Quantifying credit risk in a corporate bond

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Summary of presentation

- What is credit risk?
- Probability of default
- Recovery rate statistics
- Rating migrations
- Expected loss under default mode
- Unexpected loss under default mode
- Expected loss under migration mode
- Unexpected loss under migration mode
- Numerical example
What is credit risk?

- A bond with credit rating single A at a price of $100 is bought
- Assume that some time during the next one year the issuer files for bankruptcy, and you are paid $40 back
- The $60 loss incurred on the bond is a loss resulting from taking credit risk
- Imagine a second scenario where the bond issuer’s rating is downgraded to BB, and consequently, the bond trades at $80
- The mark to market loss of $20 incurred is the consequence of a credit event occurring
- The risk of such a loss is also classified under credit risk
- Credit risk has two components: default risk and migration risk

Probability of default

- In the simplest scenario, we saw that the issuer either repays the debt or defaults on the debt repayment
- What is the chance that the issuer will file for bankruptcy?
- Issuer’s probability of default is the measure used to quantify this
- Probability of default (PD) is the probability that the issuer will default on its contractual obligations to repay debt (measured over a 1-year horizon)
- PD can be determined either using an empirical approach or using Merton’s structural approach
Probability of default

- Empirical approach makes use of a historical data base of corporate bond defaults
- Idea is to form a static pool of companies having a particular credit rating for a given year
- Annual default rates are calculated for each static pool, which are then aggregated to compute PD for each credit rating
- Merton’s approach is based on an option pricing approach
- This makes use of the current estimates of the firm’s assets, liabilities and asset volatility, and hence, is related to the underlying dynamics of the structure of the firm
- KMV approach to determine PD is similar to Merton’s approach

Recovery rate

- When an issuer files for bankruptcy, some fraction of the face value of the debt is recovered
- One could sell the debt to vulture funds when default occurs
- Moody’s proxy the recovery rate using secondary market price of the defaulted debt 1-month after default
- If for $100 face value the secondary market trading price is $40, then recovery rate is 40%
- Empirical results suggest that industrial sector, seniority of debt, state of the economy and credit rating of the issuer 1-year prior to default are important determinants of recovery rates
Recovery rate

- In order to incorporate the variations in recovery rates, recovery rate is treated as a random variable
- Observed recovery rates have a multi-modal distribution
- No consensus on the choice of the right distribution function
- Many existing credit risk models in the market use beta distribution for modelling recovery rates
- Choosing a different distribution function other than beta (say, uniform) will have an impact on the tail part of the loss distribution
- Empirical study on recovery rate statistics has been carried out by Edward Altman and co-workers

Recovery rate statistics

<table>
<thead>
<tr>
<th>Bond seniority</th>
<th>Number of issuers</th>
<th>Median</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior secured</td>
<td>134</td>
<td>57.42%</td>
<td>52.97%</td>
<td>23.05%</td>
</tr>
<tr>
<td>Senior unsecured</td>
<td>475</td>
<td>42.27%</td>
<td>41.71%</td>
<td>26.62%</td>
</tr>
<tr>
<td>Senior subordinated</td>
<td>340</td>
<td>31.90%</td>
<td>29.68%</td>
<td>24.97%</td>
</tr>
<tr>
<td>Subordinated</td>
<td>247</td>
<td>31.96%</td>
<td>31.03%</td>
<td>22.53%</td>
</tr>
</tbody>
</table>

Rating migrations

- Probability of default considers only two possible states for bond issuer: issuer either defaults on debt payments or is solvent
- In practice, actions of rating agencies can result in either the issuer being downgraded or upgraded to a different credit rating
- Associated with each credit rating is a different PD
- One could also estimate the probability of transitioning from one credit rating to another using historical rating migration statistics
- Rating agencies (Moody’s and S&P) estimate the migration probabilities for different ratings
- This data is represented in a matrix form, and the matrix is called rating transition (or migration) matrix, which is a Markov matrix

Rating migration matrix

<table>
<thead>
<tr>
<th>Initial rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>93.66%</td>
<td>5.63%</td>
<td>0.4%</td>
<td>0.08%</td>
<td>0.03%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>AA</td>
<td>0.66%</td>
<td>91.72%</td>
<td>6.94%</td>
<td>0.49%</td>
<td>0.06%</td>
<td>0.09%</td>
<td>0.02%</td>
<td>0.01%</td>
</tr>
<tr>
<td>A</td>
<td>0.07%</td>
<td>2.25%</td>
<td>91.75%</td>
<td>5.19%</td>
<td>0.49%</td>
<td>0.20%</td>
<td>0.01%</td>
<td>0.04%</td>
</tr>
<tr>
<td>BBB</td>
<td>0.03%</td>
<td>0.25%</td>
<td>4.83%</td>
<td>89.25%</td>
<td>4.44%</td>
<td>0.80%</td>
<td>0.16%</td>
<td>0.23%</td>
</tr>
<tr>
<td>BB</td>
<td>0.03%</td>
<td>0.07%</td>
<td>0.44%</td>
<td>6.66%</td>
<td>83.22%</td>
<td>7.46%</td>
<td>1.05%</td>
<td>1.07%</td>
</tr>
<tr>
<td>B</td>
<td>0.00%</td>
<td>0.10%</td>
<td>0.32%</td>
<td>0.46%</td>
<td>5.73%</td>
<td>83.61%</td>
<td>3.84%</td>
<td>5.94%</td>
</tr>
<tr>
<td>CCC</td>
<td>0.15%</td>
<td>0.00%</td>
<td>0.30%</td>
<td>0.69%</td>
<td>1.91%</td>
<td>10.28%</td>
<td>81.22%</td>
<td>25.25%</td>
</tr>
<tr>
<td>Default</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>
Quantifying credit risk

- We are familiar with the return distribution for a stock.
- One approximates this return distribution to be normal.
- When holding a corporate bond, credit events trigger price changes, which will result in a loss or a profit.
- To quantify credit risk, we will focus only on returns that are driven by credit events.
- We will refer to the returns that can be related to a credit event generically as a loss.
- A rating upgrade will then generate a negative loss.
- The credit-related losses will have a distribution, commonly referred to as the credit loss distribution.

Typical shape of the credit loss distribution
Expected loss under default mode

- Under the default mode, the issuer either repays debt or defaults
- If the issuer defaults, the bondholder recovers a fraction of the face value outstanding
- One can model the default process $\delta$ as a Bernoulli random variable that takes the value of 0 or 1 (1 signals default)

$$P_{\text{dirty}} \xrightarrow{\delta=1} \psi \xrightarrow{\delta=0} P_{\text{clean}}$$

Deriving expected loss (default mode)

- In the default mode framework, the price of the risky debt can be written as follows
  $$\tilde{P} = P_{\text{dirty}} \times I_{[\delta=0]} + \psi \times I_{[\delta=1]}$$

- Let us denote the credit loss variable as $\tilde{\ell}$
  $$\tilde{\ell} = P_{\text{dirty}} - \tilde{P} = P_{\text{dirty}} - P_{\text{dirty}} \times I_{[\delta=0]} - \psi \times I_{[\delta=1]}$$

- Expected loss is the expected value of the random variable $\tilde{\ell}$
  $$EL = E(\tilde{\ell}) = P_{\text{dirty}} - P_{\text{dirty}} \times (1 - PD) - E(\psi \times I_{[\delta=1]})$$
Deriving expected loss (default mode)

- Computing expected loss requires taking the expectation of two random variables, the recovery rate and the default process.
- If we assume they are independent, then we have:

\[
EL = P_{dirty} \times PD - RR \times PD \\
= PD \times (P_{dirty} - RR)
\]

- Introducing the term loss on default LD=P_{dirty}RR, we have the expected loss for a nominal exposure of NE:

\[
EL = NE \times PD \times LD
\]

Unexpected loss under default mode

- By definition, unexpected loss is the standard deviation of the random variable \( \ell \).
- Once again, we will make the assumption that the default and recovery process are independent.
- After some mathematical manipulations, it can be shown that the unexpected loss for a nominal exposure NE is given by:

\[
UL = NE \times \sqrt{PD \times \sigma^2_{RR} + LD^2 \times \sigma^2_{PD}}
\]

- Here, \( \sigma^2_{PD} \) is the variance of the Bernoulli random variable \( \delta \):

\[
\sigma^2_{PD} = PD \times (1 - PD)
\]
On the independence assumption

- Is it reasonable to make the assumption that default and recovery processes are independent?
- In Merton’s structural model, default and recovery rate processes can be shown to be inversely related to each other
- In reduced-form credit risk models, default and recovery rate processes are modelled to be independent
- Empirical results suggest that these two random variables are negatively correlated
- Recent results suggest that a simple microeconomic interpretation based on supply and demand tend to explain the relationship

Expected loss under migration mode

- Under migration mode, the changes in credit rating of the issuer will have to taken into account besides default
- To estimate the profit or loss that results from rating migrations, we need to estimate price changes due to rating changes
- We can estimate these price changes if we know the yield spread differences between different credit ratings
- If $\Delta y$ is the yield spread between two credit ratings, the corresponding price change can be approximated by,

$$\text{Price change} = -P_{\text{dirty}} \times D \times \Delta y + 0.5 \times P_{\text{dirty}} \times C \times \Delta y^2$$
Expected loss under migration mode

- Since we are interested in the credit loss distribution, the loss resulting from a credit rating change will be given by,
  \[
  \text{Loss} = \Delta P = P_{\text{def}} \times D \times \Delta y - 0.5 \times P_{\text{def}} \times C \times \Delta y^2
  \]
- The expected value of the credit loss for a rating change from the \(i\)th grade to the \(k\)th grade is given by,
  \[
  \Delta P_i = P_{\text{def}} \times D \times \Delta y_i - 0.5 \times P_{\text{def}} \times C \times \Delta y_i^2
  \]
- If we model 18 rating grades including the default state, the expected loss under rating migration is given by
  \[
  EL = NE \times \sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}
  \]

Unexpected loss under migration mode

- By definition, unexpected loss under migration mode is the standard deviation of the credit loss variable
- The loss variable under migration mode for a $1 exposure is
  \[
  \tilde{\xi} = \sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}
  \]
- Here, \(\Delta \tilde{P}_{ik}\) is the random credit loss when issuer rating changes from grade \(i\) to grade \(k\)
- The expected value of \(\Delta \tilde{P}_{ik}\) is \(\Delta \tilde{P}_{ik}\) and variance \(\sigma_{ik}^2\)
- When \(k\) is equal to the default state, \(\sigma_{ik}\) is equal to \(\sigma_{\text{def}}\)
Unexpected loss under migration mode

- The variance of the loss variable for $1$ exposure is given by,

\[
\text{Var}(\tilde{\ell}) = E\left(\sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}\right) - \left[E\left(\sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}\right)\right]^2
\]

- Taking expectations and assuming the recovery rate process and default process are independent, we get the following relation:

\[
\text{Var}(\tilde{\ell}) = \sum_{k=1}^{18} p_{ik} \times (\Delta \tilde{P}_{ik}^2 + \sigma_{ik}^2) - \left[\sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}\right]^2
\]

- For nominal exposure NE, it can be shown that UL is given by,

\[
UL = NE \times \sqrt{PD \times \sigma_{ex}^2 + \sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}^2 - \left[\sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}\right]^2}
\]

Computing yield spreads

- Yield spreads between different rating grades can be derived from current market prices
- One could have the yield spreads across different rating grades by maturity
- A simpler approach, through less precise, would be to use yield spreads across rating grades without taking into account maturity of the instrument
- The table on next page shows static yield spreads that have been used in numerical examples
**Numerical example**

<table>
<thead>
<tr>
<th>Security level details of example considered</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Issuer rating grade</td>
<td>A3</td>
<td></td>
</tr>
<tr>
<td>Settlement date</td>
<td>24 Apr 2002</td>
<td></td>
</tr>
<tr>
<td>Bond maturity date</td>
<td>15 Feb 2007</td>
<td></td>
</tr>
<tr>
<td>Coupon rate</td>
<td>6.91%</td>
<td></td>
</tr>
<tr>
<td>Dirty price for $1 nominal</td>
<td>1.0533</td>
<td></td>
</tr>
<tr>
<td>Nominal exposure</td>
<td>$1,000,000</td>
<td></td>
</tr>
<tr>
<td>Modified duration</td>
<td>4.021</td>
<td></td>
</tr>
<tr>
<td>Convexity</td>
<td>19.75</td>
<td></td>
</tr>
<tr>
<td>Mean recovery rate</td>
<td>47%</td>
<td></td>
</tr>
<tr>
<td>Volatility of RR</td>
<td>25%</td>
<td></td>
</tr>
</tbody>
</table>
Under default mode,
\[
EL = NE \times PD \times LD
\]
\[
= 1000000 \times 0.001 \times 0.5833 = $583.3
\]
\[
UL = NE \times \sqrt{PD \times \sigma_{\text{DR}}^2 + LD^2 \times \sigma_{\text{RD}}^2}
\]
\[
= 1000000 \times \sqrt{0.001 \times 0.25^2 + 0.5833^2 \times 0.001 \times (1 - 0.001)} = $20,059.88
\]

Under migration mode,
\[
EL = NE \times \sum_{i=1}^{15} p_i \times \Delta P_i
\]
\[
= 1000000 \times 0.003012 = $3012
\]
\[
UL = NE \times \sqrt{PD \times \sigma_{\text{DR}}^2 + \sum_{i=1}^{15} p_i \times \Delta P_i^2 - \left[ \sum_{i=1}^{15} p_i \times \Delta P_i \right]^2}
\]
\[
= 1000000 \times \sqrt{0.001 \times 0.25^2 + 5.259 \times 10^{-4} - 0.003012^2} = $24,069.2