BANK FOR INTERNATIONAL SETTLEMENTS

Modeling credit risk in an in-house Monte Carlo simulation

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Presentation overview

I. **Why model credit losses in a simulation ?**

II. Scope of our Monte Carlo model

III. Review of the simulation steps: Observations on implementation, modeling and input-data

I. Why model credit losses in a simulation ?

- Some risk measures require knowledge of the "tail" region of the credit loss distribution of a portfolio. This is the region where severe future portfolio losses appear.

  Examples: Economic Capital / VaR and Expected Shortfall.

- Problem in credit risk: The distribution of portfolio credit losses is unknown.

- We may be able to calculate key parameters (eg, "unexpected loss") of the portfolio loss distribution. This is not enough to estimate the risk of severe losses.

Example: Economic capital (EC) / VaR

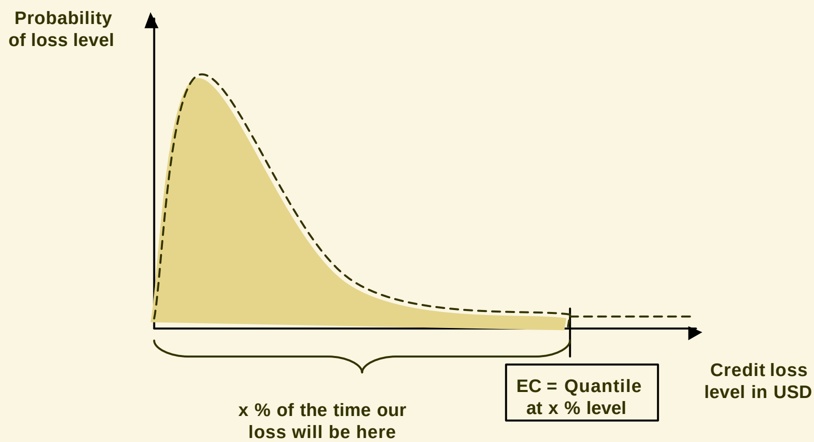
- **Definition:** The level of potential credit loss that will not be exceeded at a given confidence level within a given time horizon (usually 1y).

- Having an equal amount of capital set aside, a bank will survive the coming year with the given level of confidence.

- The severe (even if unlikely) portfolio losses determine survival.

- Severe portfolio losses appear in future states of the world where many adverse events happen at the same time.

Distribution of credit losses: Unknown



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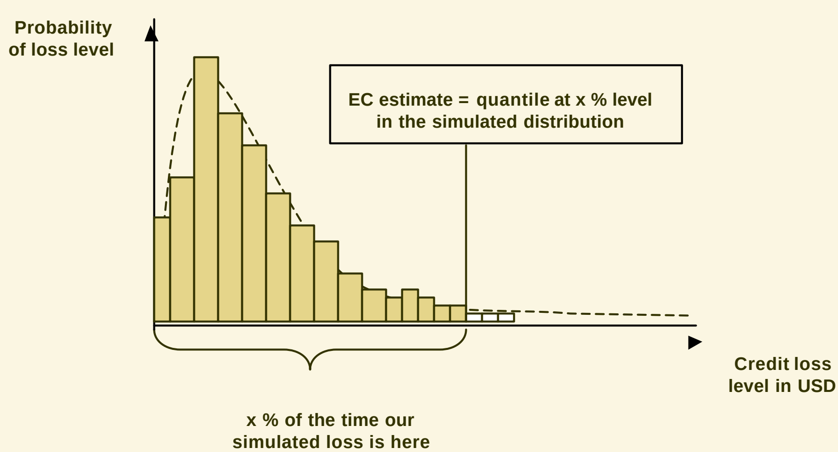
Distribution of credit losses: Simulated



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Credit loss types: Default and change of rating

- **Default:** With a certain probability an obligor defaults within 1y and

$$Loss = Exposure * LossGivenDefault$$

- **Change in credit rating:**
  - With a certain probability an obligor can change his rating within 1y.
  - For long term instruments this has the implication:

$$Loss\ or\ Gain = F(Transaction, OldRating, NewRating)$$

.

Portfolio composition: “Liquid” and “public”

- Instruments: Tendency to “liquid”
  - We will assume tradable instruments (rating migration).
  - Bonds, Swaps, FX-Forwards…
- Obligors: tendency to “publicly traded”
  - We will assign asset return correlations to obligor pairs.
  - We will assume that transactions with them have tradable character (rating migration).

➢ *These are not absolute necessities, but the approach is more intuitive if they are true.*

Presentation overview

I. Why model credit losses in a simulation ?

II. Scope of the Monte Carlo model

III. Review of the simulation steps:
- **Observations concerning implementation, modeling and input-data**

III. Review of the simulation steps

1. **Simulate the events of credit movement (to default or a new rating)**
   1.1 Individual credit movement of an obligor
   1.2 Joint credit movement of obligors

2. Simulate the loss under default
   2.1 Loss Given Default
   2.2 Exposure

3. Simulate or observe the change in value under a new rating

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1. Simulate the events of credit movement

Standard approach:

1.1 Individual credit movements: Set up internal rating grades and a 'migration matrix'. A migration matrix contains the probabilities of moving from an old rating to a new rating within 1y.

1.2 Joint credit movements: Based on asset returns. Consider the individual move as driven by a variable (asset returns) that offers 'observable' correlations between obligors.

➢ *Putting the two together will allow for a modeling of joint credit movements*

1.1 Individual credit movement: Set up internal rating grades and a migration matrix

- The easier way: Anchor internal grades to the grades of rating agencies and use agency data for the migration matrix. The internal grading of an obligor can still differ from his agency rating.
- Alternative: Define a "new" rating grade system along with a corresponding migration matrix (eg, in case of sufficient in-house data or a portfolio not fitting to rating agency systems).
- Assign to each obligor (legal entity) his current internal rating grade.

➢ *This is a first basic decision on the structure of the implementation.*
➢ *An internal grading system might already exist.*
➢ *Now we are ready to model individual credit movements.*

Example: Migration matrix for 2 rating grades

		New Rating			
		Rating "A"	Rating "B"	Default	Sum
Old Rating	Rating "A"	90%	9.9%	0.1%	100%
	Rating "B"	9%	90%	1%	100%

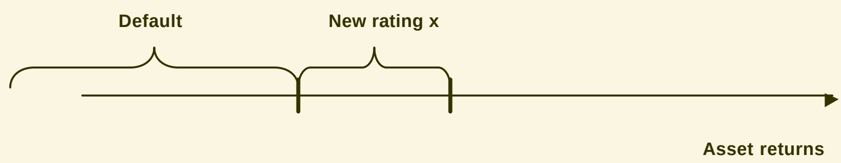
1.2 Joint credit movements: Based on Asset returns

- Observations of joint defaults or rating migrations are very scarce. Joint credit movement needs to be linked to something more observable.
- The Merton Model as background: The 1y asset returns determine default or rating migration of an obligor over 1y.
- We need a distribution assumption for multi-variate asset returns: Multi-variate normal is standard.
- We need a correlation matrix for the asset returns of our obligors: Vendor, derived from stock prices, educated guess…

1.2 Joint credit movements: Scenario generation

- For each old rating: Determine a set of ranges for asset-returns that correspond to new ratings / default (“Z-scores”).
- Generate a sample of the multi-variate (and correlated) 1y asset-returns; one asset-return for each obligor.
- For each obligor: Take the set of ranges for its old rating and observe which one is hit by the generated asset-return.

Z-Scores: Ranges for asset returns



$$P(\textit{credit movement}) = P(\textit{asset return hits range})$$

Migration matrix

Marginal distribution of asset returns

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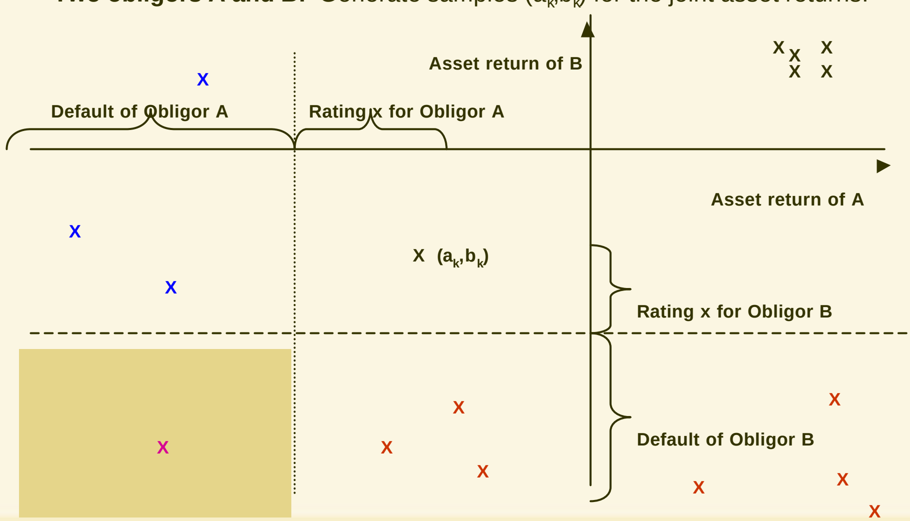
Two obligors A and B: Generate samples (a_k, b_k) for the joint asset returns.



Asset returns: Data and modeling problems

- Asset-returns are not directly observable. Asset-returns can be derived from stock prices (if there *are* stocks on the obligor).
- Deriving asset-returns from stock prices is not easy. For obligors with low leverage stock returns can serve as a proxy.
- If there are no stocks, it is important to use a consistent way of defining correlations.
- If we ‘guess’ a correlation matrix we have to ensure it *is* a correlation matrix.
- It is not clear that asset returns follow a (multi-variate) normal distribution in the adverse part of their (multi-variate) tail. The normal distribution is **not** a conservative assumption !

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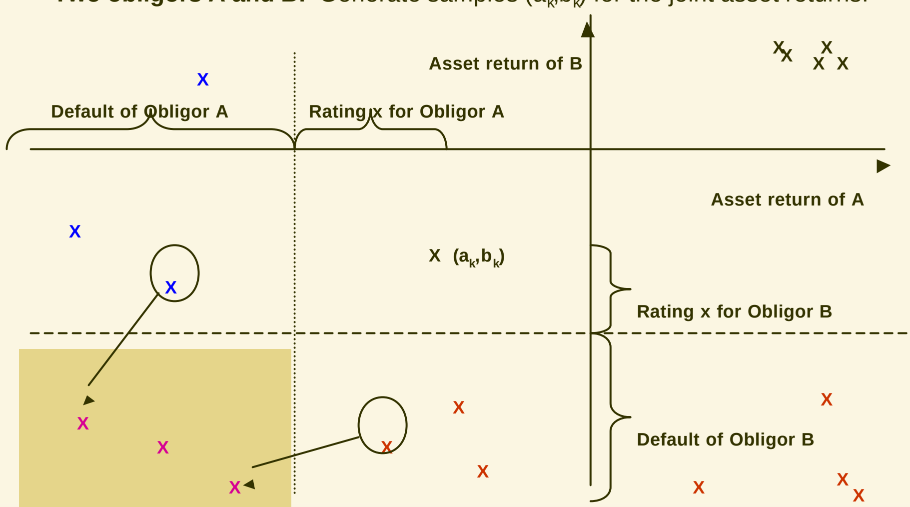
Two obligors A and B: Generate samples (a_k, b_k) for the joint asset returns.



III. Review of the simulation steps

1. Simulate the events of credit movement (to default or a new rating)
   1.1 Individual movement of an obligor
   1.2 Joint movement of obligors

2. **The loss under default**
   2.1 Loss Given Default
   2.2 Exposure

3. The change in value under a new rating

2. The loss under default

$$Loss = Exposure * LossGivenDefault$$

2.1 Loss given default (LGD): The fraction of exposure that is lost in case of default.

2.2 "Exposure": The amount at risk lost over the next 1y.

2.1 Loss given default

Standard approach: LGD is simulated as a random variable.

- Standard assumption:
  - LGD follows a Beta distribution with certain parameters.
  - The LGD of each obligor is independent of all credit movements and all other LGDs.
- The parameters for the LGD distribution will be typically assigned on a transaction level.

- *This is a second basic decision for the implementation structure: LGD parameters by seniority, instrument type, industry…*
- *Random LGD must be drawn on an obligor level.*
- *Our model does not reflect that LGD goes up when default rates go up.*

2.2 Exposure

- Our loss formula requires an input for “exposure”:

$$Loss = Exposure * LossGivenDefault$$

- The current MTM or notional are potential candidates.

- The ‘true’ amount at risk in a transaction for default at time t is

$$True\_Exp(t) = \max\{MTM(t), 0\}$$

‘True’ exposure is a random variable for future dates t (because of market factors).

- Do market-driven exposures mean double counting risk with our market risk calculation?

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Example: Portfolio A

- Asset: Bond issued by credit-risky obligor, current MTM = 100
- Liability: The same bond issued by us, current MTM = -100
- Current portfolio value = 0

- Possible market events for bond: 50% up to 120, 50% down to 80
- Possible credit event: 1% probability of default with zero recovery
- Both events independent

Confi-dence Level	"Market" VaR	"Credit" VaR	True Portfolio VaR
99.5%	0	100	120

Worst case: default and market goes up

➢ *A credit exposure of 100 will underestimate the true portfolio VaR, in any combination of market and credit VaR.*

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Example: Portfolio B

- Swap with credit-risky counterpart, current MTM = 0
- Swap hedge with risk-free counterpart, current MTM = 0
- Current portfolio value = 0

- Possible market events for swap: 50% up to 10, 50% down to -10
- Possible credit event: 1% probability of default with zero recovery
- Both events independent

Confi-dence Level	"Market" VaR	"Credit" VaR	True Portfolio VaR
99.5%	0	0	10

Worst case: default and market goes up

➢ ***A credit exposure of 0 will lead to 0 risk, in any combination of market and credit VaR.***

Example: Portfolio C

- Asset: Bond issued by credit-risky obligor, current MTM = 100
- Liability: With known current and future MTM = -100
- Current portfolio value = 0

- Possible market events for bond:
  50% up to 120, 50% down to 80
- Possible credit event:
  1% probability of default with zero recovery
- Both events independent

Confidence	"Market" VaR	"Credit" VaR	Portfolio VaR
99.5%	20	100	100

Worst case: default and market goes up

➢*Even a credit exposure of 100 will overestimate the true portfolio VaR, in any combination of market and credit VaR.*

Are market-driven exposures double counting risk ?

- There seems to be no conclusive answer. The problem comes from the fact that market and credit risk are usually separated.

- The correct way: **Simulate** credit and market risk factors in a big simulation.

A common simplification for exposure

- Determine a non-random proxy Exp(t) for True_Exp(t). It will be a profile over time.
- In the formula for default loss use Exp(0), Exp(1y) or some average over the year for "exposure"
- Typical approach for bonds and similar instruments:

$$Exp(t) = Today's\ MTM\ \ or\ the\ notional$$

as a constant over time.

- Typical approach for derivatives:

$$Exp(t) = E[True\_Exp(t)]\ or\ confidence\ level$$

…continued

- *For derivatives the 'exposure profile' over time will reflect to some extent the randomness of the 'true' exposure.*
- *Credit risk for derivatives will be noticed when their MTM is zero or negative.*
- *This approach does not lead to a correct calculation of total EC.*
- *System implication: To determine E[True_Exp(t)] we need an extra market risk system. Transactions need to be valued at future points in time under different market scenarios.*

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   - 2.1 Loss Given Default
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3. Change in value under rating migration

$$Loss\ or\ Gain = F(Transaction, OldRating, NewRating)$$

- Idea: Revalue the transaction (as it will be in 1y) under the old rating and under the new rating. The difference will be the loss or gain due to the rating migration.
- Different instruments may require different revaluation formulae.
- It should reflect the impact of credit quality on the value of the transaction

…continued

- For each rating define a credit spread (or structure of credit spreads by time, by industry etc…)
- Bonds: Intuitively, the effect will be something like

$$Bond\ Sensitivity\ in\ 1y * (new\ spread\ - old\ spread)$$

- Derivatives: The effect will depend on how likely the MTM will be positive for us.

Theoretical value adjustment for credit quality

$$Value(riskfree\ \ counterpart) - Value(risky\ counterpart)$$

$$\cong E^*\left[\sum_{j=1}^{N} DEFAULT_{[t_{j-1},t_j]} * True\_Exp(t_j) * LGD * d(t_j)\right]$$

$$\cong \sum_{j=1}^{N} \underbrace{E^*\left[DEFAULT_{[t_{j-1},t_j]} * LGD\right]}_{\text{Can be expressed through credit spreads}} * \underbrace{E^*\left[True\_Exp(t_j)\right]}_{\text{Exposure profile of the transaction}} * \underbrace{d(t_j)}_{\text{Risk-free discounting}}$$

…continued

- We get a generic formula that requires the following input as of in 1y:
  - Credit spreads associated with rating grades;
  - A profile of the transactions expected exposure over its lifetime (at least at discrete gridpoints of time):

    Exp(t) = E*(True_Exp(t));
  - Discount factors.

- For bonds the formula matches a revaluation based on their sensitivity, if the grid-points are chosen according to the bond profile.

Summary: Information needed in the simulation steps

Credit model:

- Rating system with migration matrices
- Asset correlations
- LGD characteristics
- Credit spread structure

➢ *These are input data questions: Which data sources can we use?*

Transactions:

- Exposure profiles
- Sensitivities

➢ *These are system requirements: What additional systems can we use to generate this information?*

Input data

- Migration matrices, LGD parameters and credit spreads:
  - Update frequency ?
  - Granularity: How much can we cope with ?
  - External data: How relevant for our portfolio ?
  - Historical data: How relevant for the future ?
- Asset return correlations
  - Not directly observable, deriving them from stocks is not easy
  - For some obligors there may be no stocks

Transaction data

- What are the underlying assumptions to generate exposure profiles (market risk model):
  - Long term evolvement of market data is needed;
  - Does it give us real-world or pricing-world expectations.
- Which types of sensitivity, duration etc can we get from our trading system ?
- Do we need some of this “real-time” for deal commitment ?
  - Then it should be simple, quick and stable rather than sophisticated.

Some additional problems

- Netting agreements:
  - We need exposure profiles on an obligor level.
  - Proper netting requires a good market risk system.
  - How can we still break down risk on a transaction level ?
- Special instruments
  - Options: They may mean a credit-risky commitment that our trading system does not know yet.
  - Options on credit risky bonds: The commitment may be with a different obligor.
  - Credit derivatives.