbying / trader bias
- May simply replace volatility with catastrophic jumps

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