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Mortgage Backed Securities: market overview, historical performance and valuation methods

1. Introduction

The greater importance recently given to investment returns on foreign currency reserves, compounded with the current low yields on traditional Treasury instruments, has made reserve managers consider alternate asset classes for their reserves.

As an alternative to traditional Treasury instruments, other asset classes that are typically considered by reserve managers include; agency bonds, corporate bonds, asset-backed securities (ABS), mortgage-backed securities (MBS) and even equities.

These asset classes will generally offer higher expected returns. These higher returns are however accompanied by the additional risks inherent in holding these securities. The source of the additional risk ranges from; default risk, spread risk, prepayment risk or simply the high sensitivity of equity prices to the fortunes of a particular company, sector or entire market.

Before deciding to invest in one of these alternate asset classes it is as important to fully understand the nature of the risks associated with the asset class as it is to determine the expected return characteristics of the asset class and to determine how well this asset class fits into a reserve portfolio comprising other asset classes.

This note examines the investment characteristics of mortgage-backed securities (MBSs). A MBS may be considered as a particular type of ABS[1]. The pool of assets underlying MBSs are the payments made by mortgage borrowers net of servicing fees.

MBSs have a number of qualities that make them particularly attractive as an asset class for reserve management. The size of the market is comparable to that of US Treasuries, MBSs trade in liquid markets, the credit risk inherent in agency MBSs is very low, and MBSs have offered higher risk-adjusted returns than Treasury instruments. Before investing in MBSs an invester should however understand the nature of the prepayment risk inherent in these instruments.

Section 2 gives an overview of MBSs: the participants in the mortgage market, how they are created and how they are traded. In section 3 we look at historical performance of MBSs relative to other fixed income instruments. The homeowner's prepayment option is instrumental in determining MBS performance characteristics this is described in section 4. In section 5 techniques for the valuation of MBSs are examined.

This study is primarily concerned with the MBS market in the United States. The general concepts presented here are nevertheless relevant to MBS markets in other countries. There will however be differences in the detailed functioning of each market that need to be taken into account. These differences range from different participants (and the level of explicit or implicit government guarantees on securities they issue) to the prevalence of different mortgage types in different markets.

2. Overview of mortgage markets

A mortgage is a loan secured by the collateral of some specified real estate property. The borrower (the mortgagor) makes contractual payments (usually on a monthly basis) of interest and principal to the lender (the mortgagee). Mortgages can be divided into two broad categories, those on residential properties and those on non-residential properties. Residential mortgages can furthermore be divided

[1] Mortgage-backed securities are however usually considered to be a separate asset class from asset-backed securities.

into single-family and multi-family structures. We focus our analysis on the residential mortgage market because this is the largest and most liquid segment of the market.

Mortgage markets may be divided into primary and secondary markets. In the primary market mortgage originators make loans to homeowners enabling them to finance the purchase of their home. Participants in the secondary mortgage market purchase loans from originators, thereby channelling liquidity into the primary market. Potential buyers of loans from lenders include three government-sponsored enterprises (GSEs) that were specifically created to channel liquidity into the primary market, as well as several private companies (such as the Residential Funding Corporation, a subsidiary of General Motors Acceptance Corporation).

2.1 Mortgage types

Homebuyers are given a wide choice on the type of mortgage contract to use. Here we give a brief overview of some of the main types of mortgages (see Bhattacharya et. al. [2001] for more details).

Fixed-rate level-payment mortgage (FRLPM)
This is the most common type of mortgage in the United States. The borrower makes equal monthly payments over the entire maturity of the contract (hence the term level-payment). The contract rate remains fixed over the entire maturity of the contract (hence the term fixed-rate). The size of the monthly payments is determined such that the loan is fully amortised at maturity[2]. As the loan is fully amortised over the maturity of the loan, the balance outstanding progressively decreases with maturity. The amount of interest that needs to be paid is proportional to the balance outstanding, interest payments therefore decrease with maturity. Hence, payments made in the early years predominantly service the interest charges and in the latter years payments predominantly serve to reduce the outstanding balance (see Appendix I).

Graduate payment mortgage (GPM)
These mortgages target borrowers who expect their capacity to finance their borrowing to increase with time. Both the contractual interest rate and the term of the mortgage are fixed. The monthly payments are however smaller in the initial years and larger in the latter years of a GPM. The scheduled payments in the early years may not fully cover interest payments on the outstanding balance; this leads to negative amortisation. The higher payments in the latter years are designed to fully amortise the outstanding mortgage balance over the remaining maturity of the loan.

Adjustable-rate mortgage (ARM)
The contract rate on an ARM is reset periodically in accordance with some specified index rate such as the rate on U.S. Treasury securities or LIBOR. Payments on ARM are determined so that the loan is fully amortised over its maturity. In the United Kingdom ARMs are the most common type of mortgage.

2.2 Market participants

There are many participants in the mortgage markets the majority of which have pure commercial interests in mind. There also exist a variety of government agencies and government-sponsored enterprises (GSEs) whose (original) mandate is to provide homebuyers with better access to finance.

Mortgage markets can be divided into five groups:

1. Homeowners (the borrowers)
2. Mortgage originators

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[2] Appendix I shows how to determine the size of the contractual payments.

3. Mortgage servicers
4. Mortgage insurers
5. Mortgage investors

In the primary mortgage market originators provide loans to borrowers. Mortgage originators charge an origination fee for their services. Before extending a loan the mortgage originator will perform a credit evaluation of the loan application. Three factors are used to determine if the loan will be made:

1. Capacity. Quantified by the payment-to-income (PTI) ratio of the borrower.
2. Collateral. Quantified by the loan-to-value (LTV) ratio of the property.
3. Credit. Quantified by the credit history of the borrower.

Mortgage servicing involves a variety of tasks ranging from maintaining records of outstanding balances to collecting payments and sending the proceeds to the owner of the loan. The primary source of revenue for servicers is the servicing fee, which is a fixed percentage of the outstanding mortgage balance.

Mortgage insurance protects the lender against loss in the event of default by the borrower. Insurance is normally required on loans with LTV ratios in excess of 80%. Three federal agencies, the Federal Housing Administration (FHA), the Veterans Administration (VA), and the Rural Development Administration (RDA), provide insurance on the loans they originate. The loans extended by these agencies are called non-conventional loans and their timely payment of interest and principal is guaranteed by the full faith of the U.S. government. Insurance on conventional loans is provided by regular private mortgage insurance.

The three GSEs, the Federal Home Loan Mortgage Corporation (FHLMC or Freddie Mac), the Federal National Mortgage Association (FNMA or Fannie Mae), and the Government National Mortgage Association (GNMA or Ginnie Mae) as well as several private companies buy mortgages from originators. Pools of mortgages of similar characteristics are created, securitised, and participations in these securities sold to other investors. GNMA only purchases non-conventional loans and the securities they issue are backed by the full faith of the U.S. government. Even though securities issued by FHLMC and FNMA do not carry explicit government guarantees market participants consider the credit risk associated with these instruments to be minimal.

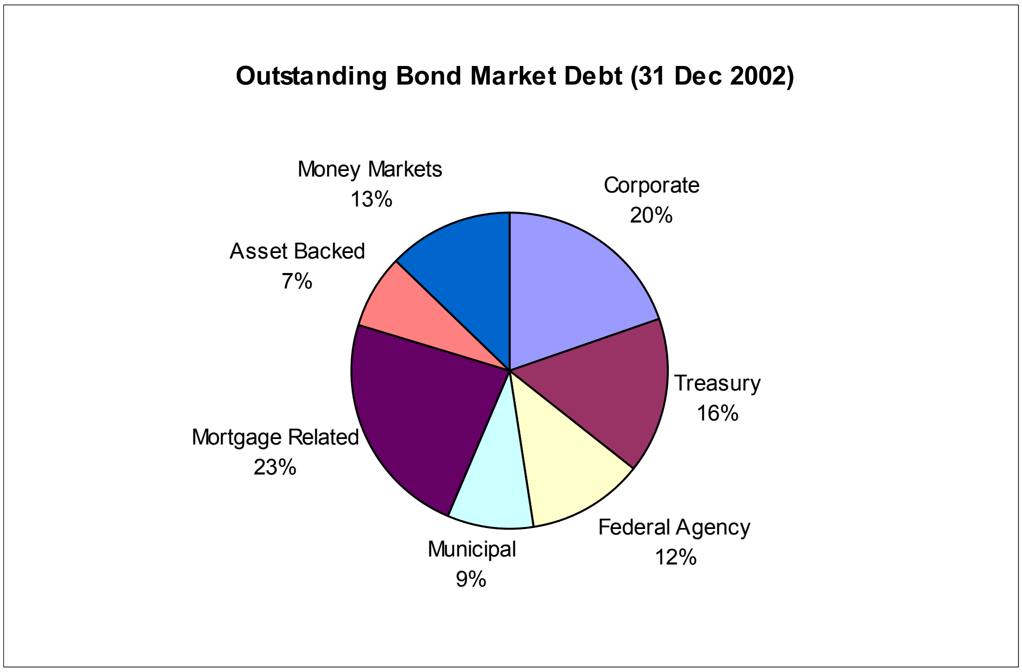
2.3 Prepayments

A mortgage contract entered into by a homebuyer and a mortgage originator will in general not remain effective up to the specified maturity date. This is because the homeowner has the option to prepay the mortgage in whole or in part at any time. Typically no prepayment penalty is imposed.

As a result of the prepayment option the exact payments made by the borrower cannot be known in advance. Consequently also the cash flow received by the holder of a MBS cannot be known in advance. Modelling the prepayment behaviour of borrowers through interest rate and economic cycles is at the crux of MBS valuation.

The prepayment option will often be exercised when interest rates fall and the borrower can refinance the mortgage at a lower rate. Under these conditions investors are in the undesirable situation of getting principal returned at par rather than holding on to a high coupon instrument in a low yield environment.

2.4 Market size



Source: The Bond Market Association.

The above chart shows that of a total $20.2trillion outstanding bond market debt at the end of 2002, almost one quarter is in MBSs. Of the $4.7trillion MBSs outstanding $3.2trillion are agency securities (GNMA $0.6trillion, FNMA $1.5trillion, and FHLMC $1.1trillion).

2.5 Types of mortgage-backed securities

There is a wide range of securities that are backed by receivables on mortgage loans. These range from simple pass-through securities to structured Collateralised Mortgage Obligations (CMOs). Different types of MBSs may perform very differently under specific environments.

Pass-through securities
Pass-through securities are created when mortgages are pooled together and undivided participations in the pool are sold. The individual loans underlying a pass-through usually have similar characteristics (loan type, maturity, and coupon rate). The security holders receive pro-rata shares of the resultant cash flow net of fees. The size of the pass-through market is over $2trillion; the majority of regularly traded pass-throughs are issued and/or guaranteed by one of the federal sponsored agencies.

Collateralised mortgage obligations (CMOs)
As with pass-through securities, the assets underlying CMOs are the receivables from an underlying pool of mortgages. The payments made on the underlying loans are however not split equally amongst all investors in the CMO. The CMO is divided into tranches, or classes, with different tranches receiving different cash flows resulting from the underlying loans. The CMO is however *self-supporting*, with the cash flows from the underlying collateral always able to meet the cash flow requirements of all CMO classes under any prepayment scenario. It is possible to have tranches that perform well during a refinancing wave and to have other tranches that perform poorly under these conditions. Issuers can in effect tailor-make mortgage securities according to investor coupon, maturity, and prepayment risk specifications.

Stripped mortgage-backed securities (SMBS)
SMBSs separate interest payments from principal payments (both scheduled and prepayment). Certificates are issued that entitle the holder to different proportions of the interest and principal cash flows resulting from the underlying mortgage pool. At the two extremes are the interest-only (IO) and principal-only (PO) certificates.

The holders of the PO certificates receive all principal payments. The sum of all principal payments is a fixed amount (the sum of all principal payments on a mortgage loan is equal to the amount borrowed). Holders of PO certificates are therefore happy when prepayment rates increase as this makes their cash flow come earlier. As a result PO certificates will perform well when interest rates fall and, as a consequence, prepayment rates increase. If interest rates rise, and prepayment rates fall, holders of PO certificates will not only have to wait longer to receive their cash but the present value of this cash flow stream will be further reduced as a result of the higher interest rate environment. PO certificates consequently perform poorly in increasing interest rate environments. PO certificates are bullish instruments.

The holders of the IO certificates receive all interest payments. Interest payments are proportional to the outstanding principal. Increases in prepayment rates result in lower outstanding balances and, as a consequence, the holder of an IO certificate receives smaller cash flows. IO certificates therefore tend to perform well when prepayment rates decrease and to perform poorly when prepayment rates increase. IO certificates are bearish instruments.

Sequential-pay classes
All principal payments are paid to the shortest maturity class until it is fully retired, then principal payments are directed to the next shortest class. This process continues until all classes are paid down. The interest paid to each class is proportional to the outstanding principal of the class; all classes therefore start receiving interest payments immediately.

Planned amortisation classes (PACs)
The inclusion of *companion classes* in a CMO structure insulates the holders of the PAC class from variations in prepayment speeds within certain limits. The purpose of companion classes is to absorb any principal payments made in excess of that promised to the PAC holders. As a result the amortisation of the PAC will only deviate from the planned schedule if prepayment speeds are either much higher or much lower than anticipated.

Further details of CMOs can be found in Collateralised Mortgage Obligations (Mortgage Research Group - Lehman Brothers).

2.6 Trading agency pass-through securities

Most agency pass-through securities trade on a to-be-announced (TBA) basis. In a TBA trade the buyer and the seller agree on general trade parameters such as agency, type, par amount and price (e.g., $100m of Ginnie Mae 30-year 7% pass-throughs at a price of 98-14), but the buyer does not know the specific pools to be delivered until two business days before the settlement date. TBA trading improves the liquidity of similar pass-through mortgage pools by making them fungible.

TBA trades of agency pass-through securities settle to a monthly schedule established by the Bond Market Association (BMA). The monthly settlement schedule was established for two main reasons:

- Pool *factors* (the fraction of original balance still outstanding, accounting for both scheduled amortisation and prepayments) are released near the beginning of the month. Dealers must await these factors to determine the outstanding balance in the pools they are delivering.
- Improving liquidity: dealers can more easily create tradable blocks if all pools for a month of trading are specified on the same day.

The last business day of each month is known as the record date, the interest for the month will be paid to the owner of the pool on the record date. If the pool was traded during the month the accrued interest up to the settlement date belongs to the previous owner and is paid on the settlement date. The owner of the pool on the record date will receive interest and principal payments corresponding to the month after a delay to allow for servicing. Holders of FNMA pass-throughs receive payment on the 25th of the subsequent month and holders of GNMA and FHLMC receive payments on the 15th of the subsequent month.

3. Performance analysis

In this section we compare the performance of indices representing various, USD-denominated, asset classes over the period from the end of April 1993 to the end of November 2002. Monthly total return data is analysed, the results are then annualised before being reported. The one-month deposit rate is used as the risk-free rate. The asset classes that are analysed are the 1-3 and 3-5 year sectors of the Treasury market, the 1-3 and 3-5 year bullet agency sectors and sectors representing mortgages with Weighted Average Lives (WALs) of 0-3 years and 3-5 years. The table below summarises the results:

	1m depo.	T. 1-3 Yrs.	T. 3-5 Yrs.	A. 1-3 Yrs.	A. 3-5 Yrs.	M. 0-3 Wal.	M. 3-5 Wal.
Av. Annual Return	4.72%	5.77%	6.64%	5.95%	6.72%	6.45%	6.89%
Excess Annual Return	-	1.05%	1.92%	1.23%	2.00%	1.72%	2.17%
Sd. of Annual Returns	0.40%	1.64%	3.53%	1.56%	2.82%	1.56%	2.26%
Sharpe Ratio	-	0.64	0.54	0.79	0.71	1.11	0.96

Data source: Merrill Lynch

The Sharpe ratio is a risk adjusted performance measure and as such is often used to compare asset classes with different risk characteristics. From the Sharpe ratio it is seen that, over the period of analysis, mortgages with WALs of 0-3 years offered the best risk adjusted performance, followed by mortgages with WALs of 3-5 years. It is noted that within an asset class the sector with the shorter maturity has a higher Sharpe ratio than the longer maturity sector.

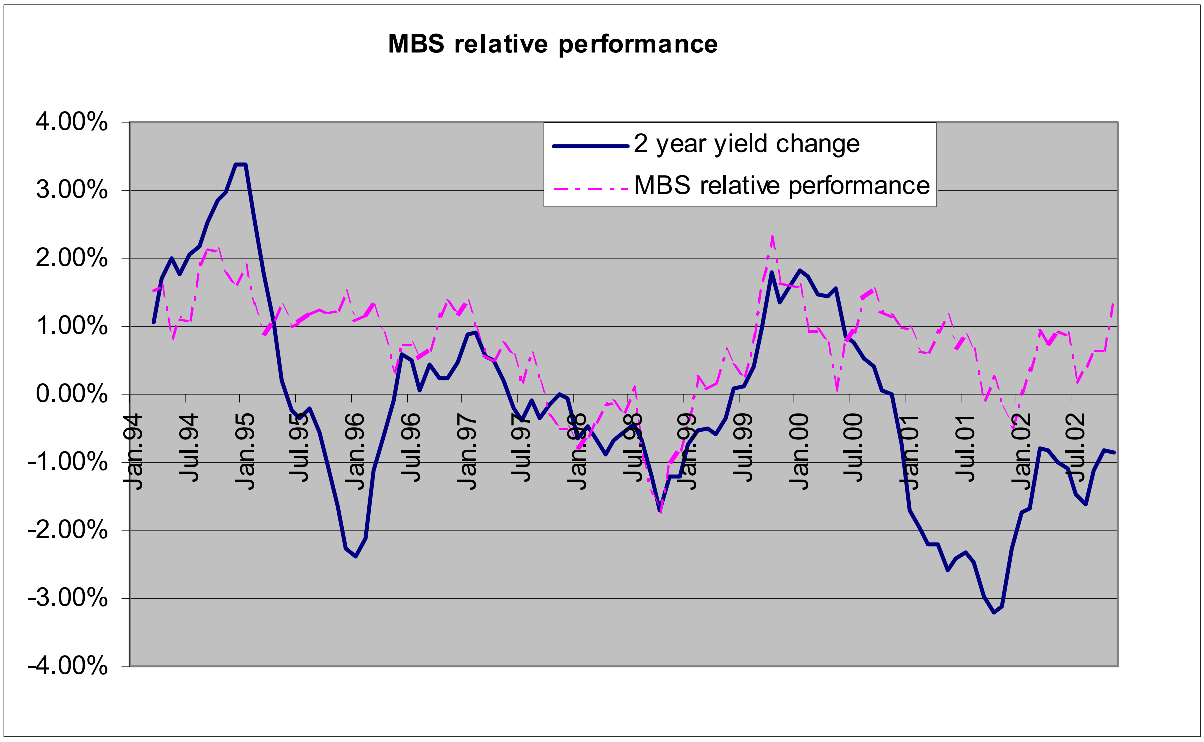
In addition to looking at performance measures, such as the Sharpe ratio, which are computed using long series of historic data, and which therefore represent performance characteristics of the asset class averaged over the economic cycle; we should consider how the asset classes perform under different phases of the interest rate cycle. Such an analysis, together with probabilities we assign to different interest rate scenarios will help us determine whether it is a good time to investing in a particular asset class.

Because of the negative convexity embedded in MBSs, an instantaneous change in yield results in MBS securities under-performing equivalent (in terms of effective duration) Treasury instruments. The MBS security will however offer a higher yield than the equivalent Treasury instrument. The extra income provided by the higher yield will compensate investors for the negative convexity of the MBS.

Other things equal: we expect MBSs to under-perform Treasuries during periods in which yields change significantly. Under these conditions the higher income provided by the MBSs is not sufficient to offset the MBS underperformance resulting from its negative convexity. This effect should be particularly pronounced when yields decrease significantly and results in what is known as “price compression” for fixed income instruments with embedded call options.

We now determine if these expectations are borne out in practice by looking at historic changes in the 2-year constant maturity Treasury yield over the previous year and the relative performance of MBSs over the corresponding period.

The graph below plots the year-to-date change of the two-year constant maturity Treasury yield and the year-to-date outperformance of the 0-3 year WAL mortgage sector relative to the 1-3 year Treasury sector.



From the graph it is seen that between October 1997 and February 1999 the year-on-year return on the MBS sector is below that of the Treasury sector. The worst underperformance of the mortgage sector was in the year leading up to October 1998, this period coincides with a rally in the Treasury market, during which the 2-year yield fell by more than 1.5%. This underperformance of the mortgage sector is consistent with price compression in a falling yield curve environment.

The graph however also shows that there are two other periods (the year leading up to January 1996 and the year leading up to September 2001) during which the two-year Treasury yield fell by more than 1.5%. Interestingly (surprisingly) during these periods the MBS sector continued to outperform.

We may conclude that even though changes in yield may be an important driver of MBS outperformance, there are other factors that influence MBS outperformance. The second half of 1998, for example, coincided with the Asian and LTCM crisis. These events resulted in investors demanding a premium to hold assets other than the most liquid Treasury instruments. It may be these considerations, rather than the decrease yields that caused the MBS sector to underperform Treasuries in this period.

One interesting point to note is that during periods in which the two-year yield increases, the MBS sector has always outperformed the Treasury sector.

4. Prepayment behaviour

A detailed description of how market practitioners model prepayments is provided by Hayre et. al. [2001]. Here we highlight some of the difficulties in modelling prepayments and stress the importance of prepayment models in the valuation of MBSs.

Modelling prepayment behaviour is essential for the valuation of MBSs. This is because the value of fixed income instruments is equal to the discounted sum of all future cash flows. For non-callable

Treasury bonds this value is simple to determine since all future cash flows are known and the appropriate discount factor to discount each cash flow can be determined from the Treasury zero-coupon curve.

Two factors complicate the valuation analysis of mortgages. Firstly, as a result of prepayments, the cash flow stream itself is not known. Secondly the discount rate appropriate to discount each payment in the cash flow stream is closely related to the cash flow stream itself. To understand this second point consider the case where interest rates go up, and consequently mortgage refinancing and prepayments decrease, the payments in the resultant (extended) cash flow stream should be discounted more aggressively as a result of the higher interest rates.

The strong relationship between prepayment rates and interest rates (which, in turn, determine the discount factors to be used to present-value future cash flows) means that we have a non-linear problem. We cannot, therefore, simply use an averaged discount curve to discount the anticipated future average cash flow stream. Instead we need to compute prepayment rates under individual realisations of possible future interest rate paths and present-value these cash flows using the discount factors appropriate for the specific interest rate path.

Prepayment projections, and in particular the refinancing component, which is the most interest rate sensitive component of prepayments, are therefore at the centre of modern MBS valuation and analysis techniques. Without a prepayment model MBS investors cannot project cash flows and consequently even basic valuation measures such as yield cannot be determined.

Hayre et. al. [2000] classify the causes of prepayment into four categories:

- Home Sales (housing turnover). Most mortgages are not assumable and the sale of the house results in prepayment of the outstanding mortgage balance.
- Refinancing. Refinancings are motivated by lower rates. Refinancing is the most volatile component of prepayments.
- Defaults. A default leads to the liquidation of the mortgage. Investors normally realize a default as a full prepayment of the outstanding balance of the loan.
- Curtailment and Full Payoffs. The homeowner may choose to pay principal at a rate faster than that stipulated in the mortgage contract in order to build up equity in their home.

Housing turnover
Home sales undergo annual cycles with peaks in the summer (school holiday) months and troughs in winter.

A home sale will usually result in the prepayment of the mortgage. However, mortgages underlying GNMA securities are assumable (the new homeowner can continue making payments on the existing mortgage), this differentiating factor should be accounted for when computing the turnover component of prepayments on GNMA securities. In particular, the new owners may choose to assume the old mortgage when current rates are above the contractual rate on the existing mortgage.

There is a *seasoning process* that affects housing turnover and prepayments in general. This process accounts for the fact that homeowners will normally be unwilling to sell a property for some time after having purchased it (because of the effort and costs involved). This process is modelled by the standard PSA prepayment ramp (see below).

Other factors that affect housing turnover are: economic growth and the business cycle; housing inflation; loan type (borrowers who select ARMs often expect to move again soon). Regional variations in demographics or economic conditions may result in different parts of the country experiencing significantly different home sales.

Refinancing
Refinancing is the most volatile component of prepayments. Because of the strong correlation between refinancing incentive and the discount factors needed to discount the cash flow stream resulting from the mortgage, modelling prepayments accurately is vital for proper MBS valuation.

Refinancing is a voluntary act of the borrower motivated by financial considerations: refinancing therefore invariably adds value to the borrower at the expense of the MBS security holder.

Traditional option analysis is of limited use in analysing refinancing because borrower behaviour represents an inefficient exercise of the prepayment option. There are various reasons for which borrowers do not exercise their prepayment option when it may seem to be in their financial interest to do so. They may simply be unaware of benefits to be had from refinancing, or their personal situation may have changed (the household income may have fallen or their creditworthiness may have deteriorated) so that they are not in a position to enter a new mortgage contract on favourable terms.

A borrower who has forgone previous opportunities to refinance is less likely to exercise future refinancing opportunities than a borrower who has consistently taken advantage of refinancing opportunities. The term *burnout* is used to describe this phenomenon, whereby a pool of mortgages that has experienced previous exposure to refinancing opportunities will, other things being equal, have a lower refinancing rate than a pool with no such prior exposure.

Burnout can be explained as the effect of changes in the composition of the pool caused by refinancing, which removes the most capable or most eager refinancers from the pool, so that the remaining borrowers have less of a tendency to refinance. MBS investors will be willing to pay a premium for pools that are “burned out”.

Quantifying prepayment rates

There are three common measures that are used to quantify the proportion of current outstanding mortgage principal that gets prepaid over a given interval:

- Single Monthly Mortality (SMM) is the proportion of the start-of-month balance that prepay in a given month.
- Constant Prepayment Rate (CPR) is the proportion of the start-of-period balance that prepay over an entire year.
- The Public Securities Association (PSA) standard prepayment model specifies a prepayment rate over the entire life of the mortgage. Prepayments are expressed as a percentage of the standard model: a prepayment rate of 200% PSA means that prepayment rates are twice as high as the rate specified by the standard (100%) PSA model.

Any one of these measures can be used to express a prepayment rate; they simply use different units to express the same quantity. Appendix III shows how these different measures are related.

Historical prepayment rates

In the absence of refinancing incentives prepayment rates tend to be about 100% PSA. The aggressive interest rate cuts seen recently in the U.S. have given rise to ample refinancing opportunities. This combined with the media effect trumpeting “historic” lows in mortgage rates has resulted in a wave of refinancing with prepayments on some pools exceeding 1000% PSA, at these high prepayment rates outstanding principal is returned to the security holder very quickly and these instruments have very low weighted average lives (WALs). The WAL of a mortgage is a measure of the average time for repayment of principal.

Appendix II shows the impact of prepayment rates on the WAL of a mortgage. Consider a 30-year, fixed-rate, level-payment, mortgage with a contractual rate of 6%:

- WAL = 19.3 years at 0% PSA
- WAL = 11.4 years at 100% PSA
- WAL = 2.3 years at 1000% PSA

5. Valuation techniques for MBSs

Some of the traditional valuation techniques used for fixed income instruments are of limited use in MBS valuation analysis. The reason for their limitation centre on the unpredictable cash flow stream associated with MBSs and in the assumptions inherent in these valuation techniques.

We first review some of the commonly used traditional valuation techniques and the assumptions inherent in them, an appreciation of which enables us to qualify the results produced by these techniques. We then describe a framework for MBS valuation modelling and some of the measures that can be obtained from the valuation model.

Traditional valuation measures

The yield on any financial instrument is the yield by which all of its associated cash flows need to be discounted in order to make the resultant present value of the security match the market price of the security. For MBSs this yield is called the *cash flow yield* (CFY). The problem in computing a CFY for an MBS is that the cash flows can only be estimated using a prepayment model.

In the computation of CFY (as in the computation of yield) it is assumed that the cash flows received before maturity can be reinvested at a rate equal to the CFY. It is also assumed that the security is held to maturity. Inherent in the computation of CFY is the assumption that the market discount cash flows received at different times at the same average CFY.

If interest rates unexpectedly decrease the yield on reinvested cash flows will be lower than that assumed when the CFY was originally computed. This results in an effective yield that is lower than the computed CFY. For MBSs this effect is amplified by the increased prepayments that result from increased refinancing when rates decrease.

Prepayment models may be used to determine the anticipated cash flow stream under the assumption that interest rates remain constant over the live of the security. The CFY determined under this assumption is known as the *static cash flow yield (SCFY)*.

Once the CFY has been computed the yield spread (over duration-matched Treasury issues) can be analysed in a historical context to determine if the MBS is historically rich or cheap. This yield spread is called the *nominal spread*. The spread differentials between discounts, currents, and premiums can also be analysed in a historical context to determine intra-MBS relative sector value.

To appreciate the limitations of the traditional yield spread as a valuation measure, consider the yield premium on two corporate bonds: the first a 20-year zero-coupon corporate bond, and the second a 10% coupon 20-year corporate bond. In determining the yield spread, the yield to maturity of each of these bonds is compared to a 20-year maturity benchmark Treasury bond. In an upward-sloping yield curve environment the yield to maturity on the zero-coupon corporate bond will be higher than that on the coupon-paying corporate bond. The opposite is true in a downward-sloping yield curve environment; in such an environment we can even envisage the situation where the yield to maturity on the zero-coupon corporate bond is below that of the coupon-paying benchmark Treasury bond. The traditional yield spread analysis is of limited value since the cash flow characteristics of the corporate bonds do not match that of the benchmark Treasury bond.

The proper thing to do is to compare non-Treasury bonds with a portfolio of Treasury securities that have the same cash flows as the non-Treasury security. In doing so we properly account for the term structure of the yield curve when we compare yields. The *static spread* of an instrument is the spread that must be added to each point of the Treasury zero-coupon curve so that when the resultant curve is used to present-value the future cash flows of the instrument we get its market price.

For MBSs, in particular, traditional nominal spread analysis is of limited use. This is because the principal of the MBS is returned long before maturity (through amortisation and prepayment) and therefore it makes little sense to compare the yield to maturity of the MBS with that of a Treasury bond of equal maturity. The static spread is more appropriate as a valuation measure for MBSs.

Valuation model

In this section we describe the general framework needed for the valuation of MBSs. A detailed description of how some of the components within this framework can be modelled is given in Appendix IV.

In building a framework to value MBSs we first look at the valuation of callable bonds. Many of the issues that need to be considered for the valuation of MBSs are the same as those needed for the valuation of callable bonds. To value MBSs we extend the valuation methodology used to value callable bonds to account for the prepayment behaviour of the borrowers, which results in a cash flow stream that is dependent of the path followed by the interest rate process.

Holding a callable bond is equivalent to holding a portfolio with a long position in an equivalent option-free bond and a short position in a call option on this bond:

$$\text{Callable bond value} = \text{non-callable bond value} - \text{call option value}$$

The value of the non-callable bond can be determined by discounting its cash flow stream using the appropriate zero-coupon yield curve. Whether, and when, the call option is exercised depends however on the evolution of interest rates[3]. We therefore need to model the evolution of interest rates in order to anticipate the exercise of the call option. Consequently, in order to value the callable bond we need a model for the evolution of interest rates.

To have a concrete example we use the binomial tree derived in the article by Black, Derman and Toy (BDT) as our model for the evolution of the short rate[4]. And we look at the valuation of a 10% coupon bond that matures in three periods. The term structure used by BDT to build their interest rate tree is given in the following table:

Year	Spot Rate	Short Rate	Yield Volatility
1	10%	10%	NA[5]
2	11%	12.01%	19%
3	12%	14.03%	18%

In the table we have included a column for the yield volatility this is superfluous to our needs but it is required for the generation of the binomial tree: the bigger the yield volatility the greater the dispersion of future short rates in the tree. The procedure to generate the tree of short rates is described in the BDT article.

The column with short rates contains the one-period forward rate. These rates are determined from the spot rate using the bootstrapping methodology. The value of the 10% coupon, 3-period non-callable bond can be determined in two ways:

- Either directly from the zero-coupon spot rate

$$\frac{\$10}{(1.1)} + \frac{\$10}{(1.11)^2} + \frac{\$10 + \$100}{(1.12)^3} = \$95.50 \quad (1)$$

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[3] The bond may also be called if the credit worthiness of the issuer improves and consequently it is able to refinance its debt at a cheaper rate.

[4] The short rate is the one-period rate at any instance in time. In the BDT model this rate is discretised onto a binomial tree. Assuming that we are on a given node in the tree, the short rate is the appropriate rate to discount payments that occur at either of the two nodes immediately to our right.

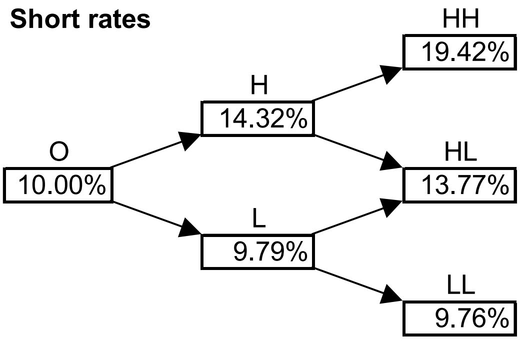
[5] The yield volatility at the first maturity point is not needed to construct the binomial tree.

- Or by successively using the forward short rate

$$\frac{\$10}{(1.1)}+\frac{\$10}{(1.1)(1.1201)}+\frac{\$10+\$100}{(1.1)(1.1201)(1.1403)}=\$95.50 \quad (2)$$

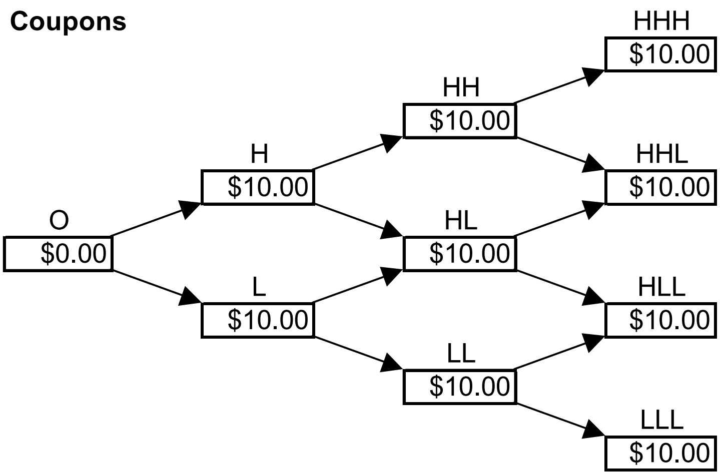
It is this second method that is used to discount cash flows through the binomial tree.

The BDT tree of short rates is shown in the following figure:



Each node in the tree corresponds to a possible future state of the world[6]. Multiple attributes, such as the short rate (as in the above figure) or the cash flow, may be specified or determined at each node. As we move from a node to one of the nodes on its right, we move forwards in time by one period. The above figure shows that at node O (time $t=0$) there is only one short rate, which is the current short rate at time $t=1$ there are two possible short rates (one labelled H and one labelled L, these letters correspond to the High and Low nodes respectively).

Before using the above short rate tree to value a callable bond we show how it can be used to value a non-callable bond and see that the result is consistent with the value obtained from equation (1) or (2). First let us specify the coupon flows that occur at each of the possible future states of the world. The following tree represents these coupon payments.



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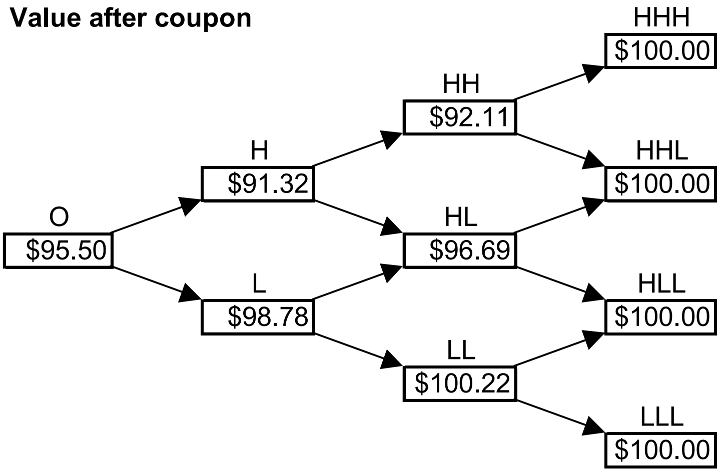
6 In fact, when path dependency is involved, the nodes in the tree are degenerate, each node representing multiple states of the world. The path taken to arrive at a node determines the exact state that the node represents. The short rate is however the same across all degenerate states of the world represented by a specific node.

The value of the bond (after coupon payment) at each node will reflect the present value of the remaining cash flows associated with the bond. We know what this value is at the last four nodes in the tree, it is simply the principal ($100) that will still be paid at each of these nodes. Below is the tree of bond values, after coupon payment, at each node in the tree:



In order to compute the other entries in the tree we need to work backwards in time (from right to left) through the tree. The value at each node is determined by discounting the bond values (plus coupon payment) from the two nodes immediately to its right. Let us see how this is done in practice for node HH and then for node H.

The two nodes immediately to the right of node HH are nodes HHH and HHL. The value of the bond at node HHH (before coupon payment) is $110. This is equal to the sum of coupon payment ($10) and the value of the bond after the coupon payment ($100).

If we are at node HH and (somehow) we know that the state of the world in one period time will be that represented by node HHH, we are then in a position to compute the present value of the known payoff we receive in one period time. The value of the bond plus coupon at node HHH ($110) is discounted at the short rate of node HH (19.42%) to determine the value of the bond at node HH after coupon payment:

$$\frac{\$110}{1.1942} = \$92.11 \tag{3}$$

In the BDT binomial tree there are equal probabilities of "up" and "down" movements at each node in the tree. As a result, a payoff of $110 at node HHH will only occur with a 50% probability and therefore it is worth only half of $92.11 (or $46.055) at node HH.

There is also a 50% probability that from node HH we end up on node HHL. Discounting the known value of the bond plus coupon from node HHL to node HH, and weighting this with the probability of moving from node HH to node HHL adds a further $46.055 to the value of the bond. The total value of the bond after coupon at node HH is therefore $92.11 (=$46.055+$46.055).

The procedure that has been used to determine the after-coupon value of the bond at node HH may now be used to determine this value at nodes HL and LL. Once these values determined we can move further back through the tree and determine the values at nodes H and L and finally right back to the origin of the tree. The after-coupon value of the bond at node H is:

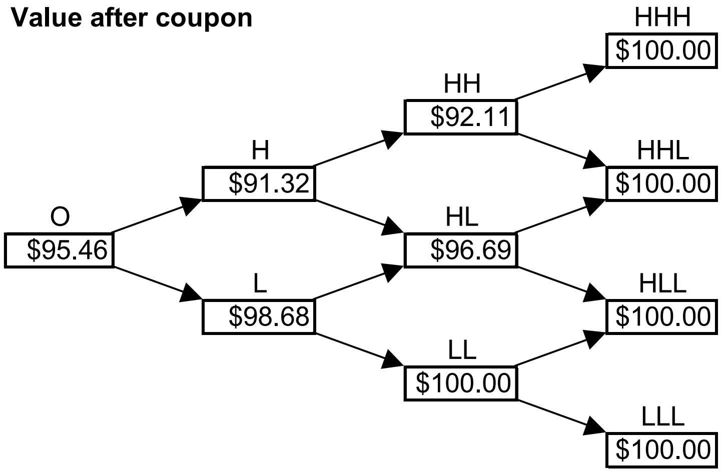
$$\frac{0.5(\$92.11 + \$10) + 0.5(\$96.69 + \$10)}{1.1432} = \$91.32 \tag{4}$$

Working backwards to the origin of the tree, we find that the value of the bond at node O is \$95.50. This matches the value obtained from equation (1) or (2).

Now we use the binomial tree to value a callable bond. The call option gives the issuer of the bond the option of calling the bond at par (\$100) immediately after each coupon payment.

The procedure for valuing the callable bond is similar to that for valuing the option free bond, there is only one small modification that needs to be made: If the after-coupon value of the bond at any node in the tree is greater than par, it will then be in the interest of the issuer to call the bond (since he can pay \$100 for a bond that is worth more than \$100). Assuming that the issuer exercises its call option optimally the after-coupon value of the bond is caped at \$100. If we look at the previous tree showing the after-coupon value of the bond, it is seen that the only node where the issuer will call the bond is node LL. Capping this value to \$100 will "feed back" through the tree and affect the bond value at all nodes leading up to node LL.

The tree below shows the after-coupon bond value for the callable bond.



The value of the callable bond is \$95.46. This is only \$0.04 below the value of the equivalent option-free bond. The value of the call option is therefore only 4 cents.

The value of the call option may seem rather low. We can however show that the computed value is consistent with the probability of the option being exercised and with financial gain to the issuer from exercising the option. The option is only exercised if node LL is reached, the probability of this occurring is 25% (a down followed by a down). The financial gain to the issuer from exercising the call option at node LL is \$100.22-\$100.00=\$0.22. The financial gain of exercising the call option weighted by the probability of the option being exercised is therefore $0.25 \times \$0.22 = \$0.055$. This gain will occur in two periods time, if we present-value this gain we get $\$0.055/(1.11)^2 = \$0.0446$, which is consistent with the computed option value.

The procedure that we have used to value bonds using the binomial tree is equivalent to selecting every possible path through the tree, computing the cash flows along each path, and discounting these cash flows (by successively using the short rate) to get a present value. The bond value is then the average across all possible paths. This procedure needs to be used if the value of the instrument is path dependent.

A tree with three periods has a total of $2^3 = 8$ different possible paths through it. Let us consider the cash flows, received by the holder of the callable bond, that occur along each of the eight possible interest rate paths. The table below lists all possible paths through the tree, the cash flows that occur at the various nodes on each path as well as the present value of the cash flows that occur along each path.

	Nodes crossed			Cash flow			
Path	t=1	t=2	t=3	t=1	t=2	t=3	PV
1	H	HH	HHH	$10	$10	$110	$90.29
2	H	HH	HHL	$10	$10	$110	$90.29
3	H	HL	HHL	$10	$10	$110	$93.93
4	H	HL	HLL	$10	$10	$110	$93.93
5	L	HL	HHL	$10	$10	$110	$97.43
6	L	HL	HLL	$10	$10	$110	$97.43
7	L	LL	HLL	$10	$110	$0	$100.17
8	L	LL	LLL	$10	$110	$0	$100.17
						Av.:	$95.46

As has been noted previously, this bond gets called at node LL, which is the second coupon payment date along paths 7 and 8, along these two paths the cash flow at bond maturity is consequently zero and the principal is returned along with the second coupon payment. The present value of the cash flows along a given path is obtained by discounting the cash flows successively using the short rate. Consequently, the PV of the cash flows received along path 1 is:

$$PV = \frac{\$10}{(1.1)} + \frac{\$10}{(1.1)(1.1432)} + \frac{\$110}{(1.1)(1.1432)(1.1942)} = \$90.29 \tag{5}$$

The average of the present values is $95.46. This matches the value of the callable bond we computed by working backwards through the binomial tree.

For each additional period added to the tree the number of possible paths doubles. A tree with 360 monthly periods is needed to value mortgages with 30 years to maturity. Such a tree has a total of 2^{360} possible paths through it. Due to the sheer size of this number it is not possible to value a callable bond as the average of its present value along all possible paths through the tree[7].

In order to get a good approximation of the price of the callable bond we could however randomly select a large number (about 1000) of paths through the tree, and compute the present value of the bond for each path. The average of these present values is our best estimate for the value of the bond[8]. This procedure in known as Monte Carlo simulation.

The value of a callable bond at each node is independent of the path the interest rate process took to get to the node[9]. This enabled us to construct the binomial tree of after-coupon bond values and use it to value the bond.

The value of some instruments however depends on the path that the interest rate process took to get to the node. This is the case of MBSs where high prepayments triggered by low interest rate environments (low paths through the tree) detract from the value of the MBS at latter nodes in the tree. For such instruments we cannot work backwards through the tree[10] to value the instrument but instead we must use Monte Carlo simulation.

Let us consider how we can use the binomial tree to value a MBS. We assume that interest rates evolve according to the same binomial tree that has been used so far. We furthermore model

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[7] It is however computationally inexpensive to value a callable bond by working backwards through a tree with 360 periods.

[8] Our best estimate thus computed will converge the "correct" value as the number of paths increases.

[9] This statement needs to be qualified: Paths 4, 6, and 7 all appear to reach node HLL. Along paths 4 and 6 the after-coupon value at this node is $100 whereas along path 7 this value is zero. Path 7 however ends at node LL when the bond is called and therefore this path does not reach node HLL.

[10] To work backwards through the tree we need to start by specifying the value of the instrument at each of the most distant nodes. This value simply cannot be specified since it is path dependent.

prepayments in a very simple way: we assume that prepayments are driven entirely by refinancing incentives, and that the only node where it is in the borrowers interest to refinance is node LL. We cannot however assume that all borrowers in a mortgage pool will refinance at node LL. This means that the payment made at node HLL, and subsequently the value of the MBS at this node, depends on the path taken to get there. The value of the MBS at node HLL will be lower (but non-zero) if we arrive at this node along the path going through node LL, where refinancing incentives exist, than if one of the two alternative paths leading to node HLL is taken. We cannot, a priori, determine the MBS value at node HLL and therefore we cannot work backwards through the tree.

Valuation of MBSs must therefore be done using Monte Carlo simulation. The procedure to be used is identical to that of valuing a callable bond using Monte Carlo simulation, the necessary steps are outlined below:

1. An interest rate path (through the binomial tree) is selected at random
2. The cash flows that occur along the selected interest rate path are determined
3. The cash flows are discounted successively using the short rate along the chosen path
4. Steps 1-3 are repeated in order to obtain the value of the MBS averaged across a representative sample of possible interest rate paths

A prepayment model is used in step 2 to determine the cash flows resulting from the MBS along the selected interest rate path.

There are various risk and return measures that may now be determined using the valuation model that we have presented here. Below we list some of these:

- Effective duration. This measure can be determined by re-valuing the MBS using an interest rate model that has been calibrated to a term structure of interest rates representative of the current term structure shifted by a small amount.
- Effective convexity. This measure can be determined in a similar manner to effective duration.
- Weighted average live. From the prepayment model we can predict how fast principal is paid down under the various interest rate scenarios used in the valuation model. In turn, this specifies the WAL of the mortgage pool.
- Total return analysis. Three components are needed to compute the total return over a specified holding period: the start-of-period value; the cash flow stream occurring during the holding period; and the end-of-period value. The valuation model described above may be used to determine each of these components.
- Option-adjusted spread (see below)

Option-adjusted spread (OAS)

The previous section describes a model that can be used to value MBSs. The price of a MBS in the market will usually be lower than its model value. This does not mean that MBSs are consistently under-priced in the market; the difference is normally viewed as a premium that investors require for assuming the prepayment risk inherent in holding a MBS[11].

The valuation model requires many assumptions in order to simplify “real world” phenomena. Assumptions are required both to model the stochastic interest rate process and also to determine

[11] Kupiec and Kah [1999] argue that the OAS is more appropriately interpreted as a spread-based measure of the unexplained portion of the MBS’ market price. They interpret consistently positive OAS measures as the result of miss-specifying the prepayment model in the risk-neutral Monte Carlo simulation process.

prepayment rates. This gives rise to *model risk*, with models of different practitioners giving different valuations.

The OAS is a measure of the yield spread that is needed to convert dollar differences between model value and market price; it is the spread that is needed to reconcile value with price. This spread is *option-adjusted* in the sense that the model cash flows reflect the prepayment option of the MBS.

To appreciate how the OAS influences the valuation of a bond, consider the valuation of the callable bond we examined earlier. Equation (5) shows how the value of this bond is determined if the interest rate process follows path 1. Discount factors of 0.909 (=1/1.1), 0.795 and 0.666 are used to discount cash flows that occur after one, two and three periods respectively. How does an OAS of 10 basis points change the valuation of this bond? To answer this question we need use this OAS to compute the new discount factors to discount the bond's future cash flows.

The OAS is a spread over the spot curve that corresponds to each interest rate path. In order to compute the correct, OAS-adjusted, discount factors for a specific interest rate path we cannot simply add the OAS to each short rate and re-compute the discount factors. The correct procedure is to first compute the spot curve corresponding to the specified interest rate path, then add the OAS to each maturity point of this spot curve, and finally compute the corresponding discount factors.

The two-year spot rate; $y_{Spot}(2)$, along path 1 can be determined from today's short rate and the short rate that is realised along path 1 one period from now:

$$\begin{aligned} &\left(1+y_{Spot}(2)\right)^2 = (1.1)(1.1432) \\ &\Rightarrow y_{Spot}(2) = 12.139\% \end{aligned} \tag{6}$$

To this spot rate we add the OAS of 10 basis points to get a two-year spot rate of 12.239%. This two-year (OAS-adjusted) spot rate corresponds to a two-year discount factor of $0.794 = (1.12239)^{-2}$. In a similar fashion we determine the one-year and three-year discount factors to be 0.908 and 0.664 respectively. Using these discount factors the value of the bond if interest rate path 1 is realised is:

$$0.908 \times \$10 + 0.794 \times \$10 + 0.664 \times \$110 = \$90.08 \tag{7}$$

At zero OAS and assuming that the interest rate process follows path 1, we had computed the value of the bond to be \$90.29. We see that an OAS of 10 basis points results in a fall of \$0.21 in the value of the bond if path 1 is realised.

Using Monte Carlo simulation we now select multiple random paths and determine the value of the bond at an OAS of 10 basis points. If the model value is still above the market price, we then know that an OAS greater than 10 basis points is needed to reconcile model value with market price. An iterative procedure is used to determine the OAS for which model value matches market price.

Scenario analysis and stress testing

Because of the multiple assumptions that are needed to compute any of the MBS valuation measures, it is important to stress test the results using scenario analysis.

In scenario analysis, the total return from holding a MBS is computed under various scenarios, this return can then be compared with that obtained from holding alternate assets over the same holding period.

Stress testing involves determining MBS performance under extreme scenarios of interest rate evolution and/or prepayment rate.

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Appendix I: Mortgage cash flow

In this appendix we analyse the cash flows of a fixed-rate, level-payment mortgage.

For a fixed-rate, level-payment mortgage scheduled contractual payments of size C are made on N payment dates. These payments cover interest payments and also serve to fully amortise the loan so that the outstanding balance after the Nth payment is zero. The size of the payments needed to fully amortise the loan depends on the number of payments to be made as well as on the contract rate, r [12], of the mortgage.

Let us define the following additional terms:

- B_0 The initial loan balance (the amount borrowed)
- B_t The balance remaining after payment on date t
- A_t The amount by which the loan amortises on date t
- I_t The interest payment made on date t

The contractual payments meet both interest payments and serve to amortise the loan:

$$C = I_{t+1} + A_{t+1} \tag{8}$$

The interest due on date $t+1$ may be determined from the contractual rate and the balance outstanding on date t:

$$I_{t+1} = rB_t \tag{9}$$

The outstanding loan balance on date $t+1$ is reduced from its previous value through amortisation:

$$B_{t+1} = B_t - A_{t+1} \tag{10}$$

Using equation (8) and then equation (9) we may write:

$$\begin{aligned} A_{t+1} &= C - I_{t+1} \\ A_{t+1} &= C - rB_t \end{aligned} \tag{11}$$

Substituting this last expression for A_{t+1} into equation (10) gives:

$$B_{t+1} = (1+r)B_t - C \tag{12}$$

Equation (12) shows that the outstanding balance on any date is equal to the future value of the previous outstanding balance, $(1+r)B_t$, less the scheduled payment, C.

We may invert equation (12) to get an expression that shows how the balance remaining on a given payment date is related to the balance remaining on the following payment date, the contract rate, and the scheduled contractual payment:

$$B_{t-1} = \frac{B_t}{1+r} + \frac{C}{1+r} \tag{13}$$

[12] The contract rate used here is the rate appropriate to the period between payments (usually one month). The rate in the mortgage contract will be the equivalent annualised rate, computed using simple compounding. If r is a monthly rate than the annualised rate in the mortgage contract will be $12r$.

For a fully amortising loan equation (13) is subject to the boundary condition $B_N = 0$. With this boundary condition we can successively back substitute to get:

$$\begin{aligned}
B_N &= 0 \\
B_{N-1} &= \frac{B_N}{1+r} + \frac{C}{1+r} = \frac{C}{1+r} \\
B_{N-2} &= \frac{B_{N-1}}{1+r} + \frac{C}{1+r} = \frac{C}{(1+r)^2} + \frac{C}{1+r} \\
B_{N-3} &= \frac{B_{N-2}}{1+r} + \frac{C}{1+r} = \frac{C}{(1+r)^3} + \frac{C}{(1+r)^2} + \frac{C}{1+r} \\
&\ldots\ldots \\
B_{N-M} &= C\sum_{i=1}^{M}\frac{1}{(1+r)^i} = C\frac{1-(1+r)^{-M}}{r} \\
B_0 \equiv B_{N-N} &= C\sum_{i=1}^{N}\frac{1}{(1+r)^i} = C\frac{1-(1+r)^{-N}}{r}
\end{aligned} \tag{14}$$

Equation (14) shows that when there are M remaining scheduled payments the outstanding balance on the loan is equal to the present value of an ordinary annuity that pays an amount C per period for the next M periods. The initial balance on the mortgage loan is simply the present value of an ordinary annuity that pays an amount C per period for the next N periods.

The payment per dollar of initial balance is:

$$\frac{C}{B_0} = \frac{r}{1-(1+r)^{-N}} \tag{15}$$

The payment per dollar of initial balance increases in r and decreases in N. If $N=1$, then $C/B_0 = 1+r$ (the payment on a one-period loan covers total principal plus interest). If $N=\infty$, then $C/B_0 = r$ (the payment covers periodic interest only).

We now determine the functional form describing the reduction in the balance outstanding. We use equation (12) to write out the first few terms that, by extension, lead to the general expression:

$$\begin{aligned}
B_1 &= (1+r)B_0 - C \\
B_2 &= (1+r)B_1 - C = (1+r)^2 B_0 - (1+r)C - C \\
B_3 &= (1+r)B_2 - C = (1+r)^3 B_0 - (1+r)^2 C - (1+r)C - C \\
&\ldots\ldots \\
B_t &= (1+r)^t B_0 - C\sum_{i=0}^{t-1}(1+r)^i = (1+r)^t B_0 - C\frac{(1+r)^t - 1}{r}
\end{aligned} \tag{16}$$

The outstanding balance may be interpreted in terms of the future value of an ordinary annuity. The first term on the right hand side of the last line of equation (16) is the future value of the initial balance of the mortgage compounded at a rate r per period. From this value we subtract the future value of an annuity paying C per period (the second term on the right hand side of the last line of equation (16)) in order to get the outstanding balance of the mortgage.

Equation (16) may be simplified by substituting from equation (15) for C. This gives:

$$B_t = B_0\left\{(1+r)^t - \frac{(1+r)^t - 1}{1-(1+r)^{-N}}\right\} \tag{17}$$

An example will clarify the material presented so far. Let us assume that a homeowner takes out a 30-year \$100,000 initial balance mortgage at a contract rate of 6% pa. At the end of 360 monthly payments the mortgage is fully paid off. The known parameters are:

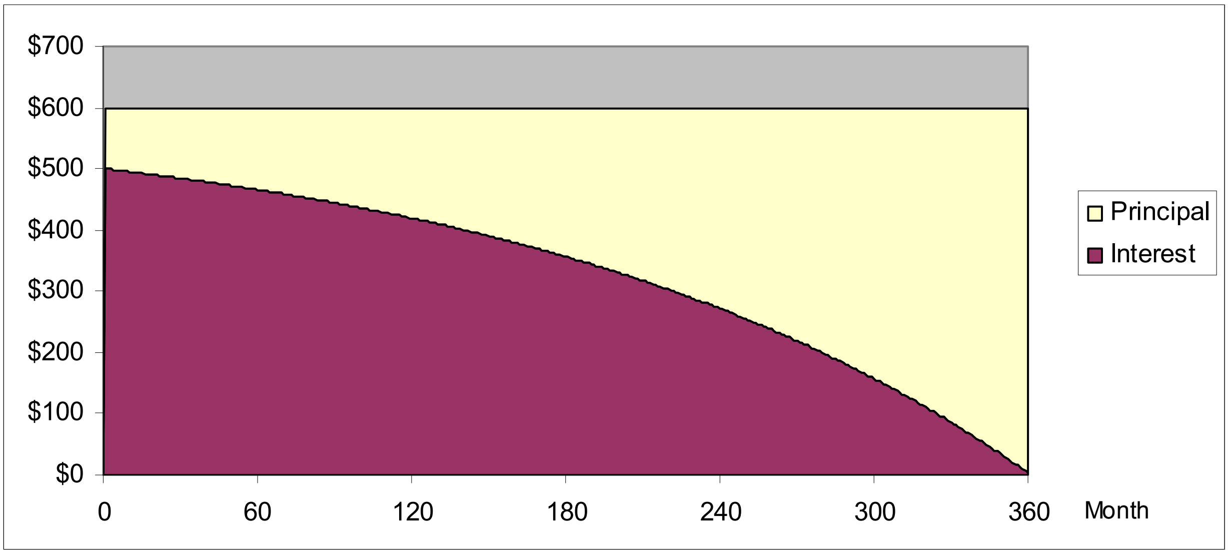
- $B_0 = \$100{,}000$
- $N = 360$
- $r = 0.06/12 = 0.005$

From equation (15) we determine the contractual payments to be \$600 per month.

The following table shows the first few month and the last few months of the mortgage contract. The table shows the balance outstanding as well as the split between interest and principal payments.

Month	Balance	Interest	Principal
1	\$99,900.45	\$500.00	\$99.55
2	\$99,800.40	\$499.50	\$100.05
3	\$99,699.85	\$499.00	\$100.55
4	\$99,598.80	\$498.50	\$101.05
…			
356	\$2,368.52	\$14.77	\$584.78
357	\$1,780.81	\$11.84	\$587.71
358	\$1,190.17	\$8.90	\$590.65
359	\$596.57	\$5.95	\$593.60
360	\$0.00	\$2.98	\$596.57

The division between interest and principal payments across the entire live of the mortgage is shown in the following graph.



From the graph it is clear that in the early years of the mortgage the payments primarily service interest payments whereas in the latter years of the mortgage the payments primarily pay down the outstanding balance on the mortgage.

Since interest payments are proportional to the outstanding balance this part of the graph also gives a good visual representation of how the outstanding balance decrease over the mortgage life.

Appendix II: Effect of prepayments on mortgage cash flows

In this appendix we show how the cash flow stream of a mortgage pool changes when prepayments are made. The notation that was introduced in Appendix I remains valid.

If a fraction η_{t+1}[13] of the mortgages remaining in the pool on date t prepay on date $t+1$ then the cash flow resulting from prepayments, P_{t+1}, is:

$$P_{t+1} = \eta_{t+1}\{(1+r)B_t - C\} \tag{18}$$

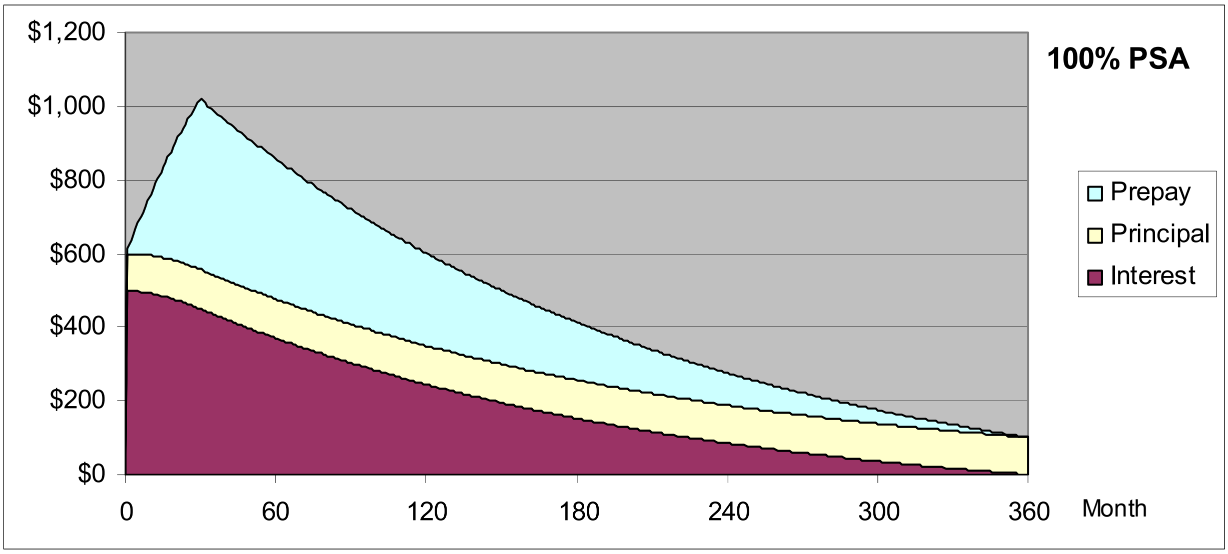
The term in curly brackets is the outstanding balance on date $t+1$ after the scheduled contractual payment but before prepayments (our notation remains consistent in that, B_t represents the balance outstanding on date t after all payments, both scheduled and prepayments).

Equation (12), which determines how the outstanding balance evolves, is now modified to account for prepayments:

$$B_{t+1} = (1+r)B_t - C - P_{t+1} \tag{19}$$

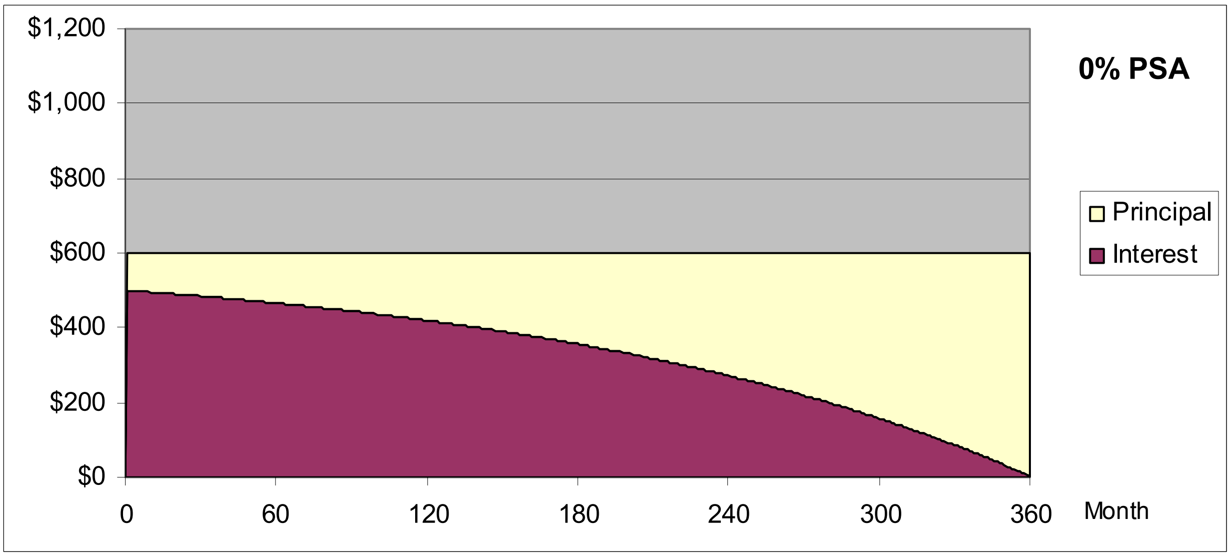
To determine how the outstanding balance evolves we need to model η_t. The prepayment rate will typically be a complex function of the mortgage characteristics, interest rate and macroeconomic environments. For the results presented here we assume that η_t follows the standard 100% PSA model, this implies a linear increase from a prepayment rate of zero at mortgage origination to a prepayment rate corresponding to 6% CPR after 30 months and which then remains constant at 6% CPR at longer maturities.

The graph below shows the total payments on any given month and how these are divided between interest payments, scheduled repayment of principal, and prepayment of principal. The characteristics of the mortgage match those in appendix I (an initial balance of $100,000, a contract rate of 6%, and a maturity of 30 years).



[13] If the period between scheduled payments are monthly periods then η is commonly called the Single Monthly Mortality (which is denoted by R_{SMM} in appendix III).

For comparison below we plot the same graph under the assumption of zero prepayment (this is the same graph at that in appendix I).



Principal payments
The total repayment of principal summed across all months is the same in both of the above graphs. In the first graph this repayment is made up of both scheduled repayments and prepayments, in the second graph the repayment of principal is made up entirely of scheduled repayments. This implies that the sum of the areas of the prepayment component and the scheduled principal repayment component in the first graph is equal to the area of the scheduled principal repayment component in the second graph.

At 100% PSA, the repayment of principal is accelerated by prepayments, reaching a peak after 30 months. In contrast, if there are no prepayments, the rate of repayment of principal keeps on increasing throughout the life of the mortgage.

Determining WAL
The speed at which the initial balance is repaid by the borrower determines the weighted average life (WAL) of a mortgage. The WAL is a measure of the average life for which the initial balance remains outstanding.

On date t the amount of principal paid is equal to the sum of scheduled amount by which the loan is amortised, A_t, plus the amount prepaid, P_t. It is this total principal that gets paid on a given date that is used to determine the weighting factors, w_t, for computing the WAL of the mortgage:

$$w_t = \frac{A_t + P_t}{B_0} \tag{20}$$

The total principal payments across all payment dates must equal the initial balance of the mortgage. Hence:

$$\sum_{t=1}^{N} w_t = 1 \tag{21}$$

The WAL is given by:

$$WAL = \sum_{t=1}^{N} t w_t \tag{22}$$

For our 30-year, 6% coupon mortgage the WAL of the non-prepaying mortgage pool is 19.3 years and the WAL of a pool that prepays at 100% PSA is 11.4 years. In the case of a mortgage pool that prepays at 1000% PSA the WAL falls to 2.3 years.

Interest payments

Even though prepayments have no effect on the total amount of principal paid by the borrower (which must equal the initial balance in all cases), they do have an effect on the total amount of interest payments made by the borrower. Faster prepayments lead to smaller outstanding balances and consequently smaller interest payments on subsequent payment dates. This effect is clearly seen by comparing the two graphs in this appendix: the area of the interest component is much smaller in the case of 100% PSA (total interest payments of $68,181) than it is in the case of no prepayments (total interest payments of $115,838).

Appendix III: Quantifying prepayment rates

The three measures commonly used to quantify prepayments were introduced in the presentation. Here we give further details of how these measures are related.

The Single Monthly Mortality (SMM) is the proportion of the start-of-month balance that prepay in a given month. Let us denote the SMM rate by R_{SMM}. The fraction of the start-of-month balance that remains in the pool after the prepayments made during the month is:

$$1 - R_{SMM}$$

If the SMM rate remains constant over a 12-month period then the fraction of the start-of-period balance that remains in the pool after the 12-month period is:

$$\left(1 - R_{SMM}\right)^{12}$$

This 12-month "survival" fraction may now be converted into the proportion of the start-of-period balance that prepay over the 12-month period. This measure is known as the Constant Prepayment Rate (CPR). Let us denote this rate by R_{CPR}:

$$R_{CPR} = 1 - \left(1 - R_{SMM}\right)^{12}$$

This equation shows us how to determine CPR from SMM. Inverting gives us an equation to obtain SMM from CPR:

$$R_{SMM} = 1 - \left(1 - R_{CPR}\right)^{1/12}$$

It is important to realise that even though CPR represents an annualised measure it is simply a scaled version of the SMM (monthly) measure. The CPR can therefore be used to specify the prepayment rate that applies to one particular month, as such the CPR may vary from month to month.

The prepayment rate specified by the PSA prepayment model is a function of the age of the mortgage. The prepayment rate increase linearly over the first 30 months of the mortgage life and remains constant thereafter. If a mortgage prepays at a rate of 100% PSA, then the CPR for the month that the mortgage in n months old is:

$$R_{CPR} = 6\% \times Min\left(1, \frac{n}{30}\right)$$

When the mortgage is 15 months old the mortgage will prepay at a CPR of 3% and when it is 40 months old it will prepay at a CPR of 6%.

If a mortgage is prepaying at a CPR of 9% when it is 15 months it is said that the mortgage is prepaying at 300% PSA (since 100% PSA corresponds to a CPR of only 3%).

It is seen that the three measures that are commonly used to quantify prepayment rates are in effect just three different units of measure to express the same underlying phenomenon.

Appendix IV: building a valuation model

In this appendix we describe how we have built a model for the valuation of securities backed by the cash flow from pools of fixed-rate, level-payment mortgages. The general framework needed to build an MBS valuation model has been described in the presentation; we avoid repeating this here and instead look into the details of the specific model that we have built.

The two components that are needed for the valuation of MBS are:

1. An interest rate model
1. A prepayment model

These are discussed in turn below.

Modelling the evolution of interest rates

A model for the evolution of interest rates is needed to determine mortgage prepayment rates. The current level of the yield curve is one of the major determinants of the rate on new mortgages (the current-coupon mortgage rate). The current-coupon rate will, in turn, determine the incentive for borrowers to refinance.

The interest rate model is also used to discount future cash flows that occur in the Monte Carlo simulation that is used to value the MBS.

We have used the Black, Derman and Toy (BDT) binomial tree to model the evolution of the short rate (the interest rate appropriate to discount cash flows that occur next period). The term structure of the yield curve that is used to calibrate the binomial tree is the current swap curve and its volatility (which has been determined from the last year of historic data). Full details on how to construct the binomial tree of short rates are given by Black, Derman and Toy [1990].

Mortgages have monthly cash flows: to be able to analyse mortgages that mature in 30 years, we need a tree with monthly periods extending out to 30 years (a total of 360 periods).

BDT specify a methodology to construct a tree of short rates. At each node in the tree we also know the shape of the entire zero-coupon yield curve (up to tree maturity). This can be determined from the short rate at the node as well as the short rates on all (accessible) nodes to its right. We use this zero-coupon yield curve to model the current coupon rate (see below).

Modelling prepayments

In our prepayment model home sales (turnover) and mortgage refinancing are the sole determinants of prepayment rates. Defaults, curtailments and full payoffs also influence prepayment rates but the effect of these is small compared to the effect of refinancing and turnover.

Let η_N represent the fraction of homeowners with mortgages remaining from date $N-1$ that choose to prepay at date N. If $\eta_{T,N}$ is the turnover component, or fraction of homeowners who prepay following the sale of their home and $\eta_{R,N}$ the fraction of homeowners who refinance, it follows that:

$$\eta_N = \eta_{T,N} + \eta_{R,N} \tag{23}$$

$\eta_{R,N}$ will primarily be driven by the financial gain (Γ_N) that the homeowner obtains from refinancing the mortgage (quantifying Γ_N is described later).

Let us assume that the reasons for prepayment occur sequentially. In the first instance prepayment follows the sale of the home, this type of refinancing is relatively insensitive to Γ_N. In the second instance prepayment follows refinancing, this type of prepayment is strongly driven by Γ_N.

As a result of the sequential nature of the factors driving prepayments, $\eta_{R,N}$ will be proportional to the fraction ($1-\eta_{T,N}$) of homeowners that remain after prepayment resulting from home sales.

Both $\eta_{T,N}$ and $\eta_{R,N}$ depend on the extent that the mortgage has aged. The reason for this is that for some time after entering into a new mortgage agreement the homeowner will be reluctant to go through this process again. Let α_N represent the extent to which the mortgage has aged; a value of 0 is representative of a new, un-aged, mortgage and a value of 1 is representative of a fully aged mortgage. We model α_N as follows:

$$\alpha_N = 1-\exp(-\Phi N) \tag{24}$$

Φ represents the speed at which mortgages age. The PSA model specifies that a mortgage is fully aged after 30 months, a value for Φ of 1/30 results in a similar aging speed.

$\eta_{T,N}$ is modelled in a very simple manner: we assume that aggregate home sales (across all mortgage pools) are constant[14]. The turnover component of prepayments is proportional to the degree to which the pool has aged:

$$\eta_{T,N} = T\alpha_N \tag{25}$$

T is the turnover speed for a fully aged mortgage.

The refinancing component of prepayments, $\eta_{R,N}$, will be influenced by the efficiency, ν_N, with which homeowners take advantage of refinancing opportunities. When a pool is exposed to refinancing opportunities the keenest refinancers in the pool will exit the pool, the remaining borrowers are less likely to take advantage of future refinancing opportunities. This process is known as *burnout* and results in the refinancing efficiency being a function of past refinancing opportunities (the efficiency decreases as the number of past refinancing opportunities increases). We do not model the effect of burnout and keep the refinancing efficiency constant ($\nu_N \equiv \nu$).

The refinancing component of prepayments is specified as follows:

$$\eta_{R,N} = (1-\eta_{T,N})\alpha_N \text{Min}\left[\text{Max}\left[\nu \times \Gamma_N, 0\right], 1\right] \tag{26}$$

Equation (26) ensures that when the refinancing gain is negative mortgage refinancing is zero (negative refinancing is not possible). When the product $\nu \times \Gamma_N$ is larger than 1 the refinancing rate is capped at $(1-\eta_{T,N})\alpha_N$, for a fully aged mortgage pool this implies that $\eta_N = \eta_{T,N} + \eta_{R,N} = 1$ and all mortgages remaining in the pool prepay on date N.

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[14] Seasonal variations in the turnover rate are not accounted for.

In order to determine the gain to be had from refinancing, we assume that when a borrower refinances he enters into a new mortgage agreement with a maturity equal to the residual maturity on his previous loan. Let us define the following terms:

- B = Outstanding loan balance
- r_c = Rate on the current loan
- r_n = Rate on the new loan (the current coupon rate)
- M = The residual maturity of the mortgage

Using equation (14) we can determine the monthly contractual payments on the current loan:

$$C = \frac{Br_c}{1-(1+r_c)^{-M}} \tag{27}$$

Discounting these contractual payments at the rate available on an equivalent new mortgage gives the present value of the current contractual payments, PV_c :

$$PV_c = C\frac{1-(1+r_n)^{-M}}{r_n} = \frac{Br_c}{1-(1+r_c)^{-M}}\frac{1-(1+r_n)^{-M}}{r_n} \tag{28}$$

We know that the present value, PV_n, of the new loan, at a coupon rate r_n, is simply equal to the outstanding balance on the loan, B .

There are various costs associated with refinancing. Some of these are fixed costs that do not vary with the size of the mortgage other costs are variable and are proportional to the size of the loan. We define F as the fixed cost and $B \times V$ as the variable cost of refinancing. We may now determine the saving that the borrower realises from refinancing:

$$SAV = PV_c - PV_n - BV - F = \frac{Br_c}{1-(1+r_c)^{-M}}\frac{1-(1+r_n)^{-M}}{r_n} - B - BV - F \tag{29}$$

The percentage (of outstanding balance) saving that the borrower realises from refinancing is:

$$\%SAV = \frac{r_c}{1-(1+r_c)^{-M}}\frac{1-(1+r_n)^{-M}}{r_n} - 1 - V - \frac{F}{B} \tag{30}$$

Either SAV or $\%SAV$ can be used to quantify the refinancing gain Γ .

The refinancing gain is a function of the current coupon rate on new mortgages, r_n . The current coupon rate is determined from the zero-coupon yield curve (whose evolution we have modelled), by first determining the yield to maturity on an ordinary annuity that matures when the mortgage matures (this will equal the yield to maturity on a non-prepaying mortgage) and then adding a spread to this yield to get the current coupon rate. The spread in part covers the servicing fees but, more importantly, it compensates investors for assuming prepayment risk[15].

The only parameter needed by the prepayment model that we have not specified how to model is the efficiency, ν , with which borrowers take advantage of refinancing opportunities. This parameter is

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[15] We have used a spread of 2.5%. On March 5 2003 the current coupon for a 30-year mortgage was 5.37% and that for a 15-year mortgage was 4.67%. These translate into spreads of 2.63% and 2.28% respectively for 30-year and 15-year mortgages (additional study is required to analyse how these spreads vary with maturity and how stable they are).

not modelled; it is instead determined through an iterative process so that the model value of the MBS matches the observed market price of the MBS.

Determining the refinancing efficiency in this manner will necessarily result in a model price that equals the market price, and therefore an OAS of zero. To the extent that the model price matches the market price, the payment made at each node in the binomial tree (which has been constructed to be in accordance with risk-neutral evolution of the short rate) is the expected value (in a risk-neutral world) of the possible payments that can be made at the node.

Our simple prepayment model will always result in an OAS of zero. We cannot therefore interpret the OAS as the risk premium that investors demand for holding prepayment risk. We may nevertheless use our prepayment model together with our valuation framework to perform scenario and risk analysis.

Determining the cash flow stream

Now that we have a model that enables us to determine the prepayments that get made during the lifetime of a mortgage pool, we use the modelled prepayment rate to determine the cash flow stream that results per \$1 initial balance in the pool.

We have defined η_N as the fraction of homeowners with mortgages remaining from date $N-1$ that prepay on date N (this is the single monthly mortality rate). Of the total original number of homeowners in the pool, prepayments result in only a fraction, γ_N, remain after prepayments made up to and including date N. We can determine the fraction of the original pool that are still in the pool on a particular date from the fraction remaining on the previous payment date and the fraction of homeowners that prepay:

$$\gamma_N = \gamma_{N-1}(1-\eta_N) \tag{31}$$

We know that $\gamma_0 = 1$ since at inception all borrowers are still in the pool.

Let us define the following additional terms:

- C The scheduled monthly contractual payments per \$1 initial balance
- $CF_{SP,N}$ The cash flow from scheduled payments on date N
- $CF_{PP,N}$ The cash flow from principal prepayment on date N
- $B_{B,N}$ The balance outstanding before prepayment, but after scheduled payments, on date N
- $B_{A,N}$ The balance outstanding after prepayments and after scheduled payments on date N
- r The contract coupon rate on the mortgage

The scheduled contractual payment made on date N can be determined from the fraction of homeowners that still remain in the pool after prepayments on date $N-1$:

$$CF_{SP,N} = \gamma_{N-1} \times C \tag{32}$$

The outstanding balance before prepayments on date N can be determined from the outstanding balance after prepayments on date $N-1$ (see equation (12)):

$$B_{B,N} = (1+r)B_{A,N-1} - C_{SP,N} \tag{33}$$

The cash flow resulting from prepayments on date N is equal to the outstanding balance before prepayments times the fraction of homeowners that prepay:

$$CF_{PP,N} = B_{B,N} \times \eta_N \tag{34}$$

The outstanding balance after prepayments on date N is equal to the outstanding balance before prepayments less the prepayments made:

$$B_{A,N} = B_{B,N} - CF_{PP,N} \tag{35}$$

Our prepayment model provides us with η_N and γ_N for all N. We also know what the values of C and r are for the specific mortgage pool that we are analysing. Furthermore, since we are determining the cash flow stream per $1 initial pool balance, we know that $B_{B,0} = B_{A,0} = 1$.

We may now use equation (33) to determine $B_{B,1}$, we then use equation (34) to determine $CF_{PP,1}$, we finally use equation (35) to determine $B_{A,1}$. Starting with the value of $B_{A,1}$ that we have just determined we may use equations (33), (34) and (35) once more to determine $B_{B,2}$, $CF_{PP,2}$ and $B_{A,2}$ respectively. This process is repeated up to maturity to give us, in particular, the entire cash flow stream associated with the mortgage pool[16].

[16] The total cash flow stream resulting on date N is equal to the sum of the scheduled payments and the principal prepayment $CF_N = CF_{SP,N} + CF_{PP,N}$.