BANK FOR INTERNATIONAL SETTLEMENTS

MBS analytics and prepayment modelling

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Presentation outline

1. Motivation
2. Approach
3. Background information
4. Components of a prepayment model
5. MBS analytics
6. Questions and answers
7. Appendices (in PDF version of presentation)

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Motivation

- Questions you should ask before buying a security:
  - What return do I expect?
  - What risks do I run?
  - What is fair value?
  - ➢ Does the asset class conform to our risk tolerance and return objective?
- No "right" answer usually (ever?) exists to any of these questions
  - Expected return depends on the probability we give to different possible outcomes
  - Risks affect people differently (liquidity risk)
- We look at ways to answer these questions for MBS

These are some of the fundamental questions that investors should ask when considering the purchase of a security and also in analyzing the securities that are already held in their portfolio.

It is very important for central banks to fully understanding the risks inherent in investing in new asset classes, such as MBS. A first step to understanding these risks is to look at the asset class in isolation, however, the potential diversification benefit should also be considered. Different asset classes have different return drivers, which results in returns being less than perfectly correlated.

It is only once we have analyzed the risks inherent in, and the potential rewards from investing in a new asset class, that we can determine whether the asset class conforms to our risk tolerance and return objectives.

Answering these "fundamental" questions can be as much an art as a science and usually involves historical analysis, numerical simulation and scenario analysis. Furthermore, different investors will have very different risk tolerances and even very different views as to what constitutes risk.

In this presentation we will show what may be done to answer these questions for MBS.

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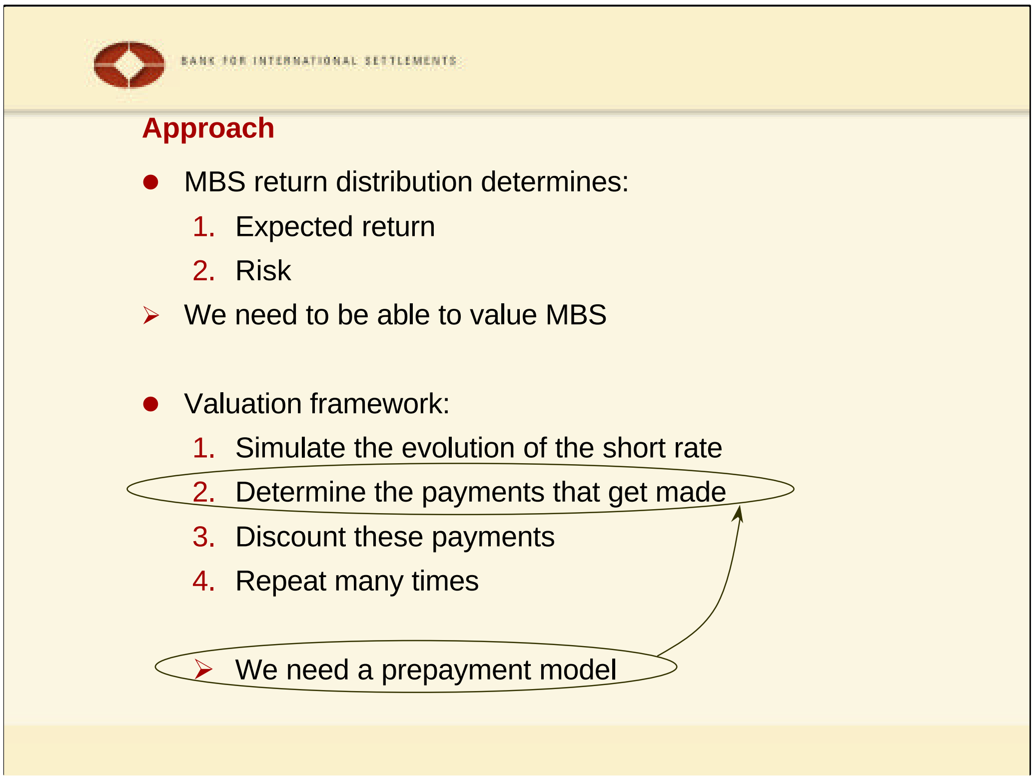

It is changes in security value (in addition to the income provided by the security) over a specified horizon that determines the total return that is realised. Furthermore, it is the distribution of possible changes in security value over the horizon that determines both expected return and risk.

In order to assess the potential return and risk of investing in MBS we need a model to value MBS.

In the previous presentation we introduced a framework to value fixed income securities with embedded options. The approach consisted of the 4 steps outlined in the slide. Each of the steps, with the exception of determining the payments (step 2), is relatively easy to perform.

For MBS pass-through securities the payments that get made by the borrowers are passed through undivided to the MBS holder. In order to be able to anticipate the payments that are received by the MBS holder we need to anticipate the payments that get made by the borrowers in the pool backing the MBS. This, in turn, depends on the prepayment behaviour of the borrowers, which is a complex process to model.

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A typical mortgage contract

- The most common mortgage type in the US is the (30-year) fixed-rate, level-payment (FRLP) mortgage
- The monthly payments on the loan are such that the loan is fully amortised at maturity
- The outstanding balance on the loan is equal to the present value of an ordinary annuity

$$B = C\frac{1-(1+r)^{-M}}{r}$$

B = Outstanding balance

M = Remaining term (months)

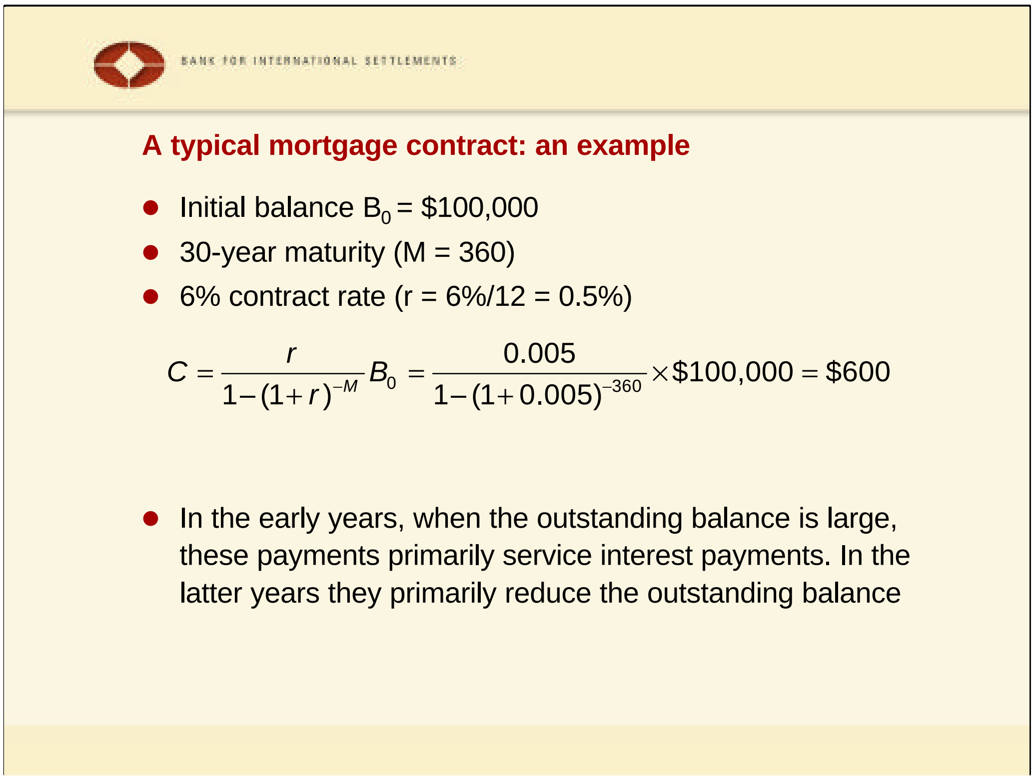
C = Contractual (monthly) payment

r = Contract rate / 12

- The borrower has the right to prepay at any time

Before we consider the reasons that cause the homeowner to prepay his mortgage, let us first consider the characteristics of a typical mortgage contract. The contractual scheduled payments that get made by the borrower and the impact of prepayment on the payment stream.

FRLP mortgages get fully amortised over the term of the loan. This means that over the term of the loan the outstanding balance gets progressively reduced so that, at the end of the loan, the outstanding balance has been reduced to zero. At any point in time the outstanding scheduled payments is a stream of fixed (equal) monthly payments (this is, by definition, an annuity) and we can use the formula for the present value of an ordinary annuity to relate the balance outstanding immediately after a payment (an *ordinary* annuity has its first payment in exactly one period’s time) to the contract rate, the size of the scheduled monthly payments and the number of payments remaining. This relationship is shown in the slide.



This slide shows an actual example in which a homeowner needs to borrow $100,000 for the purchase of his house. He decides to take out a 30-year FRLP mortgage. On the day that the mortgage contract is agreed, the rate on this type of mortgage is 6%. The equation on the previous slide can be rearranged so that we can determine the size of the contractual monthly payments on the loan: The borrower needs to make contractual monthly payments of $600, if he does this for the next 360 months the loan will be fully amortised.

The size of the contractual monthly payments is directly proportional to the initial balance on the loan, if the loan size doubles the size of the contractual payments also doubles. Furthermore, if the contract rate increases the size of the contractual payments also increases. On the other hand, if the term of the loan increases then the size of the contractual payments decreases.

In the early years of the mortgage life the size of the outstanding balance is large. The interest payments on this large balance are large, and this means that a large part of the scheduled monthly payment is directed towards servicing interest payments. On the other hand, towards the end of the life of the loan the size of the outstanding balance has been greatly reduced. The interest payments on this small balance are small, and this means that the bulk of the scheduled monthly payment is directed towards further reducing the outstanding balance.

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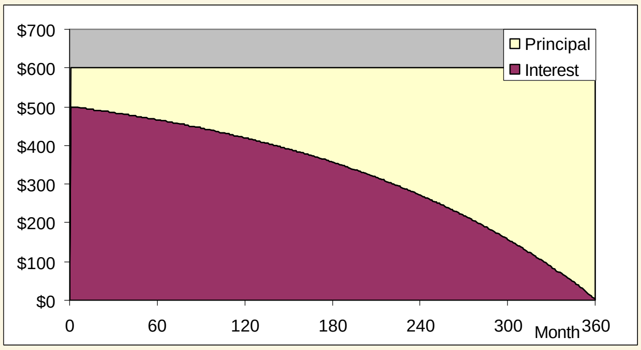
A typical mortgage contract: an example

- Initial balance: $100,000
- Maturity: 30 years
- Contract rate: 6%
- Contractual monthly payment: $600



This slide shows a graphical representation of the division of the monthly payments into scheduled repayments of principal and interest payments.

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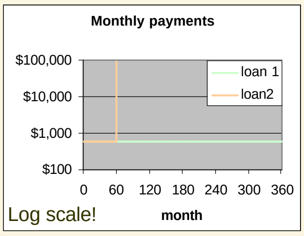
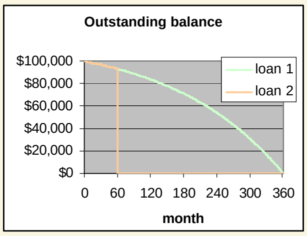
A typical mortgage contract: two examples

	Loan 1		Loan 2	
Month	Balance	Payment	Balance	Payment
0	$100,000	$0	$100,000	$0
1	$99,900	$600	$99,900	$600
2	$99,800	$600	$99,800	$600
3	$99,700	$600	$99,700	$600
4	$99,599	$600	$99,599	$600
5	$99,497	$600	$99,497	$600
6	$99,395	$600	$99,395	$600
…	…	…	…	…
59	$93,188	$600	$93,188	$600
60	$93,054	$600	$0	$93,654
61	$92,920	$600	$0	$0
…	…	…	…	…
356	$2,369	$600	$0	$0
357	$1,781	$600	$0	$0
358	$1,190	$600	$0	$0
359	$597	$600	$0	$0
360	$0	$600	$0	$0





Which lender is better off?

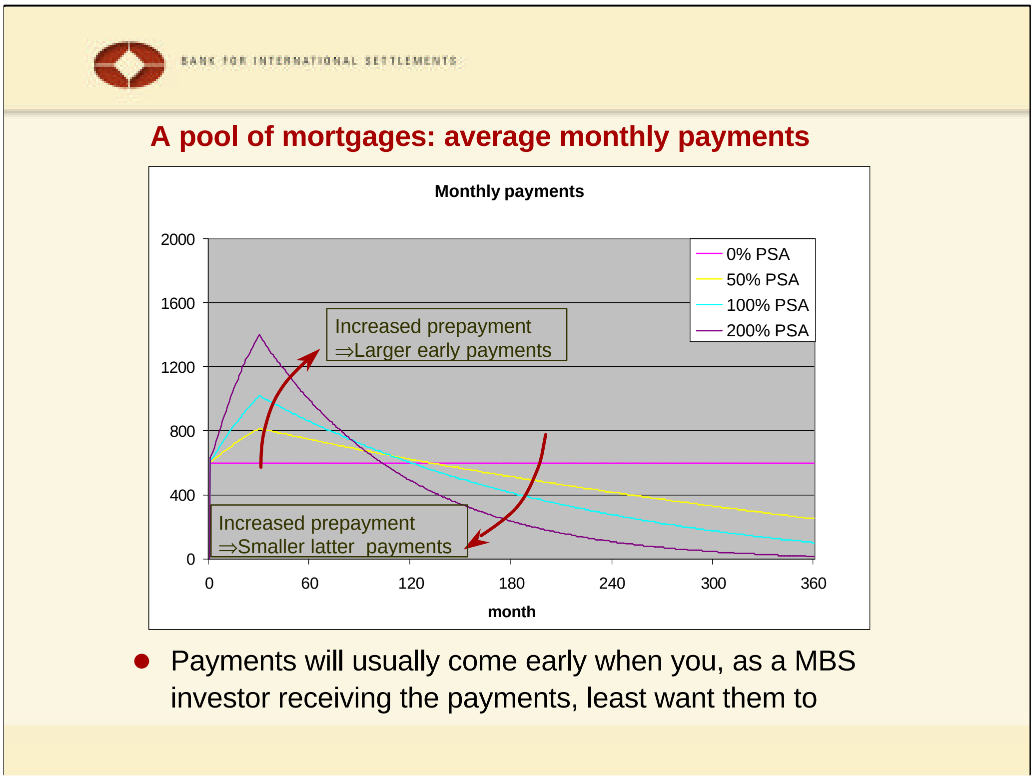
The slide shows loans to two homeowners. At the start the two loans are identical: Each borrower has taken out a 30-year, FRLP, $100,000 loan, at a contract rate of 6%. We have seen that such a loan entails scheduled monthly payments of $600.

The table shows the payments that get made by the homeowner at the end of each month and the month-end balance after amortization.

For the first 5 years the two loans remain identical. Each homeowner promptly makes his scheduled $600 monthly payments. However, at the end of the fifth year homeowner 2 exercises his option to prepay. This, in effect, brings loan 2 to an end with the return of the outstanding balance at the end of the 60^{th} month. In addition to the scheduled $600 payment made by borrower 2 at the end of month 60 he prepays the entirety ($93,054) of the outstanding balance.

The top graph shows the payment streams made by the two loans. Let us assume that these are the payments that are received by the lenders of each loan. The only difference in the payments received by the lenders is from month 60 onwards: lender 1 receives 301 (from the end of month 60 to the end of month 360) monthly payments of $600, whereas lender 2 receives a single payment of $93,654 (at the end of month 60).

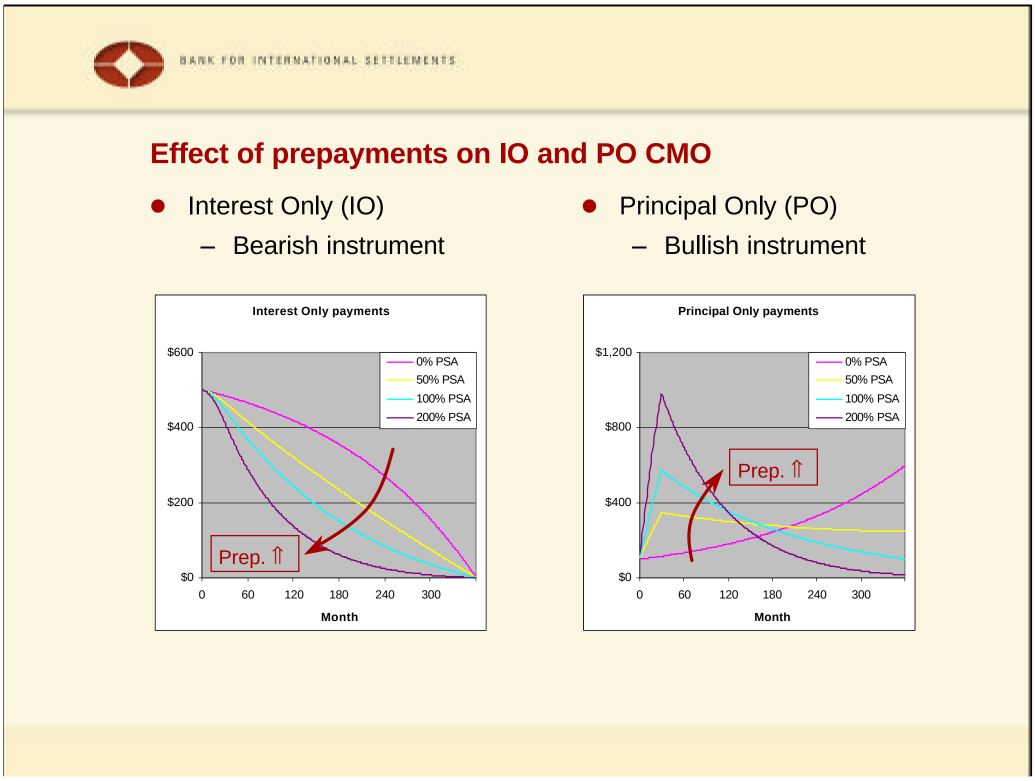
Without additional information, we cannot say which lender is better off. Lender 2 is better off if he can use the $93,654 that has been returned to him to make a 25-year FRLP loan with scheduled monthly payments greater than $600.



We have seen that the prepayment of a loan has a dramatic impact on its resultant cash flow stream. Pools of mortgages backing a MBS will normally have several hundred individual mortgages in them. The payment stream made by the pool will depend on the average prepayment rate of the underlying loans. Because not all borrowers will opt to prepay at the same moment the payments made by the pool will be much smoother than the payments made by individual loans.

The slide shows the payment stream from a pool (backed by 30-year, 6%, FRLP mortgages) at several different prepayment rates (an explanation of the units used to measure prepayment rates is given later).

Anticipating the prepayment behaviour of a single borrower is an impossible task. However, anticipating average prepayment rates of a pool of mortgages is a much more tractable problem.



We have seen that the prepayment of a single loan may, on occasions, be in the interest of the MBS holder. Mortgage refinancing will, in general, transfer value from the MBS holder to the borrower. This means that, in general, prepayments are not in the interest of the MBS holder.

In pass-through securities the payments made on the loans backing the security are passed undivided to all MBS holders. There are various types of Collateralised Mortgage Obligations (CMO) in which the payments made by the underlying loans are not split equally amongst all investors in the CMO. The CMO is however *self-supporting*, with the cash flows from the underlying collateral always able to meet the cash flow requirements of all CMO classes under any prepayment scenario. The issuer of the CMO has in effect “sliced and diced” the prepayment risk and the interest rate risk of the underlying collateral so that different traches have different exposures to each risk.

One possibility is for the payments to be divided into Interest Only (IO) payments and Principal Only (PO) payments. IO CMO tranches receive all interest payments from the underlying pool of loans. Prepayments are invariably bad news for the holder of an IO CMO class. This is because a prepayment results in a corresponding reduction in the outstanding balance and thereafter smaller interest payments are made. We will see that prepayments tend to increase with falling interest rates, IO classes therefore perform badly in a falling interest rate environments and perform well in rising interest rate environments: IO classes are thus bearish instruments.

The holder of the PO class receives all principal payments (both scheduled and prepayments) from the underlying loan. The sum of all the payments that will be received by the holder of the PO class will be the outstanding balance of the loan. High prepayments means that the payments simply arrive earlier (the total amount remains unchanged). All investors prefer $1 today as opposed to $1 tomorrow: PO classes perform well when prepayments increase which will be when rates decrease, PO classes are therefore bullish instruments.

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Quantifying prepayments

- Single Monthly Mortality (SMM)
  - A fraction R_{SMM} of borrowers in a pool prepay on a given month
  - If this rate stays constant: a fraction $1 – (1 – R_{SMM})^{12}$ prepay over a 12-month period
- Constant Prepayment Rate (CPR)
  - CPR is SMM expressed as an annual rate $R_{CPR} = 1 – (1 – R_{SMM})^{12}$
- Public Securities Association (PSA) convention
  - 100% PSA is considered the "normal" prepayment rate
    - But the "normal" prepayment rate varies with loan age
    - The CPR corresponding to 100% PSA is a function of loan age
  - Prepayment rates are expressed as a percentage of PSA

Prepayment rates are normally expressed in one of three ways. This can lead to confusion. The three different measures for prepayment can be considered as three different units to express the same underlying phenomenon, and there is a one-to-one relationship between each measure.

As the name suggests, Single Monthly Mortality (SMM) is a measure of the prepayments that occur in a pool in a single month (mortality is used in the sense that the mortgages that prepay no longer exist in the pool). The SMM Rate, R_{SMM}, is the fraction of the borrowers in a pool that prepay on a given month. A fraction $(1 – R_{SMM})$ of the start-of-month borrowers in the pool remain in the pool at the end of the month. This means that if R_{SMM} remains constant over a 12-month period, a fraction $(1 – R_{SMM})^{12}$ of the start-of-period borrowers remain in the pool at the end of the 12-month period.

If the SMM is converted into an annual rate we get a prepayment rate expressed in Constant Prepayment Rate (CPR) units (this unit of measure is also sometimes referred to as Conditional Prepayment Rate). We have seen that the start-of-period fraction that survives in the pool at the end of the 12-month period is $(1 – R_{SMM})^{12}$. The CPR is therefore related to SMM by: $R_{CPR} = 1 – (1 – R_{SMM})^{12}$.

The third "standard" unit to express prepayment rates is the Public Securities Association (PSA) convention (this unit of measure is also often referred to as the Prepayment Standard Assumption). 100% PSA specifies the "normal" prepayment rate of a mortgage pool. We will see that there is a process known as "aging"; other things equal, prepayment rates on loans that have not yet fully aged are lower than those on fully aged loans. What can be considered as the "normal" prepayment rate of a mortgage pool is therefore a function of the age of the pool (the PSA assumes that this aging process takes 30 months). The CPR corresponding to 100% PSA increases linearly with age up to 30 months, it then remains constant (at 6% CPR) for the remainder of the term of the pool.

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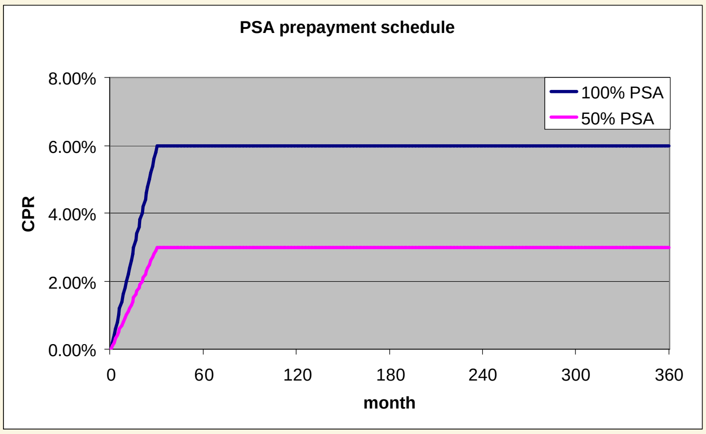
Quantifying prepayments

- The 100% (normal) and 50% PSA prepayment schedule



The graph shows the CPR corresponding to 100% PSA and 50% PSA as a function of loan age.

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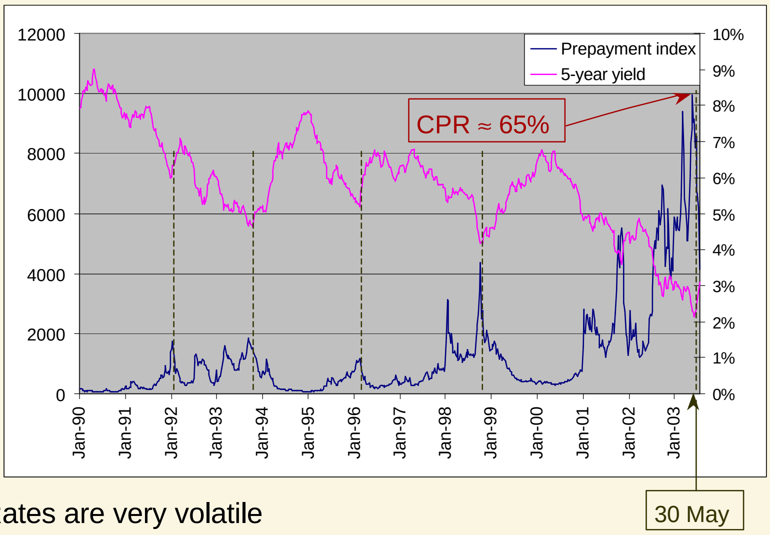
Historical prepayment rates



- Rates are very volatile
- Very high prepayment rates tend to be driven by refinancing motivated by falling mortgage rates

This graph shows an index representative of prepayment rates. It is clear from the graph that the prepayment rate can be very volatile. We also see that periods of very high prepayments are associated with periods of falling interest rates, which result in big incentives for homeowners to refinance their loan. The recent high prepayment rates are almost entirely the result of refinancing into lower rate mortgages. The very low prepayment rates in the second half of 1994 and early 1995 is the result of the negative refinancing incentive that existed as a result of the rising yields over this period.

In the next part of the presentation we look at the various causes of prepayment in order to gain an understanding of homeowner prepayment behaviour.

The graph also shows the 5-year Treasury yield. We note that an increase in refinancing activity occurs when yields fall and when yield start increasing refinancing activity falls sharply.

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The components of a prepayment model

- Reasons for prepayments
  1. Home sales (turnover)
  2. Defaults
  3. Curtailments and full payoffs
  4. Refinancing (the most volatile component)

- The dominant factor driving prepayment changes over time
- Borrower demographics change
- Prepayment rates may vary geographically
- Prepayment rates vary with mortgage type

- Modelling prepayments is not easy!

Reference: "Anatomy of Prepayments", L.S. Hayre etal, The Journal of Fixed Income, June 2000.

All prepayments fall into one of the 4 categories listed in the slide.

Home Sales (housing turnover): With the exception of GNMA, mortgages are not assumable. This means that when a house is sold the new owner cannot continue making payments on the existing mortgage, and the sale of the house results in prepayment of the outstanding mortgage balance.

Defaults: A default leads to the liquidation of the mortgage. Investors normally realise a default as a full prepayment of the outstanding balance of the loan.

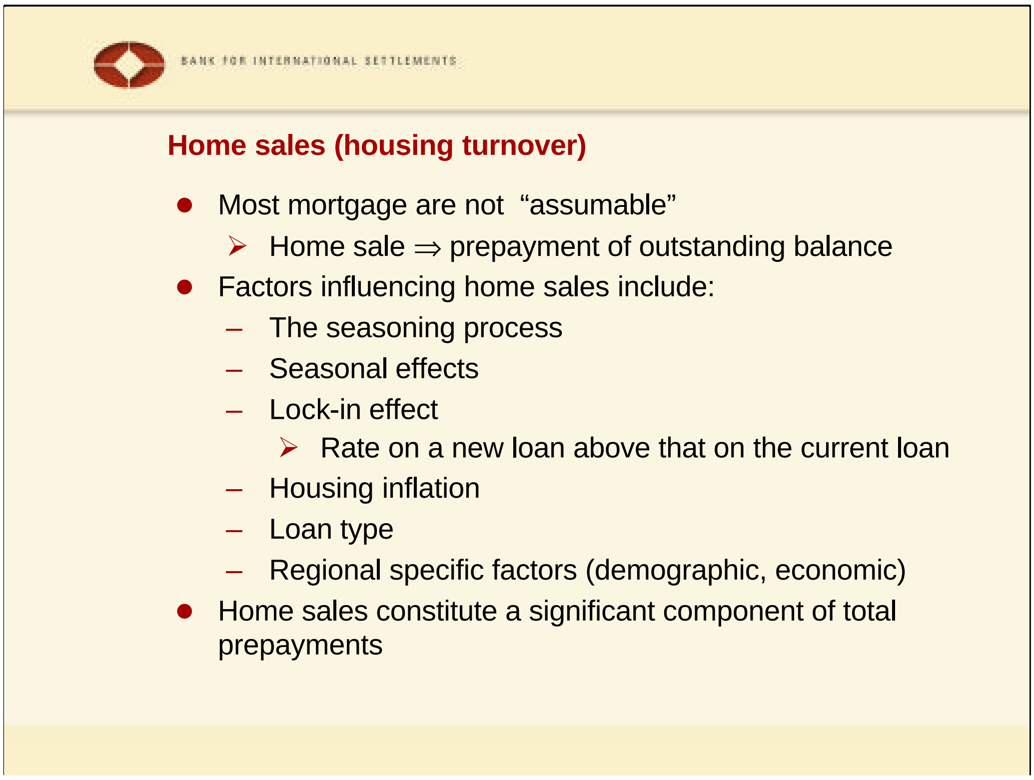
Curtailment and Full Payoffs: The homeowner may choose to pay principal at a rate faster than that stipulated in the mortgage contract in order to build up equity in their home.

Refinancing: Refinancing is normally motivated by lower rates. Refinancing is the most volatile component of prepayments.

We have already seen that prepayment rates can be very volatile. This volatility is primarily due to changes in refinancing-driven prepayments. Therefore, during periods of very high prepayment rates, refinancing will be the major factor driving prepayments. However, during periods of low prepayments, refinancing-driven prepayments will constitute only a small fraction of total prepayments.

Prepayment rates may vary considerably with type of mortgage. This is because different prepayment incentives exist for different types of mortgages. When rates come down there can be very large refinancing incentives for, 30-year, FRLP mortgages. Under the same conditions, the refinancing incentive on an Adjustable Rate Mortgage (ARM) will be much lower or even absent. Borrower "self-selection" is also a reason for different mortgage types exhibiting different prepayment rates; there is some evidence that borrowers who select balloon loans or ARM often expect to move again soon; borrowers who accept to pay higher coupons for "low-point" or "no-point" (the cost of entering into a new mortgage contract is measured in points, 1 point corresponds to 1% of the borrowed amount) loans tend to be quick refinancers.

Prepayment rates can show considerable geographical variation. This may be the result of demographic variations or it may be the result variations in economic conditions and house price inflation.



Only for Federal Housing Administration (FHA) or Veterans Administration (VA) loans may the new buyer decide to assume the obligations of the existing loan (these loans form the collateral of GNMA securities). For these loans the new owner will consider assuming the existing loan if the current coupon rate is higher than that on the existing loan. For all other loans the sale of a home results in the prepayment of the existing mortgage.

The seasoning process refers to the likelihood of a home sale increasing with the time since the mortgage was taken out. It is unlikely that a homeowner will take out a new mortgage if he is aware that he will be selling the house in the near future. This seasoning process causes the "ramp-up" process seen in the PSA standard prepayment assumption.

There will be a disincentive for homeowners to sell their house if the rate on their existing loan is below the current market rate. The borrower will not be willing to terminate the loan which he now has on favourable terms. If the homeowner needs to take out a new loan on a different house the new loan will have to be taken out at the higher rate.

House inflation may be a factor that influences housing turnover. In particular: if house prices have declined and the outstanding balance on the loan is more than the value of the property, it may not be possible for the owner to sell the house as this would entail prepayment of the outstanding balance on the existing mortgage and the proceeds from the sale of the house will not cover this payment.

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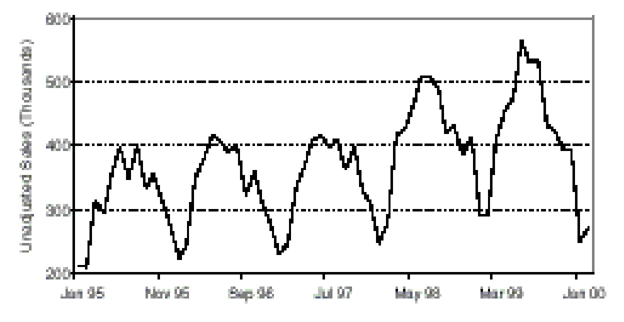
Home sales: seasonal effects



Source: National Association of Realtors.

- Turnover is much higher in the summer months
  - Weather
  - School year

This graph shows the seasonal variation of home sales. Home sales in the summer months are almost twice as high as they are in the winter months.

Parents with school-going children will resist moving during the school year (especially if the move would mean a change is school). Weather is another factor that influences home sales, generally people prefer to move in the summer when the weather is good.

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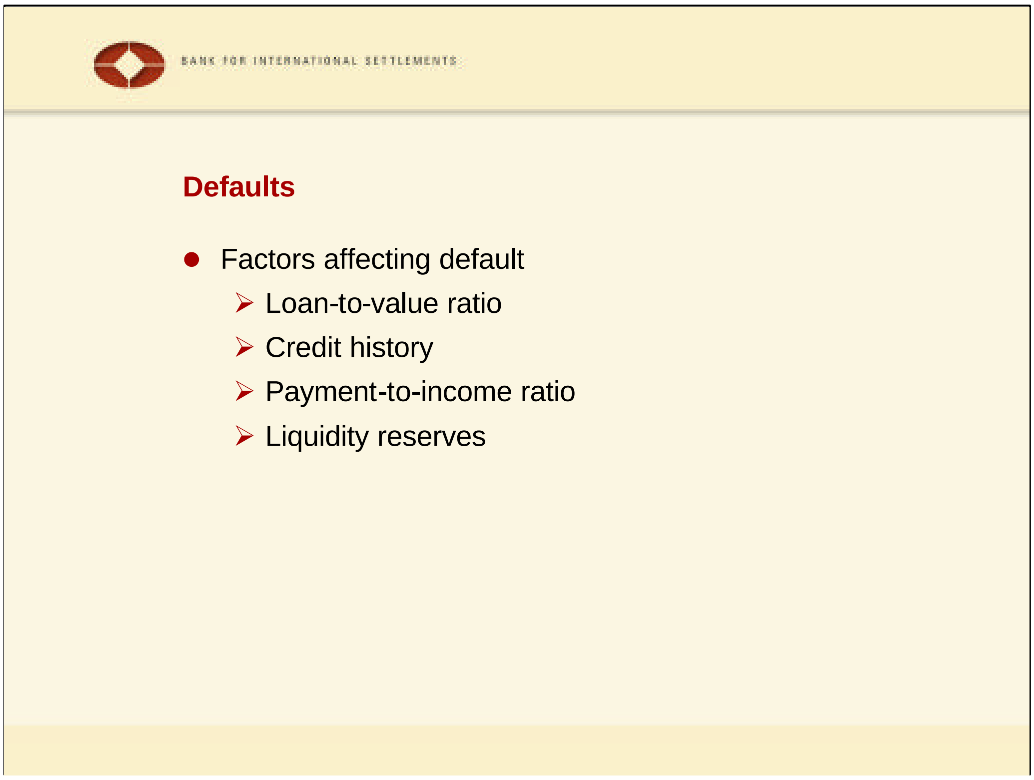
Defaults

- Default occurs when the borrower stops making scheduled monthly payments
  - The agency guarantee ensures that investors realise defaults as a full prepayment of the loan
  - Borrowers cannot choose to default if the value of their property is less than the outstanding balance of their loan
- Defaults normally represent a relatively small component of total prepayments

If a borrower stops making their scheduled monthly payments the mortgage is in default. The MBS investor does not bear the default risk: if a borrower defaults the agency that guarantees the loan will return the outstanding loan balance to the MBS investors. After this the loan belongs to the agency and it will attempt to recover what it can. To the MBS investors a default is identical to full prepayment of the outstanding loan balance.

It is wrong to believe that the borrower has a “no-penalty” option to default on the loan. Therefore, if the loan-to-value ratio of a mortgage is above 100%, the borrower cannot “walk away” leaving the agency with the loan collateral (the house). Under these circumstances the borrower can be pursued in court.



Borrowers need to qualify for mortgage loans. During this qualification process the lender will gauge the ability and willingness of borrower to repay the mortgage debt. The factors that lenders see as most important in making loan decisions are also good predictors of default rates.

The loan-to-value ratio is an important predictor of default rates. If the loan-to-value ratio is bellow 100% (there is positive equity in the property) borrowers should, in principal, not default; the alternative being selling the property and releasing the equity accumulated in it.

The loan payment-to-income ratio is a measure of how capable the borrower is at servicing the mortgage obligations.

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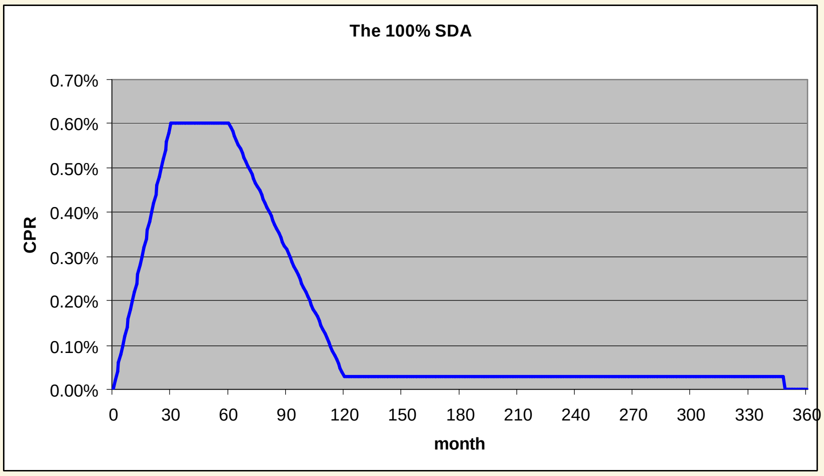
Defaults

- The standard default assumption (SDA)



At its peak the 100% SDA has CPR is 0.6%. This rate is only one tenth of the 100% PSA (CPR of 6%) for a fully aged loan. This indicates that defaults normally constitute only a small portion of total prepayments.

The general shape of the SDA curve can be explained by two factors:

1) Default rates are low in the period immediately after loan origination as a result of due diligence performed by the lender: lenders will not make loans to borrowers who they suspect are going to default.

2) The progressive build-up of equity in the property causes defaults to decrease after about 5 years.

Default rates are highest when the age of the loan is between 30 and 60 months.

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Curtailments and full payoffs

- Borrows can pay more than the scheduled monthly payment on their mortgage. Thereby building up the equity in their home at a faster rate
- Prepayment rates from curtailments vary from:
  - 0.5% CPR early in the life of a pool
  - 15% CPR towards the end of the mortgage term

Curtailments and full payoffs involve borrowers paying more than the scheduled monthly payment each month. Full payoffs occur when borrowers pay off their mortgage completely, usually when it is very seasoned and the remaining loan balance is small.

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Refinancing

- Borrowers may refinance their borrowing needs by:
  - Taking out a new loan
  - Prepaying the previous loan
- Refinancing is motivated by:
  1. Lower rates
     - Affects all borrowers at the same time
     - Results in high prepayment volatility
     - Transfers value from MBS holder to homeowner
  2. Borrower seeking to access equity in property
  3. Take advantage of improved credit rating
- Refinancing will often release the equity built up in the property
  - Supportive of consumer spending

Very high prepayment rates will invariably be the result of homeowners refinancing their mortgage to take advantage of lower rates. As interest and mortgage rates fell throughout 2002 and the first half of 2003, this type of prepayment accounted for the bulk of all prepayments over this period. As this factor driving prepayments affects all borrowers at the same time changes in yield can lead to big changes in prepayment rates. Furthermore, this type of prepayment transfers value from MBS holder to borrower.

Refinancing can also be the result of the homeowner wishing to access equity that has built up in their property and they may even be willing to enter into a new mortgage at a higher contract rate in order to gain access to this money (they can then afford to buy a new car say).

A borrower with a poor credit rating will, other things being equal, have to pay a higher mortgage rate than a borrower with a good credit rating. If the credit rating of a borrower improves he may be able to refinance his mortgage at a cheaper rate even if rates in general have not come down.

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Lower rates

- Option theory can be used to help us anticipate prepayments that are motivated by lower rate
- However, traditional option theory is of limited use in predicting these prepayments. Borrower behaviour represents an “inefficient” exercise of the prepayment option
  - The more “efficiently” borrowers refinance, the lower the value of the MBS security
- Proper modelling of this type of prepayment is a must for correct MBS valuation because:
  - It results in transfer of value from MBS holder to borrower
  - It is very volatile and represents the bulk of refinancing activity during periods of very high refinancing rates

Mortgage refinancing that has been motivated by lower rates is the most volatile component of prepayments. It also invariably transfers value from MBS holder to borrower. For these reasons it is vitally important to properly model this component of prepayment.

Option theory can be used to help us anticipate prepayments resulting from the financial incentive to refinance that is caused by rates falling below the rate on the borrower’s existing mortgage loan. Anticipating the exercise of the call option on a callable bond is relatively easy; this is because the issuer will exercise his call option in an efficient manner. However, anticipating when (and even if) mortgage borrowers will exercise their prepayment option is much more difficult; this is because the homeowner often exercises (or fails to exercise) the prepayment option in what may appear to be an irrational manner.

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Lower rates: the incentive to refinance

- What does the borrower gain from refinancing?
- Depends on:
  - The rate on the existing (old) loan (r_{old})
  - The prevailing (new) mortgage rate (r_{new})
  - The current loan balance
  - The remaining term of the loan
  - The cost of entering into a new mortgage
    - A negative incentive exits until $r_{old} - r_{new}$ is sufficiently large
- See appendix IV

$r_{old} - r_{new}$

There are various factors that influence the incentive to refinance.

It is clear that the incentive to refinance will be depend on the difference between the contract rate on the old mortgage loan and the contract rate that is available on a new mortgage loan. The further the rate on a new mortgage is bellow the rate of the old mortgage the greater the incentive to refinance.

The outstanding loan balance is also a factor. For a given reduction in the mortgage contract rate the dollar saving that can be obtained from moving from the old (high rate) mortgage to a new (low rate) mortgage will increase with the size of the loan.

As the remaining term of the loan increases a larger amount of interest-servicing payments need to be made and, as a result, longer duration loans benefit most from a reduction in the contract rate.

Even though there is no prepayment penalty there are costs involved in taking out a new mortgage. These costs act as a disincentive to refinance in exactly the same way as a penalty would. Some of these costs are proportional to the size of the loan (the origination free, for example) whereas other costs are fixed (the attorney's fee, for example). These fees normally constitute about 1% (1 point) of the borrowed amount. These costs ensure that no refinancing incentive exits until the new contract rate is sufficiently below the contract rate on the existing loan.

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Lower rates: why don't all borrowers prepay?

- Why don't all borrowers take advantage of this opportunity?
  - They may not be aware of the benefits
    - Not today (the media effect)
  - They may not be bothered
  - Their credit rating has deteriorated
  - They may have negative equity in their property
- *Burnout*: A pool that has been exposed to multiple or sustained periods during which (it appears) the borrowers should refinance is termed *burned out*
- *Burnout:* As the most eager or capable refinancers exit a pool the remaining borrowers have less of a tendency to refinance when future opportunities present themselves
  - Burned out pools are more valuable

An incentive to refinance exists if the borrower can refinance their loan at a rate that is lower than the rate on their existing loan (the difference in these rates must be sufficient to compensate the borrower for the cost of entering into a new mortgage contract).

There are various reasons for borrowers not taking advantage of this incentive: the borrower may simply be unaware of the gain to be had from refinancing (in the current climate of historically low rates this is unlikely to be a big factor); some borrowers may not want to go through the bother of refinancing; the credit rating of other borrowers may have deteriorated and the low rates available to other (more creditworthy) borrowers may not be available to them; if the existing loan-to-value ratio is above 100% the borrower may not be able to get the necessary funds to prepay his existing loan.

Any one of these explanations may be the reason for a borrower failing to prepay when it appears that there is an incentive for him to do so. The borrowers that do not prepay "survive" in the pool. This "survival bias" changes the composition of the borrowers in the pool. A pool that has, in the past, been exposed to multiple refinancing opportunities will contain a high fraction of borrowers that will forgo future refinancing "opportunities". This process is referred to as *burnout*. Prepayment rates on burned out pools will be lower than that on similar pools that have not been exposed to past refinancing opportunities.

Borrowers in a burned out pool are expected to exercise their prepayment option in a less optimal manner than borrowers in an, otherwise equivalent, un-burned out pool. For this reason MBS investors are willing to pay more for burned out pools.

More proactive (aggressive) mortgage lenders make sure that we are all well aware of potential gains to be had from refinancing. The internet offers a very cheap way for lenders to target borrowers.

This slide contains a small number of the mortgage-related (junk) email offers I have recently received.

From prepayments to cash flows

- Appendix IV shows how to develop a simple prepayment model that accounts for the most important factors driving prepayments
- Appendix IV also shows how to determine the payments that occur along each simulated interest rate path once prepayments have been modelled

BANK FOR INTERNATIONAL SETTLEMENTS

Presentation outline

1. Motivation
2. Approach
3. Background information
4. Components of a prepayment model
5. MBS analytics
6. Questions and answers
7. Appendices (in PDF version of presentation)

BANK FOR INTERNATIONAL SETTLEMENTS

MBS Analytics

- Static valuation
  - Measures of expected return
    - Yield (for bullet bonds)
    - Cash flow yield (for MBS)
  - Static spread (a measure of MBS relative value?)
- Dynamic valuation
  - The framework
  - Option adjusted spread (OAS)
  - Effective duration and convexity
  - Risk and scenario analysis

In this section we look at measures that can be used to quantify the expected return and the risk of investing in MBS. We start by looking at some of the simple static measures that have traditionally been used. These measures assume a static (zero volatility) interest rate environment. A shortcoming of this approach is that in a static interest rate environment the prepayment option is undervalued and, as a result, the MBS is overvalued.

To address this shortcoming we move to a dynamic valuation framework, which attempts to correctly account for the prepayment option under a volatile interest rate environment.

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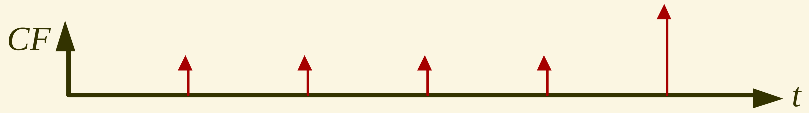
The yield on bonds



- The bond yield reconciles the **dirty** price of the bond to its cash flow stream

$$P = \frac{CF_1}{(1+y)^1} + \frac{CF_2}{(1+y)^2} + \frac{CF_3}{(1+y)^3} + \ldots \frac{CF_N}{(1+y)^N} = \sum_{t=1}^{N} \frac{CF_t}{(1+y)^t}$$

- This is the return an investor receives if:
  1. The bond is held to maturity
     - Inappropriate horizon
  2. Coupons can be reinvested at the same yield
     - Reinvestment risk

The yield of a fixed income instrument is the most basic measure of the return that an investor may expect to realise from buying it.

For bullet bonds there is no ambiguity as to what the yield is. For a particular bond (with a known future cash flow stream) here is a one-to-one relationship between market price and yield. The yield reconciles the cash flow stream of the bond with its market price.

The yield of a fixed income security can be interpreted as the return that an investor may expect to realise by buying it. For this reason the yield of a bond is often called the internal rate of return. Even if there is no ambiguity as to the value of the yield, we need to be aware of the implicit assumptions that are made when the yield is interpreted as the expected return. These are: 1) The instrument is held to maturity; 2) Coupon payments can be reinvested at the same yield.

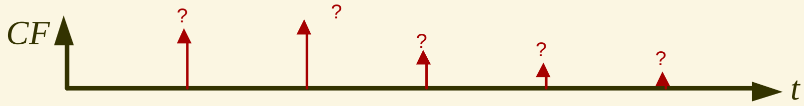
These assumptions limit the usefulness of yield as a measure of the bond's potential return.

For long-dated bonds return earned on reinvested coupons can constitute more than 80% of the bonds return. If yields fall coupons will be reinvested at a lower yield and the "expected" return will not be realised.

Investors will normally have an investment horizon that is shorter than the maturity of the instruments they hold. Over their investment horizon capital gains or losses may form the bulk of the total return.

BANK FOR INTERNATIONAL SETTLEMENTS

The MBS cash flow yield (CFY): do not expect to earn this yield!



$$P = \frac{CF_1}{(1+y)^1} + \frac{CF_2}{(1+y)^2} + \frac{CF_3}{(1+y)^3} + \dots \frac{CF_N}{(1+y)^N} = \sum_{t=1}^{N} \frac{CF_t}{(1+y)^t}$$

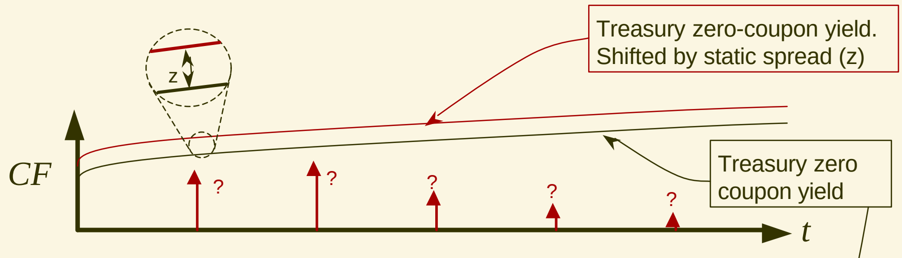
CFY

- The CFY reconciles market price with *anticipated* cash flow stream
  - The CFY depends on our prepayment assumption
- Reinvestment risk is particularly big because principal as well as interest will need to be reinvested. Furthermore, falling rates result in:
  - Larger prepayments
  - More money to reinvest
  - But at a lower rate

In addition to needing to make an assumption about the prepayment rate to predict future payments made by the MBS, interpreting CFY as a measure of the expected return from holding a MBS also implicitly assumes that the MBS is held to maturity and that the payments made by the MBS can be reinvested at the CFY. The reinvestment risk of MBS is particularly large because payments are monthly and both interest and principal need to be reinvested. Furthermore prepayment rates (and the need to reinvest these payments) will increase when rates fall and it will not be possible to realise the CFY on the reinvested earnings under these circumstances.

BANK FOR INTERNATIONAL SETTLEMENTS

Static spread (zero volatility spread): a relative return measure



$$P_{Mkt} < PV_{Static} = \frac{CF_1}{(1+y_1)^1} + \frac{CF_2}{(1+y_2)^2} + \frac{CF_3}{(1+y_3)^3} + \dots \frac{CF_N}{(1+y_N)^N} = \sum_{t=1}^{N} \frac{CF_t}{(1+y_t)^t}$$

$$P_{Mkt} = \frac{CF_1}{(1+y_1+z)^1} + \frac{CF_2}{(1+y_2+z)^2} + \frac{CF_3}{(1+y_3+z)^3} + \dots \frac{CF_N}{(1+y_N+z)^N} = \sum_{t=1}^{N} \frac{CF_t}{(1+y_t+z)^t}$$

- Does z represent adequate compensation for the prepayment risk of the MBS?
- In a volatile interest rate environment investors expect to earn less than the static spread

The static spread or zero volatility spread is the spread (z) that needs to be added to each point of the Treasury zero coupon yield curve so that the resultant yield curve may be used to reconcile MBS market price with the cash flow stream the MBS holder may expect to receive.

This spread is called the *static* spread as it is the spread, in a static yield curve environment (ie no volatility of interest rates), that an investor may expect to earn over Treasuries. The payments made by the MBS in a static yield curve environment can also be predicted relatively accurately.

Investors should take great care in using static spread as a measure of relative value. A portion of the static spread is compensation for accepting prepayment risk. Certain tranches of CMO (planned amortization class support tranches, for example) will have much larger prepayment risk than other tranches. An investor needs to ask himself if the spread offers suitable potential compensation for the (prepayment) risk inherent in the security.

In the real (volatile) interest rate environment the additional expected return from investing in MBS is less than the static spread. To see why this is so, first consider the scenario of falling yields, the return on bullet bonds (if held to maturity) will fall because of the lower yield available on reinvested coupons; the return on MBS (if held to maturity) will fall by even more because prepayments will now be larger than originally anticipated (and these need to be reinvested at low yield). Next consider the scenario of increasing yields, the return on bullet bonds (if held to maturity) will increase because of the higher yield available on reinvested coupons; the return on MBS (if held to maturity) will also increase but by a smaller amount because prepayments will now be slower than originally anticipated.

The static spread, which is the excess return that the MBS holder earns in a static yield curve environment will therefore, in general, not be realised. In general, for small changes in yield MBS outperform Treasuries but for large changes in yield MBS underperform Treasuries.

BANK FOR INTERNATIONAL SETTLEMENTS

Dynamic valuation framework

- The payments that will be received by the MBS holder are strongly affected by the future interest rate environment
- The payments are path-dependent

➢ To value MBS we need to use the valuation framework that we developed in the last presentation:
  1. Simulate the evolution of the short rate
  2. Determine payments that occur along each path
  3. Discount payments by successively using the short rate
  4. Repeat many time

We have seen that the refinancing incentive makes the size of the payments that are made to the MBS holder strongly dependent on the evolution of interest rates. MBS valuation therefore requires the framework that was developed in the previous presentation. The steps in the slide shows the 4 steps that needs to be followed.

There are two reasons why the payments made by MBS depend, not only on the current value of the short rate and on its possible future evolution, but also on the path that the interest rate process took to get to its current level. The first reason for path dependency is that payments depend on the remaining balance of the underlying mortgage pool, which depends on past prepayment activity, which depends on past refinancing incentive, which depends on past interest rate levels. The second reason for path dependency is burnout, which was discussed earlier.

Simply discounting an averaged anticipated cash flow stream using the zero coupon yield will result in a MBS value that we know is too high ($P_{Mkt} < PV_{Static}$) because it undervalues the prepayment option held by the borrower. The dynamic valuation framework that we will use next to value the MBS attempts to properly account for the prepayment option of the borrower. The MBS value determined using a dynamic valuation framework, which attempts to properly value the prepayment option embedded in the MBS, is therefore below the MBS value determined in a static framework. The difference in these two values can be regarded as the value of the prepayment option. The value of the prepayment option can also be expressed as a yield, and is the difference between the static spread and the option adjusted spread (the static spread is invariably larger than the OAS because static valuation results in a larger overestimate of MBS value than dynamic valuation does).

BANK FOR INTERNATIONAL SETTLEMENTS

Dynamic valuation framework: scenario short rate

	Path Number				
Month	**1**	**2**	**3**	**...**	**N**
1	$r_1(1)$	$r_1(2)$	$r_1(3)$	...	$r_1(N)$
2	$r_2(1)$	$r_2(2)$	$r_2(3)$	...	$r_2(N)$
3	$r_3(1)$	$r_3(2)$	$r_3(3)$	...	$r_3(N)$
...	...	...	...	...	...
t	$r_t(1)$	$r_t(2)$	$r_t(3)$	...	$r_t(N)$
...	...	...	...	...	...
358	$r_{358}(1)$	$r_{358}(2)$	$r_{358}(3)$	...	$r_{358}(N)$
359	$r_{359}(1)$	$r_{359}(2)$	$r_{359}(3)$	...	$r_{359}(N)$
360	$r_{360}(1)$	$r_{360}(2)$	$r_{360}(3)$	...	$r_{360}(N)$

- r_t corresponds to monthly compounding
- r_t is not an annualised rate ($12 \times r_t$ is the corresponding annualised value)

The table shows N simulated paths for the short rate. For each path the evolution of the short rate is simulated over 360 months. The rates in the table correspond to monthly compounding and is not annualised (the annualised value of the, monthly compounded, rate is 12r).

BANK FOR INTERNATIONAL SETTLEMENTS

Dynamic valuation framework: scenario zero rate

	Path Number				
Month	**1**	**2**	**3**	**...**	**N**
1	$z_1(1)$	$z_1(2)$	$z_1(3)$	...	$z_1(N)$
2	$z_2(1)$	$z_2(2)$	$z_2(3)$	...	$z_2(N)$
3	$z_3(1)$	$z_3(2)$	$z_3(3)$	...	$z_3(N)$
...	...	...	...	...	...
t	$z_t(1)$	$z_t(2)$	$z_t(3)$	...	$z_t(N)$
...	...	...	...	...	...
358	$z_{358}(1)$	$z_{358}(2)$	$z_{358}(3)$	...	$z_{358}(N)$
359	$z_{359}(1)$	$z_{359}(2)$	$z_{359}(3)$	...	$z_{359}(N)$
360	$z_{360}(1)$	$z_{360}(2)$	$z_{360}(3)$	...	$z_{360}(N)$

$$(1+z_t(n))^t = (1+r_1(n))\times(1+r_2(n))\times(1+r_3(n))\times\ldots\times(1+r_t(n))$$

From the path followed by the short rate we can determine the zero coupon yield that should be used to discount payments that occur at any point along each path. Let $z_t(n)$ be the zero coupon yield corresponding to time t along path n (once again, this yield corresponds to monthly compounding and is not annualized).

Dynamic valuation framework: scenario cash flow

	Path Number				
Month	**1**	**2**	**3**	**...**	**N**
1	$C_1(1)$	$C_1(2)$	$C_1(3)$	...	$C_1(N)$
2	$C_2(1)$	$C_2(2)$	$C_2(3)$	...	$C_2(N)$
3	$C_3(1)$	$C_3(2)$	$C_3(3)$	...	$C_3(N)$
...	...	...	...	...	...
t	$C_t(1)$	$C_t(2)$	$C_t(3)$	...	$C_t(N)$
...	...	...	...	...	...
358	$C_{358}(1)$	$C_{358}(2)$	$C_{358}(3)$	...	$C_{358}(N)$
359	$C_{359}(1)$	$C_{359}(2)$	$C_{359}(3)$	...	$C_{359}(N)$
360	$C_{360}(1)$	$C_{360}(2)$	$C_{360}(3)$	...	$C_{360}(N)$

- Different prepayment models result in:
  - Different cash flows
  - Different model values (and different OAS)

Along any interest rate path we can determine the refinancing incentive. With this refinancing incentive a prepayment model enables us to determine the cash flows that are made to the MBS holder along each of the interest rate paths.

BANK FOR INTERNATIONAL SETTLEMENTS

Dynamic valuation framework

$$PV_2(3) = \frac{C_2(3)}{(1+z_2(3))^2}$$

	Path Number				
Month	**1**	**2**	**3**	**...**	**N**
1	$PV_1(1)$	$PV_1(2)$	$PV_1(3)$	...	$PV_1(N)$
2	$PV_2(1)$	$PV_2(2)$	$PV_2(3)$	...	$PV_2(N)$
3	$PV_3(1)$	$PV_3(2)$	$PV_3(3)$	...	$PV_3(N)$
...	...	...	...	...	...
t	$PV_t(1)$	$PV_t(2)$	$PV_t(3)$	...	$PV_t(N)$
...	...	...	...	...	...
358	$PV_{358}(1)$	$PV_{358}(2)$	$PV_{358}(3)$	...	$PV_{358}(N)$
359	$PV_{359}(1)$	$PV_{359}(2)$	$PV_{359}(3)$	...	$PV_{359}(N)$
360	$PV_{360}(1)$	$PV_{360}(2)$	$PV_{360}(3)$	...	$PV_{360}(N)$
Sum	PV(1)	PV(2)	PV(3)	...	PV(N)

- Model value < PV_{Static}
- Model value ≠ P_{Mkt}
- In general:

PV_{Static} > Model value > P_{Mkt}

$$\text{Model value} = \sum_{n=1}^{N} PV(n) \Big/ N$$

$$PV(3) = \sum_{i=1}^{360} PV_i(3)$$

The present value of a single cash flow that occurs along one of the simulated interest rate paths can now be determined by discounting it using the appropriate zero coupon yield. The value of the MBS for one of the simulated interest rate paths is equal to the sum of the PVs of all the cash flows that get made along the simulated path. And the model value for the MBS is the average of the MBS values for each of the simulated interest rate paths.

As this dynamic valuation framework "properly" accounts for the value of the prepayment option the model value will be a better representation of the "true" value of the MBS (the market price **is** the true value) than the static MBS value, PV_{Static}, described earlier. In particular, we expect the model value to be less than PV_{Static}, and the market price below the model value.

BANK FOR INTERNATIONAL SETTLEMENTS

Dynamic valuation framework: OAS

$$PV_2(3) = \frac{C_2(3)}{(1+z_2(3)+K)^2}$$

	Path Number				
Month	**1**	**2**	**3**	**...**	**N**
1	$PV_1(1)$	$PV_1(2)$	$PV_1(3)$	...	$PV_1(N)$
2	$PV_2(1)$	$PV_2(2)$	$PV_2(3)$	...	$PV_2(N)$
3	$PV_3(1)$	$PV_3(2)$	$PV_3(3)$	...	$PV_3(N)$
...	...	...	...	...	...
t	$PV_t(1)$	$PV_t(2)$	$PV_t(3)$	...	$PV_t(N)$
...	...	...	...	...	...
358	$PV_{358}(1)$	$PV_{358}(2)$	$PV_{358}(3)$	...	$PV_{358}(N)$
359	$PV_{359}(1)$	$PV_{359}(2)$	$PV_{359}(3)$	...	$PV_{359}(N)$
360	$PV_{360}(1)$	$PV_{360}(2)$	$PV_{360}(3)$	...	$PV_{360}(N)$
Sum	PV(1)	PV(2)	PV(3)	...	PV(N)

- Model value = P_{Mkt}
- When:
  - K = OAS

$$\text{Model value} = \sum_{n=1}^{N} PV(n) \Big/ N$$

$$PV(3) = \sum_{i=1}^{360} PV_i(3)$$

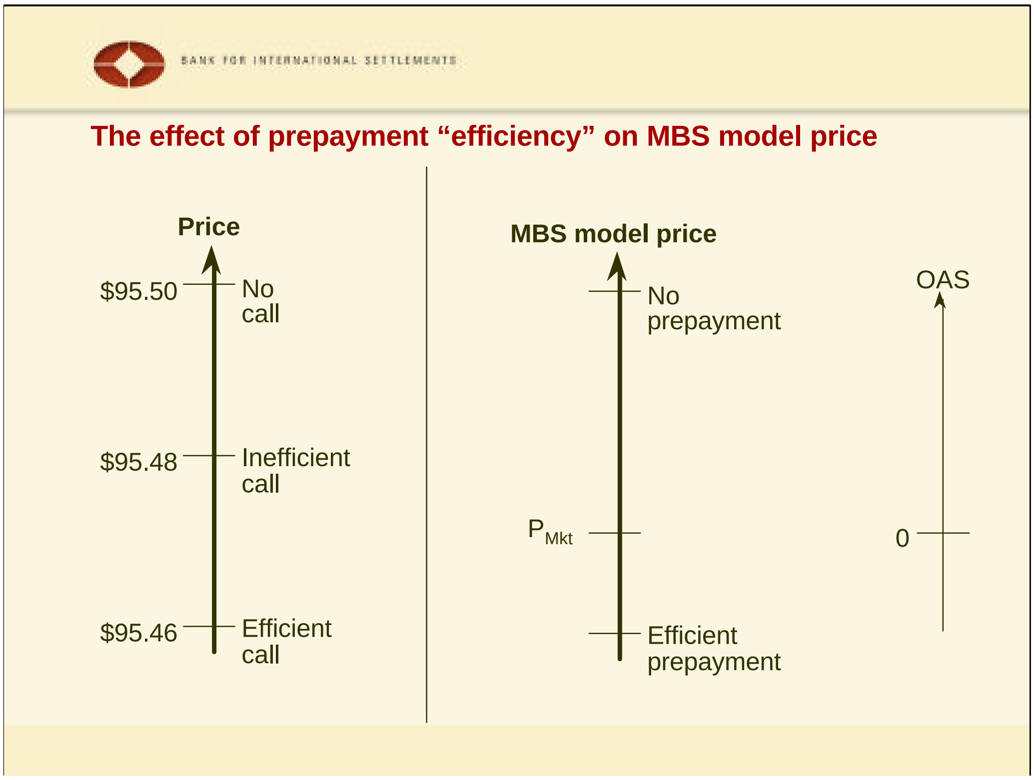
We cannot possibly expect the MBS value that is the result of our dynamic valuation framework to exactly match the observed market price of the MBS. This is because there are many assumptions that have gone into the modelling of prepayments: different prepayment models result in different model values.

In order to be able to reconcile model value with observed market price we add a spread to each zero coupon yield before using the resultant yield to discount the simulated cash flow stream. This spread is the Option-Adjusted Spread (OAS). This spread is termed *option-adjusted* because the dynamic valuation model produces a MBS valuation that already accounts for (the value is adjusted for) the prepayment option of the borrower.

Interpretation of OAS

- The OAS can be interpreted as a premium that investors require for assuming prepayment risk inherent in holding a MBS
- OAS is security-specific
- Different prepayment models produce different OAS
  - OAS may be interpreted as a spread-based measure of the unexplained portion of the MBS market price
- Consider two extremes:
  1. Model assumes no prepayments
     - High MBS price (above market price)
     - High OAS
  2. Model assumes "optimal" prepayments
     - Low MBS price (below market price)
     - Negative OAS

We have seen that the OAS is a spread over the Treasury zero coupon curve that is needed to reconcile model value with market price. The OAS of MBS is normally positive, this means that the valuation model overvalues the MBS. The fact that you can buy MBS at a price that is lower than the model value is normally interpreted as compensation to investors for assuming prepayment risk.



The left hand side of the slide shows how the price of a callable bond varies as we change our assumption as to how efficiently the issuer exercises its call option (see presentation: "A framework for valuing fixed income securities"). In that presentation we saw that if the issuer efficiently exercises its call option the bond price is $95.46. We also saw that if we assume that the issuer calls half of the outstanding bonds then the bond price is $95.48. If we assume that the issuer is completely inefficient at exercising it call option (it never calls) then the value of the bond will be the same as that of a bullet bond: $95.50.

Efficient exercises of the call option therefore lowers the value of the bond and transfers value from the bond holder (the option writer) to the

bond issuer (the option holder).

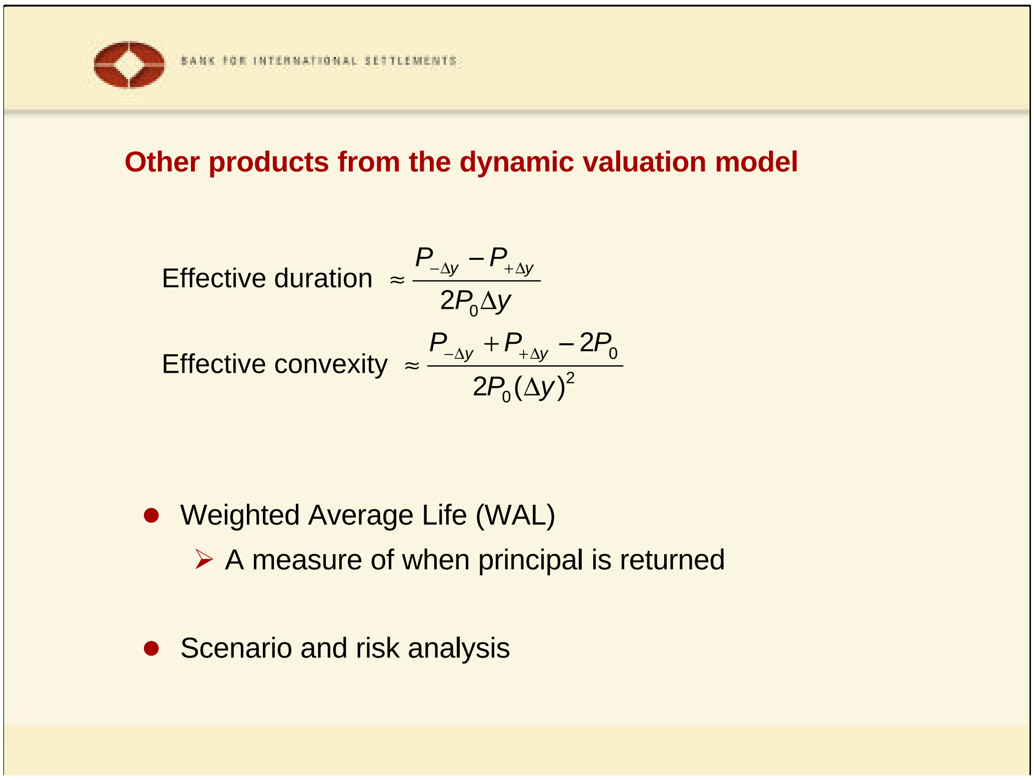
We can view the effect, on MBS price, of changes in the efficiency with which borrowers exercise their prepayment option in a very similar fashion: the more efficiently the borrower exercises his prepayment option the lower the value of the MBS.

Consider two extremes:

First a (very unrealistic) prepayment model that assumes that borrowers never prepay their mortgage. Using this assumption in our MBS valuation framework will overvalue the MBS (the model price will be above the market price). This is because the market known that the homeowners will be more efficient in the exercise of their prepayment option, an action which normally transfers value from MBS holder to homeowner.

Next an (equally unrealistic) prepayment model that assumes that borrowers will fully prepay their mortgage whenever this detracts value from the MBS holder. Using this assumption in our MBS valuation framework will undervalue the MBS (the model price will be below the market price). This is because the market knows that the prepayment behaviour of homeowners will not be as bad for the MBS holder as the prepayment model assumes.

Between these two extreme prepayment behaviours there is a prepayment behaviour for which the model value equals market price.



The standard measure of duration is modified duration. The limitation of modified duration is that it assumes that when interest rates change the cash flow of the security does not change.

To determine effective duration we first use our valuation model to value the MBS under the current term structure of the yield curve. Let P_0 to be the MBS model value under these conditions.

The entire yield curve is then shifted up by a small amount (Δy). In the usual way, we then simulate multiple possible evolutions of the short rate which are consistent with this "shocked" term structure of interest rates. We value the MBS in the usual way by modelling prepayments along the simulated interest rate paths, discounting and averaging. let $P_{+\Delta y}$ be the value of the MBS under this shocked yield curve. This exercise is then repeated for a yield curve that is shifted down by a small amount (Δy).

The equation in the slide shows how to compute effective duration (and effective convexity) from the the MBS model values under the current and shifted yield curve scenarios.

Effective convexity is computed in a similar manner.

Appendix II describes the WAL of an MBS.

With the valuation model that we have developed we can perform scenario analysis. Under a given scenario we can use the valuation model to determine the payments that are made by the MBS over a forthcoming period, we can also also use the valuation model to value the MBS at the end of this period. With this information we can determine the total return offered by the MBS under the specified scenario. This return can be compared with that that would be provided by Treasuries under the same scenario.

In a similar manner we can use our valuation model to determine the return distribution over a specified period, from the return distribution we can extract information such as the expected return and VaR. An alternative is to compute the distribution of the difference in return between MBS and a "benchmark" asset class such as Treasuries (measures such as tracking error can then be obtained from this distribution).

BANK FOR INTERNATIONAL SETTLEMENTS

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Appendix I: Mortgage cash flow

In this appendix we analyse the cash flows of a fixed-rate, level-payment mortgage.

For a fixed-rate, level-payment mortgage scheduled contractual payments of size C are made on N payment dates. These payments cover interest payments and also serve to fully amortise the loan so that the outstanding balance after the Nth payment is zero. The size of the payments needed to fully amortise the loan depends on the number of payments to be made as well as on the contract rate, r [1], of the mortgage.

Let us define the following additional terms:

- B_0 The initial loan balance (the amount borrowed)
- B_t The balance remaining after payment on date t
- A_t The amount by which the loan amortises on date t
- I_t The interest payment made on date t

The contractual payments meet both interest payments and serve to amortise the loan:

$$C = I_{t+1} + A_{t+1} \tag{1}$$

The interest due on date $t+1$ may be determined from the contractual rate and the balance outstanding on date t:

$$I_{t+1} = rB_t \tag{2}$$

The outstanding loan balance on date $t+1$ is reduced from its previous value through amortisation:

$$B_{t+1} = B_t - A_{t+1} \tag{3}$$

Using equation (1) and then equation (2) we may write:

$$\begin{aligned} A_{t+1} &= C - I_{t+1} \\ A_{t+1} &= C - rB_t \end{aligned} \tag{4}$$

Substituting this last expression for A_{t+1} into equation (3) gives:

$$B_{t+1} = (1+r)B_t - C \tag{5}$$

Equation (5) shows that the outstanding balance on any date is equal to the future value of the previous outstanding balance, $(1+r)B_t$, less the scheduled payment, C.

We may invert equation (5) to get an expression that shows how the balance remaining on a given payment date is related to the balance remaining on the following payment date, the contract rate, and the scheduled contractual payment:

$$B_{t-1} = \frac{B_t}{1+r} + \frac{C}{1+r} \tag{6}$$

---

[1] The contract rate used here is the rate appropriate to the period between payments (usually one month). The rate in the mortgage contract will be the equivalent annualised rate, computed using simple compounding. If r is a monthly rate than the annualised rate in the mortgage contract will be $12r$.

For a fully amortising loan equation (6) is subject to the boundary condition $B_N = 0$. With this boundary condition we can successively back substitute to get:

$$\begin{aligned}
B_N &= 0 \\
B_{N-1} &= \frac{B_N}{1+r} + \frac{C}{1+r} = \frac{C}{1+r} \\
B_{N-2} &= \frac{B_{N-1}}{1+r} + \frac{C}{1+r} = \frac{C}{(1+r)^2} + \frac{C}{1+r} \\
B_{N-3} &= \frac{B_{N-2}}{1+r} + \frac{C}{1+r} = \frac{C}{(1+r)^3} + \frac{C}{(1+r)^2} + \frac{C}{1+r} \\
&\ldots\ldots \\
B_{N-M} &= C\sum_{i=1}^{M} \frac{1}{(1+r)^i} = C\frac{1-(1+r)^{-M}}{r} \\
B_0 \equiv B_{N-N} &= C\sum_{i=1}^{N} \frac{1}{(1+r)^i} = C\frac{1-(1+r)^{-N}}{r}
\end{aligned} \tag{7}$$

Equation (7) shows that when there are M remaining scheduled payments the outstanding balance on the loan is equal to the present value of an ordinary annuity that pays an amount C per period for the next M periods. The initial balance on the mortgage loan is simply the present value of an ordinary annuity that pays an amount C per period for the next N periods.

The payment per dollar of initial balance is:

$$\frac{C}{B_0} = \frac{r}{1-(1+r)^{-N}} \tag{8}$$

The payment per dollar of initial balance increases in r and decreases in N. If $N = 1$, then $C/B_0 = 1+r$ (the payment on a one-period loan covers total principal plus interest). If $N = \infty$, then $C/B_0 = r$ (the payment covers periodic interest only).

We now determine the functional form describing the reduction in the balance outstanding. We use equation (5) to write out the first few terms that, by extension, lead to the general expression:

$$\begin{aligned}
B_1 &= (1+r)B_0 - C \\
B_2 &= (1+r)B_1 - C = (1+r)^2 B_0 - (1+r)C - C \\
B_3 &= (1+r)B_2 - C = (1+r)^3 B_0 - (1+r)^2 C - (1+r)C - C \\
&\ldots\ldots \\
B_t &= (1+r)^t B_0 - C\sum_{i=0}^{t-1}(1+r)^i = (1+r)^t B_0 - C\frac{(1+r)^t - 1}{r}
\end{aligned} \tag{9}$$

The outstanding balance may be interpreted in terms of the future value of an ordinary annuity. The first term on the right hand side of the last line of equation (9) is the future value of the initial balance of the mortgage compounded at a rate r per period. From this value we subtract the future value of an annuity paying C per period (the second term on the right hand side of the last line of equation (9)) in order to get the outstanding balance of the mortgage.

Equation (9) may be simplified by substituting from equation (8) for C. This gives:

$$B_t = B_0 \left\{ (1+r)^t - \frac{(1+r)^t - 1}{1-(1+r)^{-N}} \right\} \tag{10}$$

An example will clarify the material presented so far. Let us assume that a homeowner takes out a 30-year \$100,000 initial balance mortgage at a contract rate of 6% pa. At the end of 360 monthly payments the mortgage is fully paid off. The known parameters are:

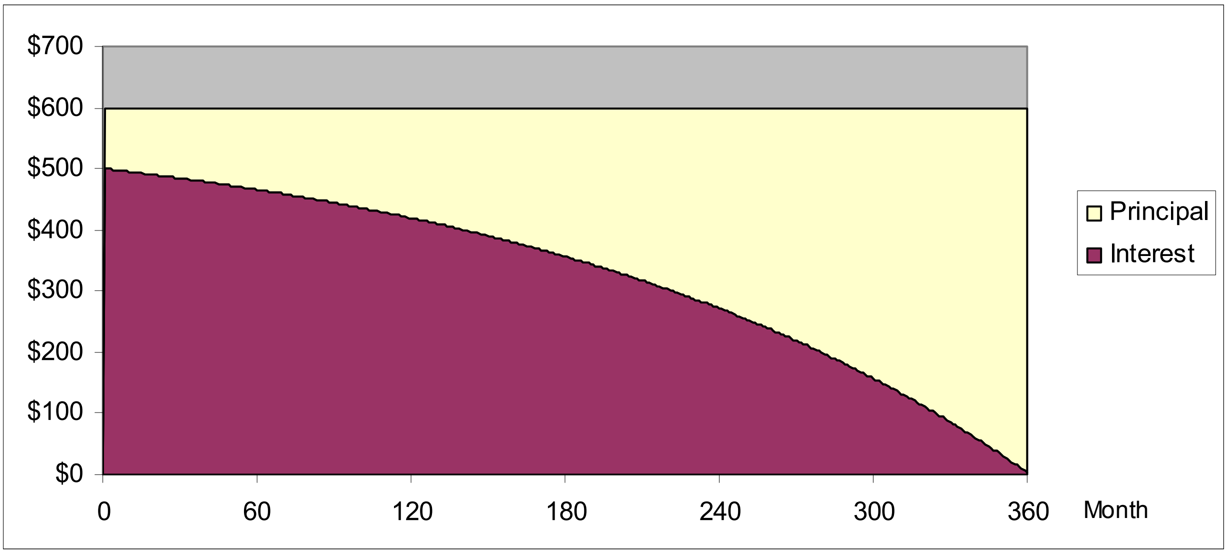
- $B_0 = \$100{,}000$
- $N = 360$
- $r = 0.06/12 = 0.005$

From equation (8) we determine the contractual payments to be $600 per month.

The following table shows the first few month and the last few months of the mortgage contract. The table shows the balance outstanding as well as the split between interest and principal payments.

Month	Balance	Interest	Principal
1	$99,900.45	$500.00	$99.55
2	$99,800.40	$499.50	$100.05
3	$99,699.85	$499.00	$100.55
4	$99,598.80	$498.50	$101.05
…			
356	$2,368.52	$14.77	$584.78
357	$1,780.81	$11.84	$587.71
358	$1,190.17	$8.90	$590.65
359	$596.57	$5.95	$593.60
360	$0.00	$2.98	$596.57

The division between interest and principal payments across the entire live of the mortgage is shown in the following graph.



From the graph it is clear that in the early years of the mortgage the payments primarily service interest payments whereas in the latter years of the mortgage the payments primarily pay down the outstanding balance on the mortgage.

Since interest payments are proportional to the outstanding balance this part of the graph also gives a good visual representation of how the outstanding balance decrease over the mortgage life.

Appendix II: Effect of prepayments on mortgage cash flows

In this appendix we show how the cash flow stream of a mortgage pool changes when prepayments are made. The notation that was introduced in Appendix I remains valid.

If a fraction η_{t+1}[2] of the mortgages remaining in the pool on date t prepay on date $t+1$ then the cash flow resulting from prepayments, P_{t+1}, is:

$$P_{t+1} = \eta_{t+1}\left\{(1+r)B_t - C\right\} \tag{11}$$

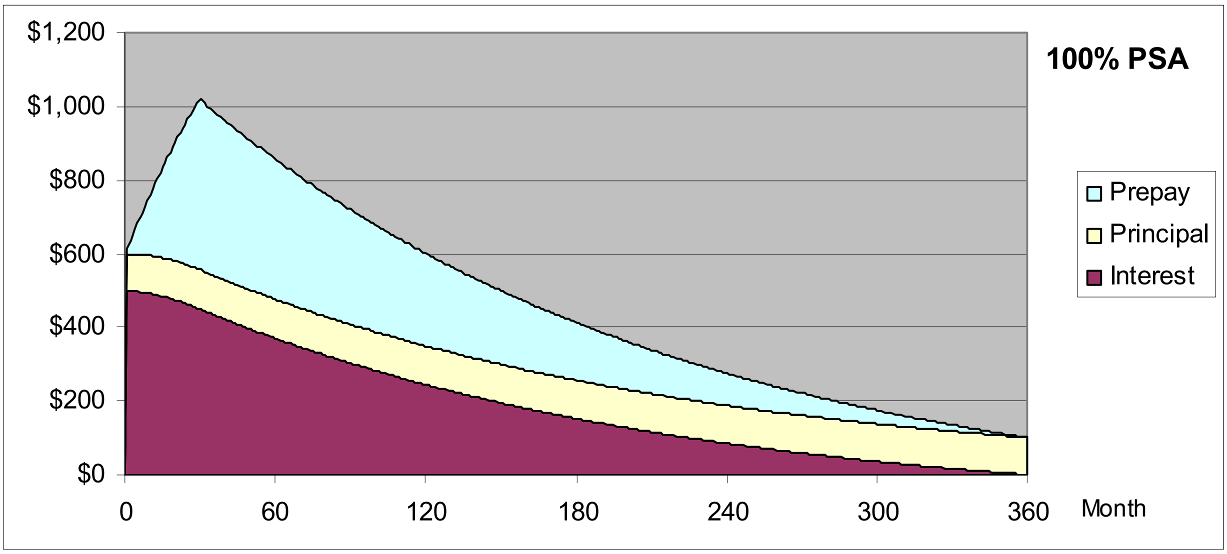
The term in curly brackets is the outstanding balance on date $t+1$ after the scheduled contractual payment but before prepayments (our notation remains consistent in that, B_t represents the balance outstanding on date t after all payments, both scheduled and prepayments).

Equation (5), which determines how the outstanding balance evolves, is now modified to account for prepayments:

$$B_{t+1} = (1+r)B_t - C - P_{t+1} \tag{12}$$

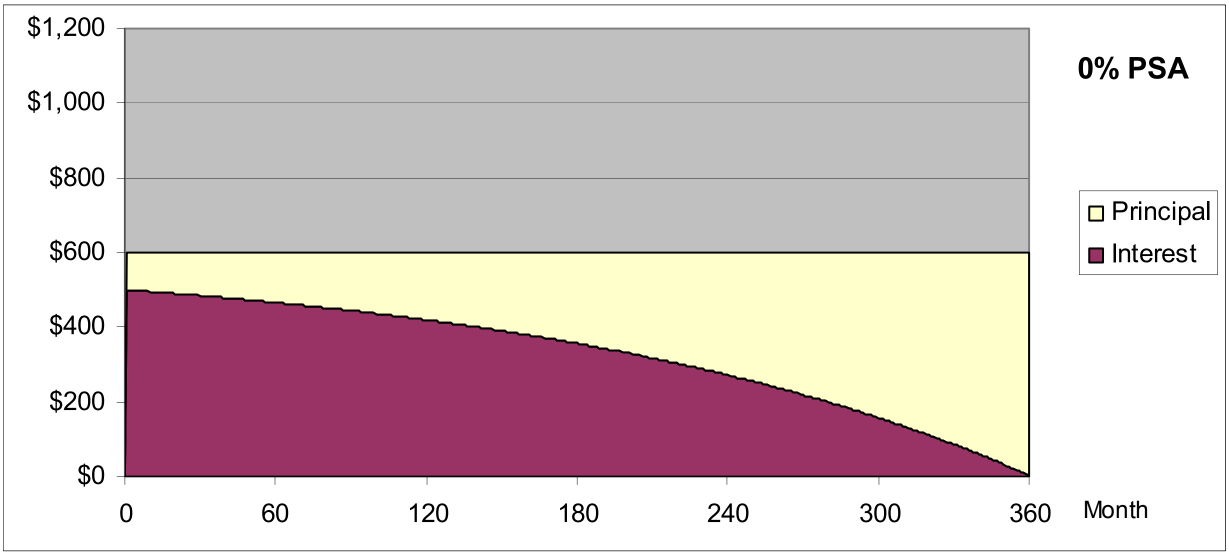
To determine how the outstanding balance evolves we need to model η_t. The prepayment rate will typically be a complex function of the mortgage characteristics, interest rate and macroeconomic environments. For the results presented here we assume that η_t follows the standard 100% PSA model, this implies a linear increase from a prepayment rate of zero at mortgage origination to a prepayment rate corresponding to 6% CPR after 30 months and which then remains constant at 6% CPR at longer maturities.

The graph below shows the total payments on any given month and how these are divided between interest payments, scheduled repayment of principal, and prepayment of principal. The characteristics of the mortgage match those in appendix I (an initial balance of \$100,000, a contract rate of 6%, and a maturity of 30 years).



[2] If the period between scheduled payments are monthly periods then η is commonly called the Single Monthly Mortality (which is denoted by R_{SMM} in appendix III).

For comparison below we plot the same graph under the assumption of zero prepayment (this is the same graph at that in appendix I).



Principal payments
The total repayment of principal summed across all months is the same in both of the above graphs. In the first graph this repayment is made up of both scheduled repayments and prepayments, in the second graph the repayment of principal is made up entirely of scheduled repayments. This implies that the sum of the areas of the prepayment component and the scheduled principal repayment component in the first graph is equal to the area of the scheduled principal repayment component in the second graph.

At 100% PSA, the repayment of principal is accelerated by prepayments, reaching a peak after 30 months. In contrast, if there are no prepayments, the rate of repayment of principal keeps on increasing throughout the life of the mortgage.

Determining WAL
The speed at which the initial balance is repaid by the borrower determines the weighted average life (WAL) of a mortgage. The WAL is a measure of the average life for which the initial balance remains outstanding.

On date t the amount of principal paid is equal to the sum of scheduled amount by which the loan is amortised, A_t, plus the amount prepaid, P_t. It is this total principal that gets paid on a given date that is used to determine the weighting factors, w_t, for computing the WAL of the mortgage:

$$w_t = \frac{A_t + P_t}{B_0} \tag{13}$$

The total principal payments across all payment dates must equal the initial balance of the mortgage. Hence:

$$\sum_{t=1}^{N} w_t = 1 \tag{14}$$

The WAL is given by:

$$WAL = \sum_{t=1}^{N} t w_t \tag{15}$$

For our 30-year, 6% coupon mortgage the WAL of the non-prepaying mortgage pool is 19.3 years and the WAL of a pool that prepays at 100% PSA is 11.4 years. In the case of a mortgage pool that prepays at 1000% PSA the WAL falls to 2.3 years.

Interest payments

Even though prepayments have no effect on the total amount of principal paid by the borrower (which must equal the initial balance in all cases), they do have an effect on the total amount of interest payments made by the borrower. Faster prepayments lead to smaller outstanding balances and consequently smaller interest payments on subsequent payment dates. This effect is clearly seen by comparing the two graphs in this appendix: the area of the interest component is much smaller in the case of 100% PSA (total interest payments of $68,181) than it is in the case of no prepayments (total interest payments of $115,838).

Appendix III: Quantifying prepayment rates

The three measures commonly used to quantify prepayments were introduced in the presentation. Here we give further details of how these measures are related.

The Single Monthly Mortality (SMM) is the proportion of the start-of-month balance that prepay in a given month. Let us denote the SMM rate by R_{SMM}. The fraction of the start-of-month balance that remains in the pool after the prepayments made during the month is:

$$1 - R_{SMM}$$

If the SMM rate remains constant over a 12-month period then the fraction of the start-of-period balance that remains in the pool after the 12-month period is:

$$\left(1 - R_{SMM}\right)^{12}$$

This 12-month "survival" fraction may now be converted into the proportion of the start-of-period balance that prepay over the 12-month period. This measure is known as the Constant Prepayment Rate (CPR). Let us denote this rate by R_{CPR}:

$$R_{CPR} = 1 - \left(1 - R_{SMM}\right)^{12}$$

This equation shows us how to determine CPR from SMM. Inverting gives us an equation to obtain SMM from CPR:

$$R_{SMM} = 1 - \left(1 - R_{CPR}\right)^{1/12}$$

It is important to realise that even though CPR represents an annualised measure it is simply a scaled version of the SMM (monthly) measure. The CPR can therefore be used to specify the prepayment rate that applies to one particular month, as such the CPR may vary from month to month.

The prepayment rate specified by the PSA prepayment model is a function of the age of the mortgage. The prepayment rate increase linearly over the first 30 months of the mortgage life and remains constant thereafter. If a mortgage prepays at a rate of 100% PSA, then the CPR for the month that the mortgage in n months old is:

$$R_{CPR} = 6\% \times Min\left(1, \frac{n}{30}\right)$$

When the mortgage is 15 months old the mortgage will prepay at a CPR of 3% and when it is 40 months old it will prepay at a CPR of 6%.

If a mortgage is prepaying at a CPR of 9% when it is 15 months it is said that the mortgage is prepaying at 300% PSA (since 100% PSA corresponds to a CPR of only 3%).

It is seen that the three measures that are commonly used to quantify prepayment rates are in effect just three different units of measure to express the same underlying phenomenon.

Appendix IV: building a valuation model

In this appendix we describe how we have built a model for the valuation of securities backed by the cash flow from pools of fixed-rate, level-payment mortgages. The general framework needed to build an MBS valuation model has been described in the presentation; we avoid repeating this here and instead look into the details of the specific model that we have built.

The two components that are needed for the valuation of MBS are:

1. An interest rate model
1. A prepayment model

These are discussed in turn below.

Modelling the evolution of interest rates

A model for the evolution of interest rates is needed to determine mortgage prepayment rates. The current level of the yield curve is one of the major determinants of the rate on new mortgages (the current-coupon mortgage rate). The current-coupon rate will, in turn, determine the incentive for borrowers to refinance.

The interest rate model is also used to discount future cash flows that occur in the Monte Carlo simulation that is used to value the MBS.

We have used the Black, Derman and Toy (BDT) binomial tree to model the evolution of the short rate (the interest rate appropriate to discount cash flows that occur next period). The term structure of the yield curve that is used to calibrate the binomial tree is the current swap curve and its volatility (which has been determined from the last year of historic data). Full details on how to construct the binomial tree of short rates are given by Black, Derman and Toy [1990].

Mortgages have monthly cash flows: to be able to analyse mortgages that mature in 30 years, we need a tree with monthly periods extending out to 30 years (a total of 360 periods).

BDT specify a methodology to construct a tree of short rates. At each node in the tree we also know the shape of the entire zero-coupon yield curve (up to tree maturity). This can be determined from the short rate at the node as well as the short rates on all (accessible) nodes to its right. We use this zero-coupon yield curve to model the current coupon rate (see below).

Modelling prepayments

In our prepayment model home sales (turnover) and mortgage refinancing are the sole determinants of prepayment rates. Defaults, curtailments and full payoffs also influence prepayment rates but the effect of these is small compared to the effect of refinancing and turnover.

Let η_N represent the fraction of homeowners with mortgages remaining from date $N-1$ that choose to prepay at date N. If $\eta_{T,N}$ is the turnover component, or fraction of homeowners who prepay following the sale of their home and $\eta_{R,N}$ the fraction of homeowners who refinance, it follows that:

$$\eta_N = \eta_{T,N} + \eta_{R,N} \tag{16}$$

$\eta_{R,N}$ will primarily be driven by the financial gain (Γ_N) that the homeowner obtains from refinancing the mortgage (quantifying Γ_N is described later).

Let us assume that the reasons for prepayment occur sequentially. In the first instance prepayment follows the sale of the home, this type of refinancing is relatively insensitive to Γ_N. In the second instance prepayment follows refinancing, this type of prepayment is strongly driven by Γ_N.

As a result of the sequential nature of the factors driving prepayments, $\eta_{R,N}$ will be proportional to the fraction ($1-\eta_{T,N}$) of homeowners that remain after prepayment resulting from home sales.

Both $\eta_{T,N}$ and $\eta_{R,N}$ depend on the extent that the mortgage has aged. The reason for this is that for some time after entering into a new mortgage agreement the homeowner will be reluctant to go through this process again. Let α_N represent the extent to which the mortgage has aged; a value of 0 is representative of a new, un-aged, mortgage and a value of 1 is representative of a fully aged mortgage. We model α_N as follows:

$$\alpha_N = 1 - \exp(-\Phi N) \tag{17}$$

Φ represents the speed at which mortgages age. The PSA model specifies that a mortgage is fully aged after 30 months, a value for Φ of 1/30 results in a similar aging speed.

$\eta_{T,N}$ is modelled in a very simple manner: we assume that aggregate home sales (across all mortgage pools) are constant[3]. The turnover component of prepayments is proportional to the degree to which the pool has aged:

$$\eta_{T,N} = T\alpha_N \tag{18}$$

T is the turnover speed for a fully aged mortgage.

The refinancing component of prepayments, $\eta_{R,N}$, will be influenced by the efficiency, ν_N, with which homeowners take advantage of refinancing opportunities. When a pool is exposed to refinancing opportunities the keenest refinancers in the pool will exit the pool, the remaining borrowers are less likely to take advantage of future refinancing opportunities. This process is known as *burnout* and results in the refinancing efficiency being a function of past refinancing opportunities (the efficiency decreases as the number of past refinancing opportunities increases). We do not model the effect of burnout and keep the refinancing efficiency constant ($\nu_N \equiv \nu$).

The refinancing component of prepayments is specified as follows:

$$\eta_{R,N} = (1 - \eta_{T,N})\alpha_N \mathrm{Min}\left[\mathrm{Max}\left[\nu \times \Gamma_N, 0\right], 1\right] \tag{19}$$

Equation (19) ensures that when the refinancing gain is negative mortgage refinancing is zero (negative refinancing is not possible). When the product $\nu \times \Gamma_N$ is larger than 1 the refinancing rate is capped at $(1 - \eta_{T,N})\alpha_N$, for a fully aged mortgage pool this implies that $\eta_N = \eta_{T,N} + \eta_{R,N} = 1$ and all mortgages remaining in the pool prepay on date N.

In order to determine the gain to be had from refinancing, we assume that when a borrower refinances he enters into a new mortgage agreement with a maturity equal to the residual maturity on his previous loan. Let us define the following terms:

- B = Outstanding loan balance
- r_c = Rate on the current loan
- r_n = Rate on the new loan (the current coupon rate)
- M = The residual maturity of the mortgage

Using equation (7) we can determine the monthly contractual payments on the current loan:

$$C = \frac{Br_c}{1 - (1 + r_c)^{-M}} \tag{20}$$

[3] Seasonal variations in the turnover rate are not accounted for.

Discounting these contractual payments at the rate available on an equivalent new mortgage gives the present value of the current contractual payments, PV_c:

$$PV_c = C\frac{1-(1+r_n)^{-M}}{r_n} = \frac{Br_c}{1-(1+r_c)^{-M}}\frac{1-(1+r_n)^{-M}}{r_n} \tag{21}$$

We know that the present value, PV_n, of the new loan, at a coupon rate r_n, is simply equal to the outstanding balance on the loan, B.

There are various costs associated with refinancing. Some of these are fixed costs that do not vary with the size of the mortgage other costs are variable and are proportional to the size of the loan. We define F as the fixed cost and $B \times V$ as the variable cost of refinancing. We may now determine the saving that the borrower realises from refinancing:

$$SAV = PV_c - PV_n - BV - F = \frac{Br_c}{1-(1+r_c)^{-M}}\frac{1-(1+r_n)^{-M}}{r_n} - B - BV - F \tag{22}$$

The percentage (of outstanding balance) saving that the borrower realises from refinancing is:

$$\%SAV = \frac{r_c}{1-(1+r_c)^{-M}}\frac{1-(1+r_n)^{-M}}{r_n} - 1 - V - \frac{F}{B} \tag{23}$$

Either SAV or $\%SAV$ can be used to quantify the refinancing gain Γ.

The refinancing gain is a function of the current coupon rate on new mortgages, r_n. The current coupon rate is determined from the zero-coupon yield curve (whose evolution we have modelled), by first determining the yield to maturity on an ordinary annuity that matures when the mortgage matures (this will equal the yield to maturity on a non-prepaying mortgage) and then adding a spread to this yield to get the current coupon rate. The spread in part covers the servicing fees but, more importantly, it compensates investors for assuming prepayment risk[4].

The only parameter needed by the prepayment model that we have not specified how to model is the efficiency, ν, with which borrowers take advantage of refinancing opportunities. This parameter is not modelled; it is instead determined through an iterative process so that the model value of the MBS matches the observed market price of the MBS.

Determining the refinancing efficiency in this manner will necessarily result in a model price that equals the market price, and therefore an OAS of zero. To the extent that the model price matches the market price, the payment made at each node in the binomial tree (which has been constructed to be in accordance with risk-neutral evolution of the short rate) is the expected value (in a risk-neutral world) of the possible payments that can be made at the node.

Our simple prepayment model will always result in an OAS of zero. We cannot therefore interpret the OAS as the risk premium that investors demand for holding prepayment risk. We may nevertheless use our prepayment model together with our valuation framework to perform scenario and risk analysis.

Determining the cash flow stream

Now that we have a model that enables us to determine the prepayments that get made during the lifetime of a mortgage pool, we use the modelled prepayment rate to determine the cash flow stream that results per \$1 initial balance in the pool.

We have defined η_N as the fraction of homeowners with mortgages remaining from date $N-1$ that prepay on date N (this is the single monthly mortality rate). Of the total original number of

[4] We have used a spread of 2.5%. On March 5 2003 the current coupon for a 30-year mortgage was 5.37% and that for a 15-year mortgage was 4.67%. These translate into spreads of 2.63% and 2.28% respectively for 30-year and 15-year mortgages (additional study is required to analyse how these spreads vary with maturity and how stable they are).

homeowners in the pool, prepayments result in only a fraction, γ_N, remain after prepayments made up to and including date N. We can determine the fraction of the original pool that are still in the pool on a particular date from the fraction remaining on the previous payment date and the fraction of homeowners that prepay:

$$\gamma_N = \gamma_{N-1}(1-\eta_N) \tag{24}$$

We know that $\gamma_0 = 1$ since at inception all borrowers are still in the pool.

Let us define the following additional terms:

- C The scheduled monthly contractual payments per \$1 initial balance
- $CF_{SP,N}$ The cash flow from scheduled payments on date N
- $CF_{PP,N}$ The cash flow from principal prepayment on date N
- $B_{B,N}$ The balance outstanding before prepayment, but after scheduled payments, on date N
- $B_{A,N}$ The balance outstanding after prepayments and after scheduled payments on date N
- r The contract coupon rate on the mortgage

The scheduled contractual payment made on date N can be determined from the fraction of homeowners that still remain in the pool after prepayments on date $N-1$:

$$CF_{SP,N} = \gamma_{N-1} \times C \tag{25}$$

The outstanding balance before prepayments on date N can be determined from the outstanding balance after prepayments on date $N-1$ (see equation (5)):

$$B_{B,N} = (1+r)B_{A,N-1} - C_{SP,N} \tag{26}$$

The cash flow resulting from prepayments on date N is equal to the outstanding balance before prepayments times the fraction of homeowners that prepay:

$$CF_{PP,N} = B_{B,N} \times \eta_N \tag{27}$$

The outstanding balance after prepayments on date N is equal to the outstanding balance before prepayments less the prepayments made:

$$B_{A,N} = B_{B,N} - CF_{PP,N} \tag{28}$$

Our prepayment model provides us with η_N and γ_N for all N. We also know what the values of C and r are for the specific mortgage pool that we are analysing. Furthermore, since we are determining the cash flow stream per \$1 initial pool balance, we know that $B_{B,0} = B_{A,0} = 1$.

We may now use equation (26) to determine $B_{B,1}$, we then use equation (27) to determine $CF_{PP,1}$, we finally use equation (28) to determine $B_{A,1}$. Starting with the value of $B_{A,1}$ that we have just determined we may use equations (26), (27) and (28) once more to determine $B_{B,2}$, $CF_{PP,2}$ and $B_{A,2}$ respectively. This process is repeated up to maturity to give us, in particular, the entire cash flow stream associated with the mortgage pool[5].

[5] The total cash flow stream resulting on date N is equal to the sum of the scheduled payments and the principal prepayment $CF_N = CF_{SP,N} + CF_{PP,N}$.