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# Simulating the loss distribution of a corporate bond portfolio


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## Summary of presentation

- Why do a simulation?
- On the computational complexity
- The simulation set up
- Computing credit risk measures
- Numerical results
- Student t distribution and its properties
- Loss simulation under Student t distribution
- Relative credit risk measures
- Conclusion

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## Why do a simulation?

- So far we have tried to quantify the credit risk of a corporate bond portfolio through two quantities: Expected and unexpected loss
- Expected and unexpected loss are the first two moments of the credit loss distribution
- If the credit loss distribution were to be close to normal, we can infer higher order moments and tail risk measures using EL & UL
- The credit loss distribution is very different from a normal distribution and has a long fat tail
- For a distribution with long fat tail, the mean (EL) and standard deviation of loss (UL) do not capture the downside risks

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## Why do a simulation?

- When holding a portfolio of corporate bonds, we may like to know how much we could lose under an extreme scenario
- Stated differently, we may want to know what is the value at risk at a certain level of confidence or the average loss that one could expect beyond a certain confidence level
- Such risk measures quantify the risk in the tail part of the loss distribution and are referred to as tail risk measures
- Computing tail risk measures for the credit loss distribution will require doing a simulation because it is not possible to extract this information using the analytically derived UL value
- One can also compute EL and UL directly from simulations

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## On the computational complexity

- Popular belief is that performing a Monte Carlo simulation is computationally demanding
- Contrary to this popular belief, performing a Monte Carlo simulation can be computationally more attractive than using an analytical approach to quantify credit risk measures
- This is because computational complexity of Monte Carlo simulation increases linearly with the number of variables
- In contrast, computational complexity increases exponentially in the number of variables for discrete probability tree methods
- 2-bond portfolio has $18^2 = 324$ credit states
- 10-bond portfolio has $18^{10} = 3.57 \times 10^{12}$ states

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## The simulation set up

- To generate the credit loss for one run of the Monte Carlo simulation, we need to go through the following steps
  - Simulate correlated random numbers that model the joint distribution of asset returns of the obligors in the portfolio
  - Infer the implied credit rating of each obligor based on simulated asset returns
  - Compute the potential loss in value based on the implied credit rating, and in those cases where the asset return value signals an obligor default, compute a random loss on default value by sampling from a beta distribution
- Repeating the above steps will allow us to simulate the loss distribution

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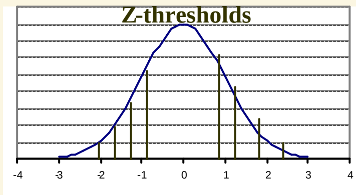
## The simulation set up

Assign PD, LD, RR, $\sigma_{RR}$ and asset return correlation → Generate a random vector from $N(0,\Sigma)$ → Check for credit events using asset return vector → Compute the credit loss when credit events occur → Compute portfolio credit loss for each simulation → Repeat




Compute the credit loss when credit events occur: $\ell_1, \ell_2, \ell_3$

Compute portfolio credit loss for each simulation: $\ell_P = \ell_1 + \ell_2 + \ell_3$

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## Computing credit risk measures

- From the simulated credit losses, different aggregate credit risk measures can be derived
- The expected and unexpected portfolio losses are given by,

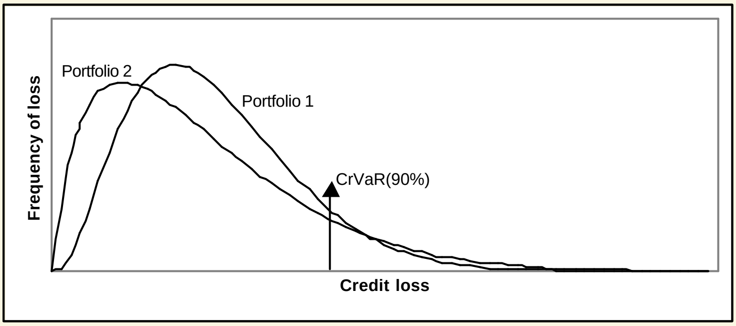
$$EL_P = \frac{1}{N}\sum_{k=1}^{N} \ell_P(k)$$

$$UL_P = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}\left(\ell_P(k) - EL_P\right)^2}$$

- If simulated losses are sorted, credit VaR at 90% confidence level will be the 1000$^{th}$ worst-case loss if N=10,000

## Expected shortfall risk (ESR)

- Credit VaR does not capture the severity of loss in the worst-case scenarios in which the loss exceeds CrVaR




## Expected shortfall risk (ESR)

- Examining the loss exceedence beyond the desired confidence level at which CrVaR is estimated is important to gauge the loss severity in the tail part of the loss distribution
- Expected shortfall risk, which is sometimes referred to as conditional VaR, provides an estimate of the loss severity
- A simple interpretation of ESR is that it measures the average loss in the worst p% scenarios where, (100-p)% denotes the confidence level at which CrVaR is estimated

$$ESR_{p\%} = E\left[\ell_P(k) \,\middle|\, \ell_P(k) > CrVaR_{p\%}\right]$$

$$ESR_{90\%} = \frac{1}{(1-0.9)N} \times \sum_{k=0.9N+1}^{N} \ell_P(k)$$

## 23-bond portfolio as on 24 April 2002

| S. No. | Issuer | Ticker | Industry | Issuer rating | Nominal USD mm | Dirty price | Maturity | Coupon | KMV's EDF (bp) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Health Care Reit | HCN | INR | Ba1 | 20.0 | 99.91 | 15.Aug.07 | 7.500% | 3 |
| 2 | Hilton Hotels | HLT | CCL | Ba1 | 20.0 | 104.13 | 15.May.08 | 7.625% | 29 |
| 3 | Apple Computer | AAPL | COT | Ba2 | 20.0 | 100.97 | 15.Feb.04 | 6.500% | 144 |
| 4 | Delta Air Lines | DAL | TRA | Ba3 | 20.0 | 99.42 | 15.Dec.09 | 7.900% | 147 |
| 5 | Alcoa Inc | AA | BAC | A1 | 20.0 | 105.24 | 01.Jun.06 | 5.875% | 21 |
| 6 | ABN Amro Bank | AAB | BNK | Aa3 | 20.0 | 109.18 | 31.May.05 | 7.250% | 10 |
| 7 | Abbey Natl Plc | ABBEY | BNK | Aa3 | 20.0 | 108.43 | 17.Nov.05 | 6.690% | 33 |
| 8 | Alliance Capital | AC | FIN | A2 | 20.0 | 100.29 | 15.Aug.06 | 5.625% | 158 |
| 9 | Aegon Nv | AGN | INR | A1 | 20.0 | 110.42 | 15.Aug.06 | 8.000% | 10 |
| 10 | Abbott Labs | ABT | CNC | Aa3 | 20.0 | 104.54 | 01.Jul.06 | 5.625% | 7 |
| 11 | Caterpillar Inc | CAT | BAC | A2 | 20.0 | 105.98 | 01.May.06 | 5.950% | 24 |
| 12 | Coca Cola Enter | CCE | CNC | A2 | 20.0 | 102.04 | 15.Aug.06 | 5.375% | 88 |
| 13 | Countrywide Home | CCR | FIN | A3 | 20.0 | 101.25 | 01.Aug.06 | 5.500% | 149 |
| 14 | Colgate-Palm Co | CL | CNC | Aa3 | 20.0 | 101.43 | 29.Apr.05 | 3.980% | 4 |
| 15 | Hershey Foods Co | HSY | CNC | A1 | 20.0 | 105.61 | 01.Oct.05 | 6.700% | 2 |
| 16 | IBM Corp | IBM | COT | A1 | 20.0 | 99.66 | 01.Oct.06 | 4.875% | 26 |
| 17 | Johnson Controls | JCI | COT | A3 | 20.0 | 100.30 | 15.Nov.06 | 5.000% | 24 |
| 18 | JP Morgan Chase | JPM | BNK | Aa3 | 20.0 | 108.62 | 01.Jun.05 | 7.000% | 42 |
| 19 | Bank One NA ILL | ONE | BNK | Aa3 | 20.0 | 101.50 | 26.Mar.07 | 5.500% | 19 |
| 20 | Oracle Corp | ORCL | COT | A3 | 20.0 | 105.33 | 15.Feb.07 | 6.910% | 58 |
| 21 | Pub Svc EL & Gas | PEG | UTL | A3 | 20.0 | 104.94 | 01.Mar.06 | 6.750% | 39 |
| 22 | Procter & Gamble | PG | CNC | Aa3 | 20.0 | 101.76 | 30.Apr.05 | 4.000% | 4 |
| 23 | PNC Bank NA | PNC | BNK | A3 | 20.0 | 102.26 | 01.Aug.06 | 5.750% | 24 |

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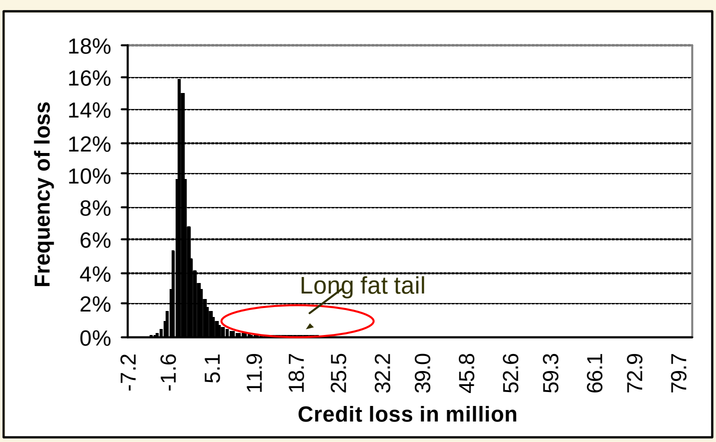
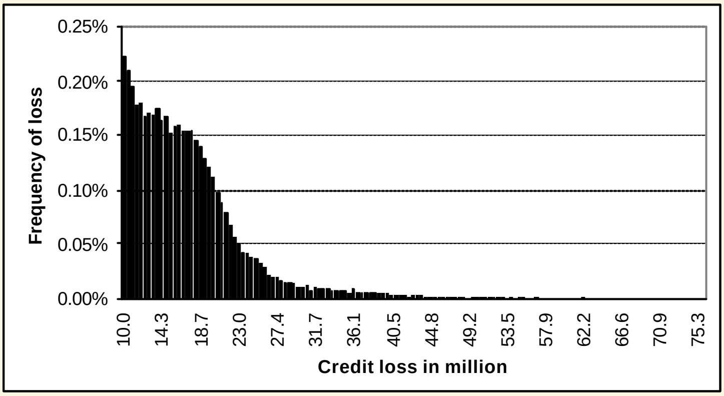
## Simulated credit loss distribution



## Simulated loss distribution around tail region




## Computing loss correlation under migration mode

| Portfolio credit risk measures under migration mode based on simulated loss distribution | | |
|---|---|---|
| **Description** | **Amount (million)** | **Relative to portfolio size** |
| Expected loss | $ 1.626 | 34.1 bp |
| Unexpected loss | $ 4.238 | 88.9 bp |
| CrVaR at 90% confidence | $ 4.905 | 102.9 bp |
| ESR at 90% confidence | $ 11.452 | 240.3 bp |

Using analytical formula: EL=34.0 bp; UL=88.8 bp

## Relaxing the normal distribution assumption

- We assumed that joint distribution of asset returns are normal
- How good is this assumption?
- In general, even if the marginal distribution of random variables is normal, their joint distribution need not be multinormal
- A well-known feature of financial data returns is that they exhibit leptokurtosis and fat tails
- A Student t distribution captures some of these features including tail dependence
- Tail dependence captures the extent to which the dependence (or correlation) between random variables arises from extreme observations



## Student t distribution

- Student t distributions falls under the category of normal mixture distributions
- A useful property of such distribution functions is that they inherit the correlation matrix of the multivariate normal distribution
- Specifically, we have the following relation for multivariate t

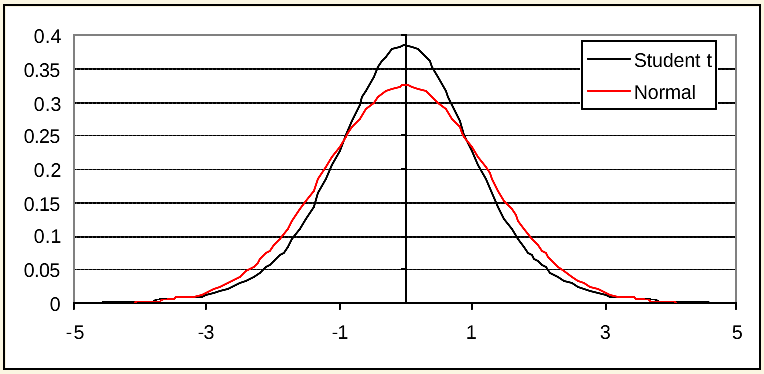
$$\vec{x} = s \cdot \vec{u} \qquad where,\ s = \sqrt{\frac{\nu}{\omega}}$$

Chi-square r.v.

$$Corr(x_i, x_k) = Corr(u_i, u_k)$$

- For ν>2 the Student t distribution will have zero mean vector and covariance matrix $\frac{\nu}{\nu-2}\Sigma$

## Student t distribution

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## Student t distribution properties

$$f_\nu(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi} \times \Gamma(\nu/2)} \times \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

$$f_\nu(x_1, x_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{x_1^2 + x_2^2 - 2\rho x_1 x_2}{(1-\rho^2)\nu}\right)^{-(\nu+2)/2}$$

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## Loss simulation under Student t

- ***Step 1***: Compute the Cholesky factor *L* of the matrix *C* where *C* is the *nxn* asset return correlation matrix
- ***Step 2***: Simulate *n* independent standard normal random variates $z_1, z_2, \cdots, z_n$ and set $\vec{u} = L\vec{z}$
- ***Step 3***: Simulate a random variate $\omega$ from chi-square distribution with $\nu$ degrees of freedom that is independent of the normal random variates and set $s = \frac{\sqrt{\nu}}{\sqrt{\omega}}$
- ***Step 4***: Set which represents the desired *n*-dimensional *t* variate with $\nu$ degrees of freedom and correlation matrix *C*
- Repeating the steps 2 to 4 will allow us to generate the sequence of multivariate *t*-distributed random variables

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## Credit risk using Student t distribution

| **Portfolio credit risk measures under migration mode based on simulated loss distribution** | | |
|---|---|---|
| **Description** | **Amount (million)** | **Relative to portfolio size** |
| Expected loss | $ 1.621 | 34.0 bp |
| Unexpected loss | $ 5.009 | 105.1 bp |
| CrVaR at 90% confidence | $ 4.602 | 96.6 bp |
| ESR at 90% confidence | $ 12.211 | 256.2 bp |

## Relative credit risk measures

- Consider the case where a portfolio manager holds a subset of bonds in the benchmark in order to replicate a corporate portfolio
- Given that the portfolio manager holds only a subset of bonds, how well does the portfolio replicate benchmark characteristics?
- We could potentially consider computing a relative risk measure such as tracking error to quantify the risk
- Unfortunately, tracking error captures primarily the relative risk arising from market risk factors
- The dominant source of relative risk of a corporate bond portfolio against its benchmark comes from exposure mismatches to different issuers



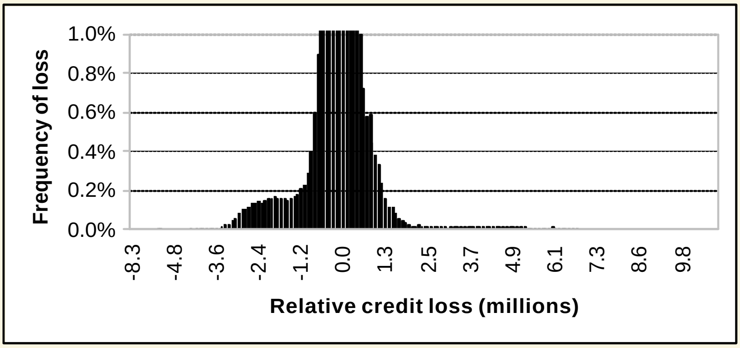
## Relative credit risk

- The relative credit risk between the portfolio and the benchmark can be seen as the credit risk of an active portfolio, whose nominal exposure to the ith bond is given by,

$$NE_{i,A} = \left( NE_{i,P} - NE_{i,B} \times \frac{M_P}{M_B} \right)$$

- Given the above nominal exposures, it is straightforward to generate the relative credit loss distribution
- From the simulations, we can compute the various relative risk measures, which are the risk measures of the active portfolio

## Relative credit loss distribution






## Credit risk measures

| Credit risk measures under migration mode and multivariate t-distribution for asset returns | | | |
|---|---|---|---|
| **Description** | **Portfolio held** | **Benchmark** | **Relative credit rik** |
| %EL | 27.1 bp | 34.0 bp | -7.0 bp |
| %UL | 123.4 bp | 105.6 bp | 86.8 bp |
| $\%CrVaR_{90\%}$ | 79.5 bp | 96.6 bp | 33.8 bp |
| $\%ESR_{90\%}$ | 252.6 bp | 256.2 bp | 112.5 bp |

# Conclusion

- Risk measures for a corporate bond portfolio are very different from those for a government bond portfolio
- Estimating the tail risk becomes important here because the loss distribution has a long fat tail
- Important credit risk measures include EL, UL, CrVaR, ESR
- Relative credit risk requires constructing an active portfolio and then computing the various credit risk measures of this portfolio
- Tracking error is not a relevant relative risk measure for a corporate bond portfolio (or an emerging market bond portfolio)
- It is possible to formulate optimisation problems for portfolio construction and rebalancing