Further down the line

Andrew Smith and Michael Thomas examine various methods of yield-curve fitting and extrapolation for long-term interest rate modelling

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With Solvency II nearly upon us, the problem of identifying 'long-term' and 'risk-free' interest rates has become a major issue for both UK and European life insurers. Different approaches to extrapolating the yield curve can give significantly different results and it is not obvious which approaches are best.

For example, Figure 1 shows euro yield curve extrapolations generated using four different methods at the end of 2008. The yields are all compounded continuously and we have transformed the horizontal axis to show the limit as term tends to infinity.

The lowest two curves are taken from the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) paper CP40 and have been extended to infinity. The highest curve is not specific to 2008, but is a long-term curve based on what CP40 calls the 'macro-economic approach'. The Wilson curve is also fitted to 2008 swap data, but is constrained to coincide with the macroeconomic approach at the long end.

European Central Bank (ECB) approach

The ECB approach uses a curve-fit based on a formula developed by Svensson (1994). Figure 2 shows the fitted curves. At the end of 2008, the long-term limit is below 0.5%, substantially less than what most actuaries would consider reasonable.

QIS4 methodology

CEIOPS' fourth quantitative impact study

(QIS4) specified an approach to derive yield curves from swap rates by assuming that the forward curve is constant between every pair of data points. Figure 3 shows the fitted yield curves.

This approach appears to show more acceptable behaviour at the long end than the Svensson approach, although the yield curves lack the Svensson formula's smoothness.

How should long-term interest rates behave in theory? The literature contains many stationary yield-curve models and they all behave asymptotically according to the familiar actuarial formula that Stoodley (1934) first proposed:

$$\textit{Forward yield } (T) = \textit{forward yield } (\text{infinity}) + \frac{\kappa\lambda}{\kappa + e^{\lambda T}}$$

In 1996, Dybvig, Ingersoll and Ross showed the surprising result that, within any arbitrage-free interest rate model, the long-term forward rates should converge towards a limiting 'long rate' assumption which is constant over time.

Components of the long rate

To calibrate the terms in Stoodley's

Table 1 — Determination of the long rate

	Volatility multiplier			
	1 (Historic)	1.5	2	3
Mean cash return	5.10%	5.10%	5.10%	5.10%
Term premium	1.10%	1.65%	2.20%	3.30%
Convexity effect	(0.40%)	(0.90%)	(1.60%)	(3.60%)
Long rate	5.80%	5.85%	5.70%	4.80%
Stoodley exponent				
Vasicek		10.99%	6.93%	4.05%
Cox Ingersoll Ross		12.10%	8.93%	6.17%

formula, we need to estimate the parameters in a stationary yield-curve model. The long rate can generally be expressed in the following manner:

Long rate = mean cash return *plus a term premium* (being the premium, on an arithmetic basis, associated with buying long-term bonds) *less a convexity effect* (the difference between arithmetic and geometric returns)

Of these components, the mean cash return is the most significant and the most difficult to estimate, with historical averages being the main guide. The term premium and convexity effect can be estimated from historic bond market returns according to the following formulae:

Term premium = *volatility multiplier* × *historical term premium*
Convexity effect = *volatility multiplier*^{2} × *historical convexity effect*

The volatility multiplier captures the relative volatility of long-term bonds when compared to the historical bond portfolio. This is a key judgmental input which relates directly to the strength of mean reversion. In an environment which permits parallel yield-curve shifts, suggesting no mean reversion, the volatility multiplier would be infinite. We provide results assuming finite volatility multiplier values of 1.5, 2 and 3 respectively.

Application to historical data

Dimson, Staunton and Marsh (2002) list arithmetic and geometric mean returns for cash and bonds over an 101-year period for a number of currencies. This study provides ideal material for long-rate calibration. We assume that the bonds underlying the study have an average term of 10 years, although the analysis is not sensitive to this assumption.

The mean cash returns are found to vary between countries, with the highest (arithmetic) return in Denmark at 7.1% and the lowest in Switzerland at 3.3%. The average over all economies is 5.1%.

The historical term premium on bonds also varies between countries. The average over all countries is broadly 1.1%. The historical convexity effect is generally small at around -0.4% for most countries.

Table 1 demonstrates determination of the long rate for various volatility multiplier values based on the above historical

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Figure 1 — Alternative yield-curve extrapolations (31 December 2008)

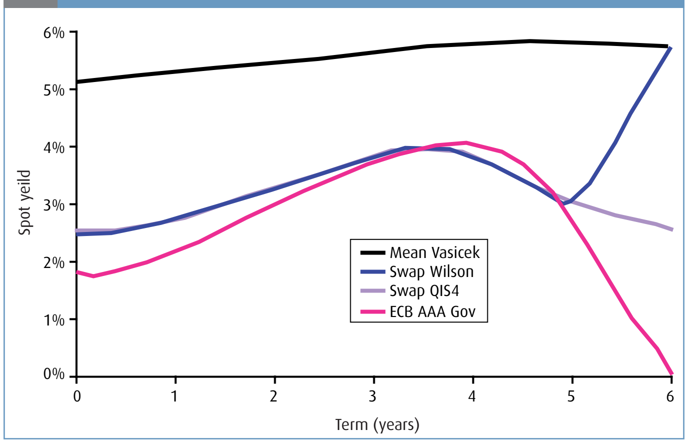
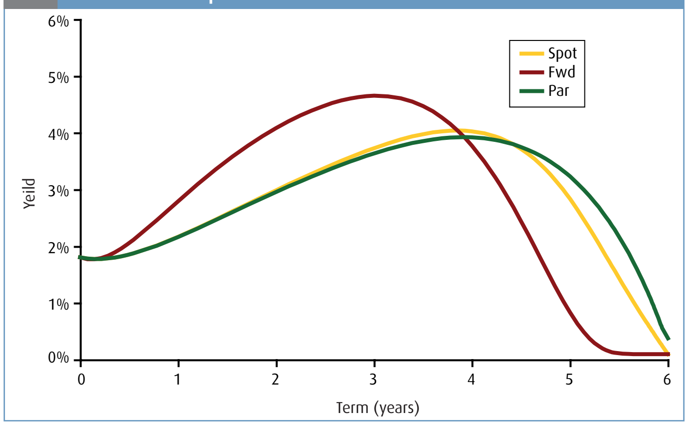


Figure 2 — Yield-curve extrapolation based on the Svensson model and 31 December 2008 ECB parameters



Further down the line

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estimates. Table 1 also shows the implied exponent λ for fitting Stoodley's formula, the calculation of which is model-dependent. We show results for two well-known models, and note that choice of model has a relatively minor effect.

Figure 4 shows the implied average curves for Vasicek's model, assuming a volatility multiplier of two. As these curves relate to long-term averages, bond returns can be considered on both an arithmetic and geometric-mean basis. The geometric mean is equivalent to a mean forward curve. We have also calculated the spot and par curves consistent with that mean forward curve, which confirm the usual hypothesis of an upward sloping par curve.

Is there a model that can fit the macro-economic analysis and still replicate actual bond market prices?

Smith & Wilson (2000) published a model for bond prices using linear combinations of spline functions with long-term yield constraints. This model is particularly attractive from a calibration perspective as it reduces to a series of linear equations which can be easily solved for any finite series of bond prices. The fit collapses to Stoodley's formula beyond the last observable yield.

» The Smith-Wilson model was significantly more efficient than any of the alternative extrapolation approaches «

Mare & Thomas (2007) have demonstrated the superior predictive capability of the model when applied to a real hedging problem. Their work compared the efficiency of a number of alternative models for hedging long-term interest rate risks, including the Svensson, Nelson-Siegel and Smith-Wilson models. They found that the Smith-Wilson model was significantly more efficient than any of the alternative extrapolation approaches.

Figure 5 shows the Smith-Wilson fit at the end of 2008, imposing the 5.7% long rate and 6.93% Stoodley exponent derived from Vasicek's model.

The yield curves in Figure 5 fit all of the observed swap prices exactly, with a forward curve that is smoother than the QIS4 method. The turning point at 50 years represents an assumed return from the current low-interest environment to the 20th century average.

Conclusions

The wide variety of results from different extrapolation approaches raises some interesting questions for life insurers. Industry responses to CP40 have been supportive of a macro-economic approach, but a challenge lies in identifying credible models which can fit actual bond prices and extrapolate toward a macro-economic long rate.

We highlight a potential approach that has been used in practice for a number of years. Further, we note that well-constructed macro-economic approaches can also offer significant advantages for hedging long-dated insurance cashflows.

A full list of references can be found on the online version of this article at *www.the-actuary.org.uk/871266*

Figure 3 — Yield-curve extrapolation based on QIS4 piecewise constant forward rates

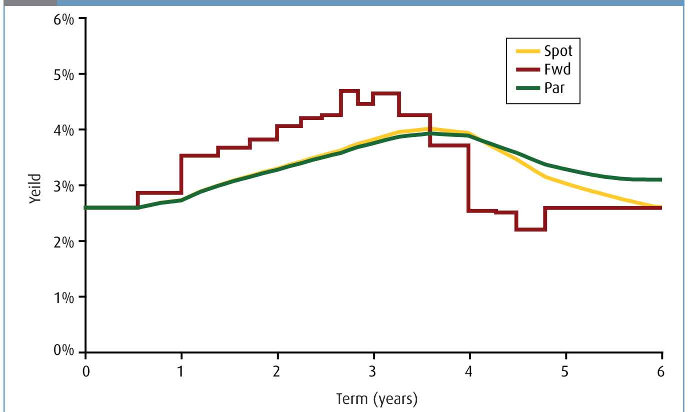


Figure 4 — Vasicek parameterisation for volatility multiplier = 2

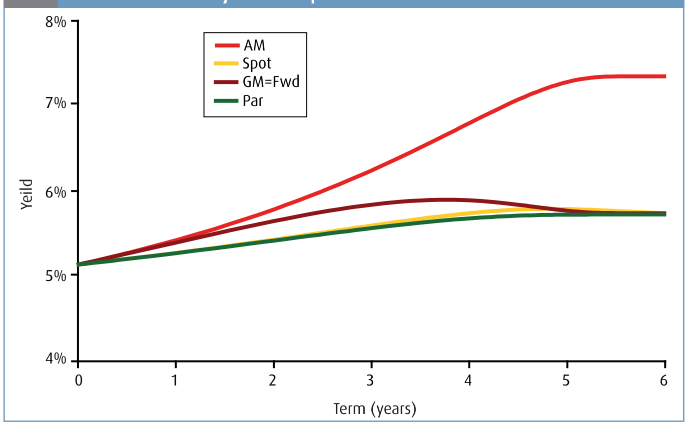
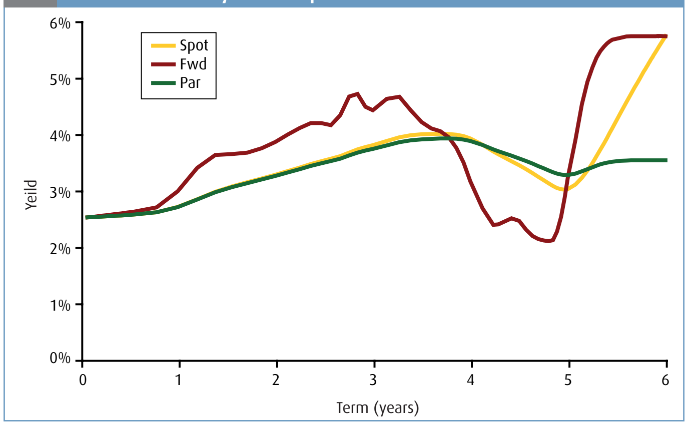


Figure 5 — Smith-Wilson parameterisation for volatility multiplier = 2



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