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Quantifying credit risk in a corporate bond

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Summary of presentation

- What is credit risk?
- Probability of default
- Recovery rate statistics
- Rating migrations
- Expected loss under default mode
- Unexpected loss under default mode
- Expected loss under migration mode
- Unexpected loss under migration mode
- Numerical example

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What is credit risk?

- A bond with credit rating single A at a price of $100 is bought
- Assume that some time during the next one year the issuer files for bankruptcy, and you are paid $40 back
- The $60 loss incurred on the bond is a loss resulting from taking credit risk
- Imagine a second scenario where the bond issuer's rating is downgraded to BB, and consequently, the bond trades at $80
- The mark to market loss of $20 incurred is the consequence of a credit event occurring
- The risk of such a loss is also classified under credit risk
- Credit risk has two components: default risk and migration risk

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Probability of default

- In the simplest scenario, we saw that the issuer either repays the debt or defaults on the debt repayment
- What is the chance that the issuer will file for bankruptcy?
- Issuer's probability of default is the measure used to quantify this
- Probability of default (PD) is the probability that the issuer will default on its contractual obligations to repay debt (measured over a 1-year horizon)
- PD can be determined either using an empirical approach or using Merton's structural approach

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Probability of default

- Empirical approach makes use of a historical data base of corporate bond defaults
- Idea is to form a static pool of companies having a particular credit rating for a given year
- Annual default rates are calculated for each static pool, which are then aggregated to compute PD for each credit rating
- Merton's approach is based on an option pricing approach
- This makes use of the current estimates of the firm's assets, liabilities and asset volatility, and hence, is related to the underlying dynamics of the structure of the firm
- KMV approach to determine PD is similar to Merton's approach

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Recovery rate

- When an issuer files for bankruptcy, some fraction of the face value of the debt is recovered
- One could sell the debt to vulture funds when default occurs
- Moody's proxy the recovery rate using secondary market price of the defaulted debt 1-month after default
- If for $100 face value the secondary market trading price is $40, then recovery rate is 40%
- Empirical results suggest that industrial sector, seniority of debt, state of the economy and credit rating of the issuer 1-year prior to default are important determinants of recovery rates

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Recovery rate

- In order to incorporate the variations in recovery rates, recovery rate is treated as a random variable
- Observed recovery rates have a multi-modal distribution
- No consensus on the choice the the right distribution function
- Many existing credit risk models in the market use beta distribution for modelling recovery rates
- Choosing a different distribution function other than beta (say, uniform) will have an impact on the tail part of the loss distribution
- Empirical study on recovery rate statistics has been carried out by Edward Altman and co-workers

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Recovery rate statistics

Recovery rate statistics on defaulted securities (1978-2001)				
Bond seniority	**Number of issuers**	**Median**	**Mean**	**Standard deviation**
Senior secured	134	57.42%	52.97%	23.05%
Senior unsecured	475	42.27%	41.71%	26.62%
Senior subordinated	340	31.90%	29.68%	24.97%
Subordinated	247	31.96%	31.03%	22.53%

Source: E. Altman, A. Resti and A. Sironi, "Analyzing and Explaining Default Recovery Rates", *A report submitted to The International Swaps & Derivatives Association*, December 2001.

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Rating migrations

- Probability of default considers only two possible states for bond issuer: issuer either defaults on debt payments or is solvent
- In practice, actions of rating agencies can result in either the issuer being downgraded or upgraded to a different credit rating
- Associated with each credit rating is a different PD
- One could also estimate the probability of transitioning from one credit rating to another using historical rating migration statistics
- Rating agencies (Moody's and S&P) estimate the migration probabilities for different ratings
- This data is represented in a matrix form, and the matrix is called rating transition (or migration) matrix, which is a Markov matrix

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Rating migration matrix

			Rating at year end					
Initial rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	93.66%	5.83%	0.40%	0.08%	0.03%	0.00%	0.00%	0.00%
AA	0.66%	91.72%	6.94%	0.49%	0.06%	0.09%	0.02%	0.01%
A	0.07%	2.25%	91.75%	5.19%	0.49%	0.20%	0.01%	0.04%
BBB	0.03%	0.25%	4.83%	89.25%	4.44%	0.80%	0.16%	0.23%
BB	0.03%	0.07%	0.44%	6.66%	83.22%	7.46%	1.05%	1.07%
B	0.00%	0.10%	0.32%	0.46%	5.73%	83.61%	3.84%	5.94%
CCC	0.15%	0.00%	0.30%	0.89%	1.91%	10.28%	61.22%	25.25%
Default	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

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Quantifying credit risk

- We are familiar with the return distribution for a stock
- One approximates this return distribution to be normal
- When holding a corporate bond, credit events trigger price changes, which will result in a loss or a profit
- To quantify credit risk, we will focus only on returns that are driven by credit events
- We will refer to the returns that can be related to a credit event generically as a loss
- A rating upgrade will then generate a negative loss
- The credit-related losses will have a distribution, commonly referred to as the credit loss distribution

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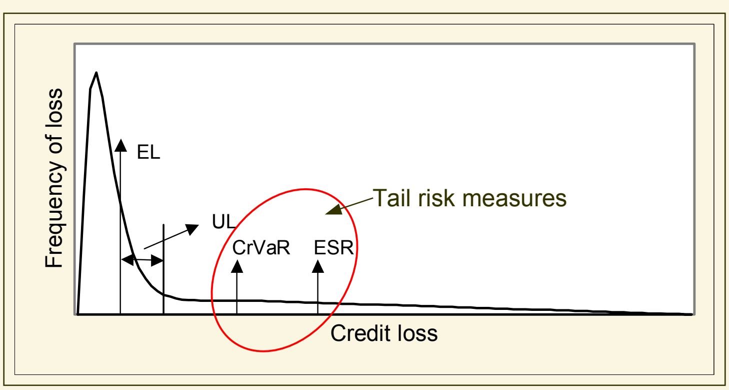
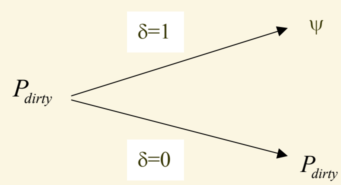
Typical shape of the credit loss distribution



Expected loss under default mode

- Under the default mode, the issuer either repays debt or defaults
- If the issuer defaults, the bondholder recovers a fraction of the face value outstanding
- One can model the default process δ as a Bernoulli random variable that takes the value of 0 or 1 (1 signals default)



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Deriving expected loss (default mode)

- In the default mode framework, the price of the risky debt can be written as follows

$$\tilde{P} = P_{dirty} \times I_{[\delta=0]} + \psi \times I_{[\delta=1]}$$

- Let us denote the credit loss variable as $\tilde{\ell}$

$$\tilde{\ell} = P_{dirty} - \tilde{P} = P_{dirty} - P_{dirty} \times I_{[\delta=0]} - \psi \times I_{[\delta=1]}$$

- Expected loss is the expected value of the random variable $\tilde{\ell}$

$$EL = E(\tilde{\ell}) = P_{dirty} - P_{dirty} \times (1 - PD) - E(\psi \times I_{[\delta=1]})$$

Deriving expected loss (default mode)

- Computing expected loss requires taking the expectation of two random variables, the recovery rate and the default process
- If we assume they are independent, then we have

$$\begin{aligned} EL &= P_{dirty} \times PD - RR \times PD \\ &= PD \times (P_{dirty} - RR) \end{aligned}$$

- Introducing the term loss on default LD=P_{dirty}-RR, we have the expected loss for a nominal exposure of NE

$$EL = NE \times PD \times LD$$

Unexpected loss under default mode

- By definition, unexpected loss is the standard deviation of the random variable $\tilde{\ell}$
- Once again, we will make the assumption that the default and recovery process are independent
- After some mathematical manipulations, it can be shown that the unexpected loss for a nominal exposure NE is given by

$$UL = NE \times \sqrt{PD \times \sigma_{RR}^2 + LD^2 \times \sigma_{PD}^2}$$

- Here, σ_{PD}^2 is the variance of the Bernoulli random variable δ

$$\sigma_{PD}^2 = PD \times (1 - PD)$$

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On the independence assumption

- Is it reasonable to make the assumption that default and recovery processes are independent?
- In Merton's structural model, default and recovery rate processes can be shown to be inversely related to each other
- In reduced-form credit risk models, default and recovery rate processes are modelled to be independent
- Empirical results suggest that these two random variables are negatively correlated
- Recent results suggest that a simple microeconomic interpreta-tion based on supply and demand tend to explain the relationship

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Expected loss under migration mode

- Under migration mode, the changes in credit rating of the issuer will have to taken into account besides default
- To estimate the profit or loss that results from rating migrations, we need to estimate price changes due to rating changes
- We can estimate these price changes if we know the yield spread differences between different credit ratings
- If Δy is the yield spread between two credit ratings, the corres-ponding price change can be approximated by,

$$Price\ change = -P_{dirty} \times D \times \Delta y + 0.5 \times P_{dirty} \times C \times \Delta y^2$$

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Expected loss under migration mode

- Since we are interested in the credit loss distribution, the loss resulting from a credit rating change will be given by,

$$Loss \equiv \Delta P = P_{dirty} \times D \times \Delta y - 0.5 \times P_{dirty} \times C \times \Delta y^2$$

- The expected value of the credit loss for a rating change from the *i*th grade to the *k*th grade is given by,

$$\Delta P_{ik} = P_{dirty} \times D \times \Delta y_{ik} - 0.5 \times P_{dirty} \times C \times \Delta y_{ik}^2$$

- If we model 18 rating grades including the default state, the expected loss under rating migration is given by

$$EL = NE \times \sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}$$

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Unexpected loss under migration mode

- By definition, unexpected loss under migration mode is the standard deviation of the credit loss variable
- The loss variable under migration mode for a $1 exposure is

$$\tilde{\ell} = \sum_{k=1}^{18} p_{ik} \times \Delta \tilde{P}_{ik}$$

- Here, $\Delta \tilde{P}_{ik}$ is the random credit loss when issuer rating changes from grate *i* to grade *k*
- The expected value of $\Delta \tilde{P}_{ik}$ is ΔP_{ik} and variance σ_{ik}^2
- When *k* is equal to the default state, σ_{ik} is equal to σ_{RR}

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Unexpected loss under migration mode

- The variance of the loss variable for $1 exposure is given by,

$$Var(\tilde{\ell}) = E\left(\sum_{k=1}^{18} p_{ik} \times \Delta\tilde{P}_{ik}^2\right) - \left[E\left(\sum_{k=1}^{18} p_{ik} \times \Delta\tilde{P}_{ik}\right)\right]^2$$

- Taking expectations and assuming the recovery rate process and default process are independent, we get the following relation:

$$Var(\tilde{\ell}) = \sum_{k=1}^{18} p_{ik} \times (\Delta P_{ik}^2 + \sigma_{ik}^2) - \left[\sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}\right]^2$$

- For nominal exposure NE, it can be shown that UL is given by,

$$UL = NE \times \sqrt{PD \times \sigma_{RR}^2 + \sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}^2 - \left[\sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}\right]^2}$$

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Computing yield spreads

- Yield spreads between different rating grades can be derived from current market prices
- One could have the yield spreads across different rating grades by maturity
- A simpler approach, through less precise, would be to use yield spreads across rating grades without taking into account maturity of the instrument
- The table on next page shows static yield spreads that have been used in numerical examples

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Rating grade	Rating description	Spread vs Govt
1	Aaa / AAA	15 bp
2	Aa1 / AA+	30 bp
3	Aa2 / AA	45 bp
4	Aa3 / AA-	60 bp
5	A1 / A+	75 bp
6	A2 / A	90 bp
7	A3 / A-	105 bp
8	Baa1 / BBB+	130 bp
9	Baa2 / BBB	155 bp
10	Baa3 / BBB-	180 bp
11	Ba1 / BB+	230 bp
12	Ba2 / BB	280 bp
13	Ba3 / BB-	330 bp
14	B1 / B+	430 bp
15	B2 / B	530 bp
16	B3 / B-	630 bp
17	Caa-C / CCC	780 bp

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Numerical example

Security level details of example considered	
Description	**Value**
Issuer rating grade	A3
Settlement date	24 Apr 2002
Bond maturity date	15 Feb 2007
Coupon rate	6.91%
Dirty price for $1 nominal	1.0533
Nominal exposure	$1,000,000
Modified duration	4.021
Convexity	19.75
Mean recovery rate	47%
Volatility of RR	25%

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- Under default mode,

$$EL = NE \times PD \times LD$$
$$= 1000000 \times 0.001 \times 0.5833 = \$\ 583.3$$

$$UL = NE \times \sqrt{PD \times \sigma_{RR}^2 + LD^2 \times \sigma_{PD}^2}$$
$$= 1000000 \times \sqrt{0.001 \times 0.25^2 + 0.5833^2 \times 0.001 \times (1 - 0.001)} = \$\ 20,059.88$$

- Under migration mode,

$$EL = NE \times \sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}$$
$$= 1000000 \times 0.003012 = \$\ 3012$$

$$UL = NE \times \sqrt{PD \times \sigma_{RR}^2 + \sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}^2 - \left[\sum_{k=1}^{18} p_{ik} \times \Delta P_{ik}\right]^2}$$
$$= 1000000 \times \sqrt{0.001 \times 0.25^2 + 5.259 \times 10^{-4} - 0.003012^2} = \$\ 24,069.2$$